Holography and anomaly cancellation in gauge-Higgs unification

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Universality in anomaly flow
by Y. Hosotani
PTEP 2022, 073B01 (2205.00154)

Anomaly flow by an Aharonov-Bohm phase
by Funatsu, Hatanaka, Hosotani, Orikasa, Yamatsu
PTEP 2022, 043B04 (2202.01393)

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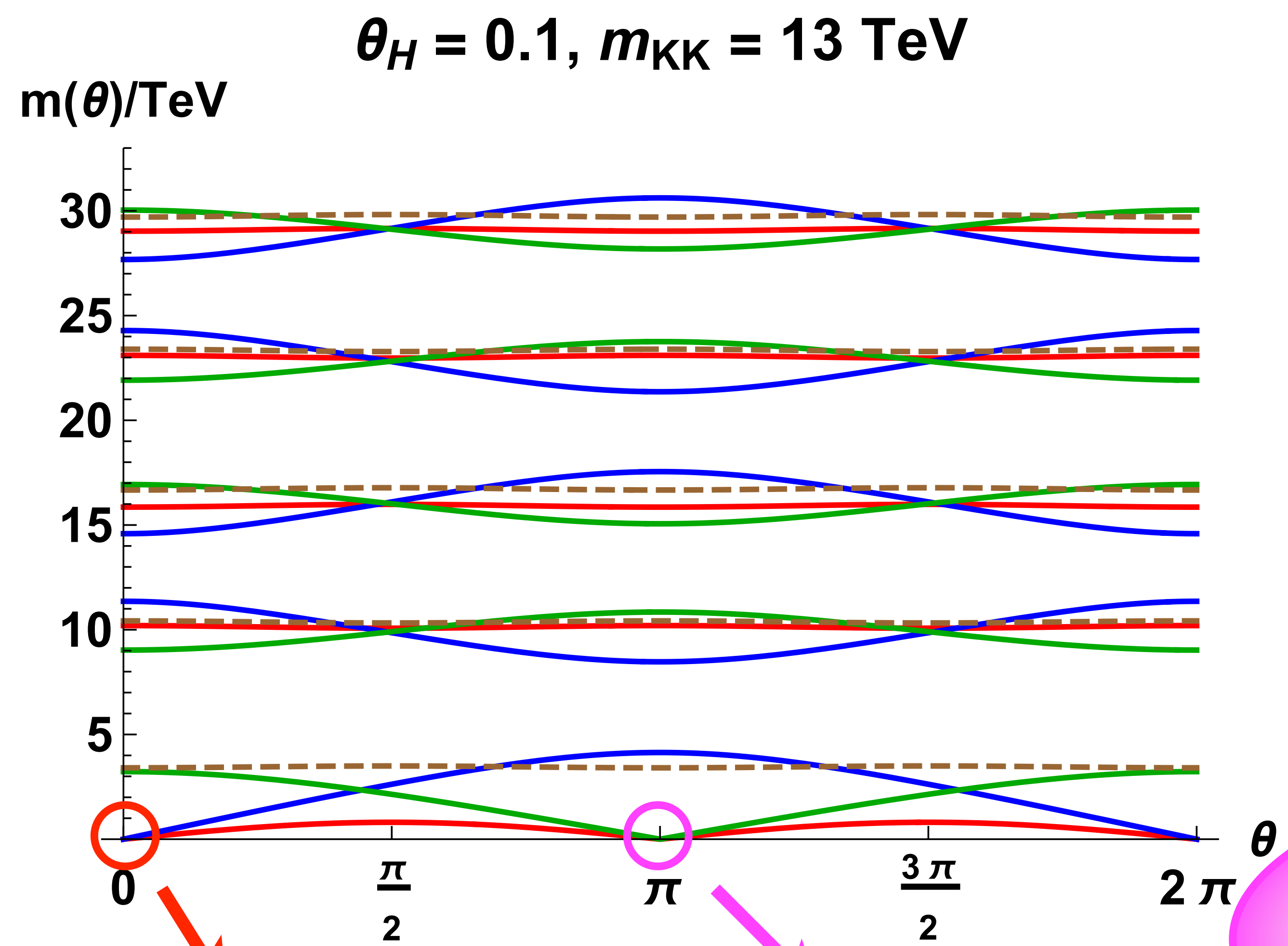
Introduction

Chiral fermion ⟺ **Chiral anomaly**



$SU(2)_L \times U(1)_Y$ → $SU(2)_R \times U(1)_{Y'}$

top: chiral ⟶ vectorlike

GUT inspired
$SU(3) \times SO(5) \times U(1)$
GHU in RS

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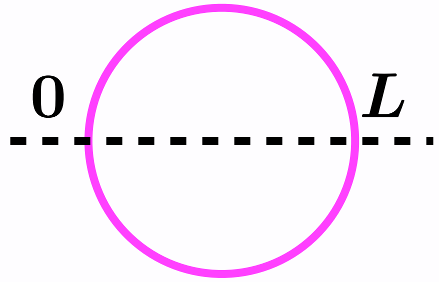
What is the fate of chiral anomaly?

$SU(2)$ GHU in $M^4 \times (S^1/Z_2)$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^{-1}$$



$$\Psi(x, y_j - y) = P_j \gamma^5 \Psi(x, y_j + y)$$

$$(y_0, y_1) = (0, L)$$

$$P_0 = P_1 = \tau^3 \qquad SU(2) \Rightarrow U(1)$$

AB phase θ_H

$$P e^{i g_A \int_0^{2L} dy \langle A_y \rangle} = e^{i \theta_H \tau^2}$$

$$\begin{pmatrix} A^1_\mu \\ A^3_\mu \end{pmatrix} = \sum_{n=-\infty}^{\infty} B^{(n)}_\mu(x) \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \sin \frac{ny}{R} \\ \cos \frac{ny}{R} \end{pmatrix}$$

$$m_{B^{(n)}} = \frac{1}{R} \left| n + \frac{\theta_H}{\pi} \right|$$

Massless mode

θ_H	
$\theta_H = 0$	$B^{(0)}_\mu$
π	$B^{(-1)}_\mu$
2π	$B^{(-2)}_\mu$

Spectrum

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} = \sum_n \psi_R^{(n)}(x) \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \cos \frac{ny}{R} \\ \sin \frac{ny}{R} \end{pmatrix}$$

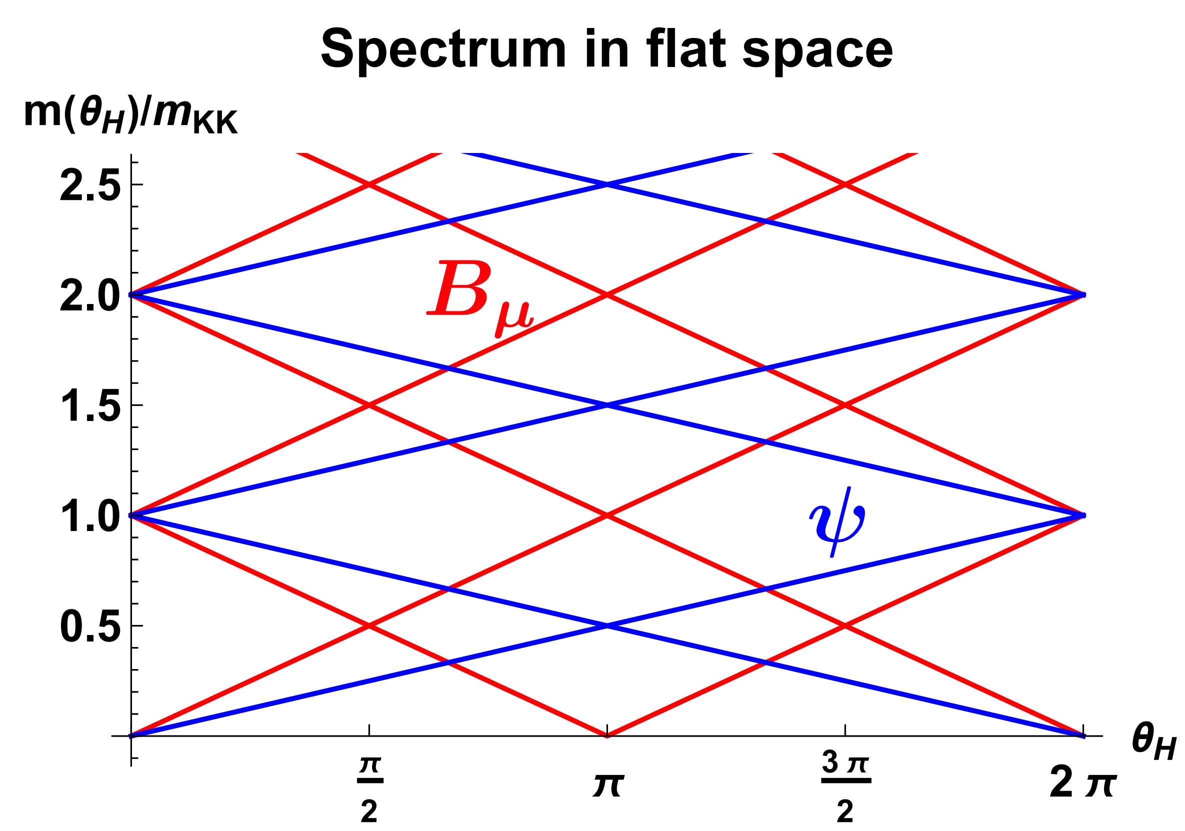
$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = \sum_n \psi_L^{(n)}(x) \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \sin \frac{ny}{R} \\ -\cos \frac{ny}{R} \end{pmatrix}$$

$$m_{\psi^{(n)}} = \frac{1}{R} \left| n + \frac{\theta_H}{2\pi} \right|$$

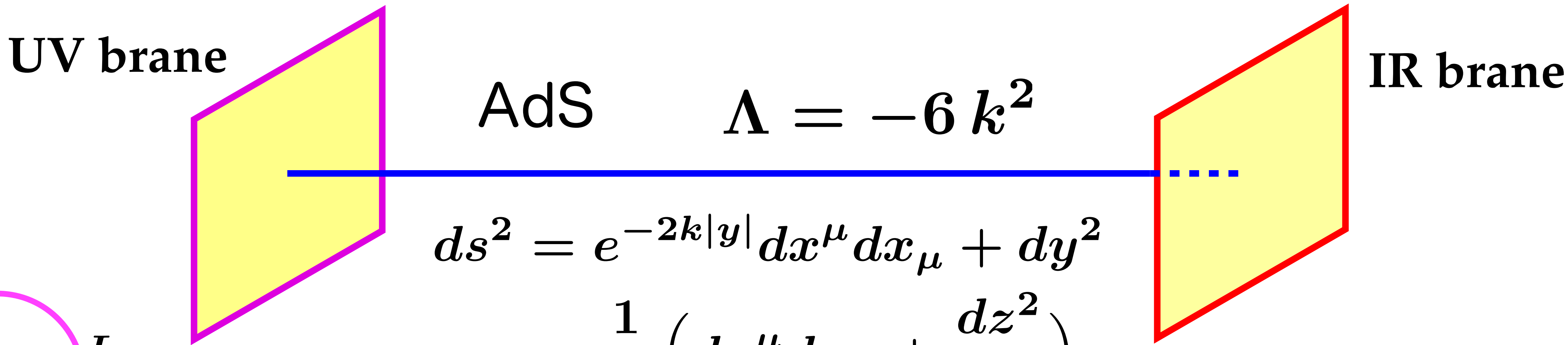
Massless mode

$\theta_H = 0 : \quad u_R \, , \, d_L$

chiral

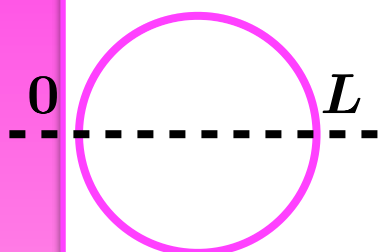


$SU(2)$ GHU in Randall-Sundrum



$$ds^2 = e^{-2k|y|} dx^\mu dx_\mu + dy^2$$

$$= \frac{1}{z^2}\Big(dx^\mu dx_\mu + \frac{dz^2}{k^2} \Big)$$



$$z = e^{ky} \ , \ 1 \le z \le z_L = e^{kL} \ , \ 0 \le y \le L$$

$$\begin{pmatrix} A_\mu^1 \\ A_\mu^3 \end{pmatrix} = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} Z_\mu^{(n)}(x) \begin{pmatrix} h_n(y) \\ k_n(y) \end{pmatrix}$$

$$\check{\Psi} = \frac{1}{z^4}\Psi$$

$$\begin{pmatrix} \check{u}_R \\ \check{d}_R \end{pmatrix} = \sqrt{k} \sum_{n=0}^{\infty} \chi_R^{(n)}(x) \begin{pmatrix} f_{Rn}(y) \\ g_{Rn}(y) \end{pmatrix}$$

$$\begin{pmatrix} \check{u}_L \\ \check{d}_L \end{pmatrix} = \sqrt{k} \sum_{n=0}^{\infty} \chi_L^{(n)}(x) \begin{pmatrix} f_{Ln}(y) \\ g_{Ln}(y) \end{pmatrix}$$

Spectrum

$$Z_\mu^{(n)} \; : \; SC'(1;\lambda_n) + \lambda_n \sin^2\theta_H = 0$$

$$\chi^{(n)} \; : \; S_L S_R(1;\lambda_n, c) + \sin^2\frac{\theta_H}{2} = 0$$

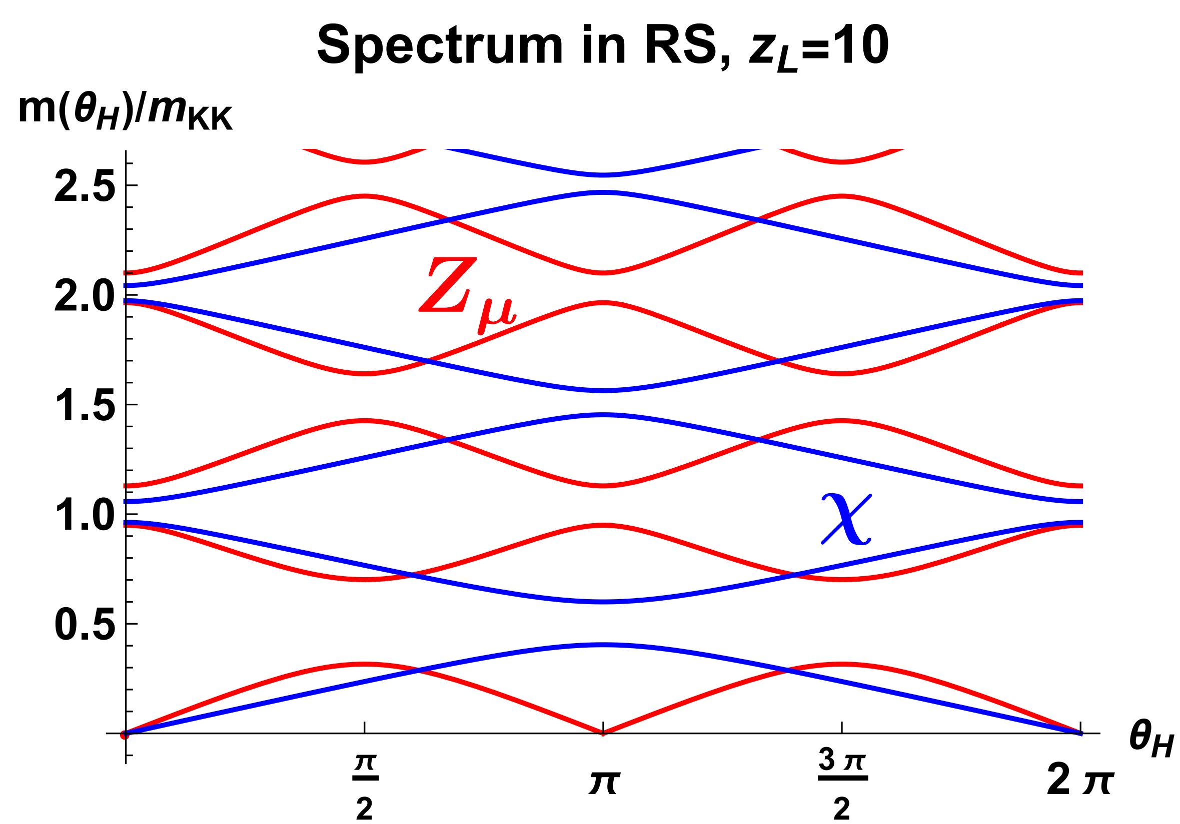
$(m_n = k\lambda_n)$

$$C(z;\lambda) = \frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L)$$

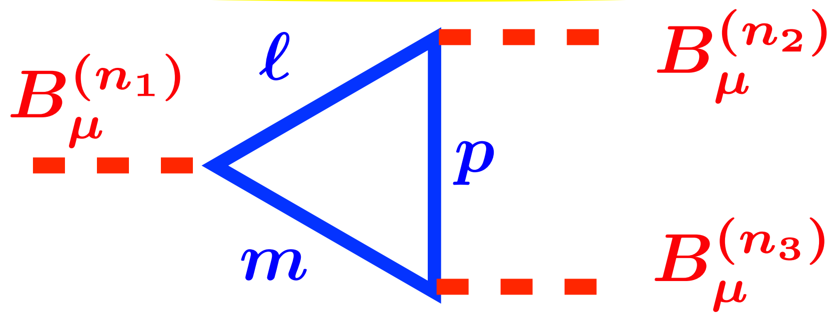
$$S(z;\lambda) = -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L)$$

$$F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v)$$

$$\begin{pmatrix} C_L \\ S_L \end{pmatrix}(z;\lambda,c) = \pm\frac{\pi}{2}\lambda\sqrt{z z_L}F_{c+\frac{1}{2},c\mp\frac{1}{2}}(\lambda z, \lambda z_L)$$



Gauge couplings and anomaly in flat space



$$\frac{g_4}{2} \sum_{n,m,\ell=-\infty}^{\infty} B_\mu^{(n)} \Big\{ s^R_{nm\ell}\, \psi_R^{(m)\dagger} \bar{\sigma}^\mu \psi_R^{(\ell)} + s^L_{nm\ell}\, \psi_L^{(m)\dagger} \sigma^\mu \psi_L^{(\ell)} \Big\}$$

$\delta_{n,m+\ell}$ $\qquad$ $\delta_{n,m+\ell}$

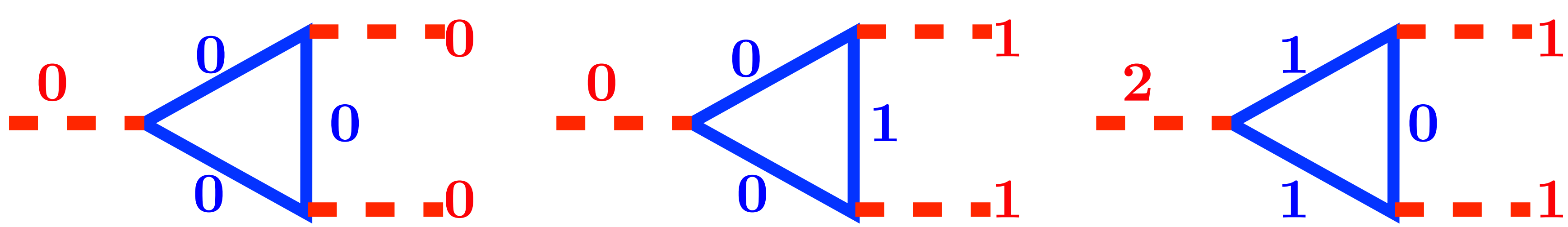
Anomaly coefficient

$$\partial_\mu j^\mu_{(n)} \Rightarrow -\left(\frac{g_4}{2}\right)^3 \sum_\ell \sum_m \frac{b_{n\ell m}}{16\pi^2} B^{(\ell)}_{\mu\nu} \widetilde{B}^{(m)\mu\nu}$$

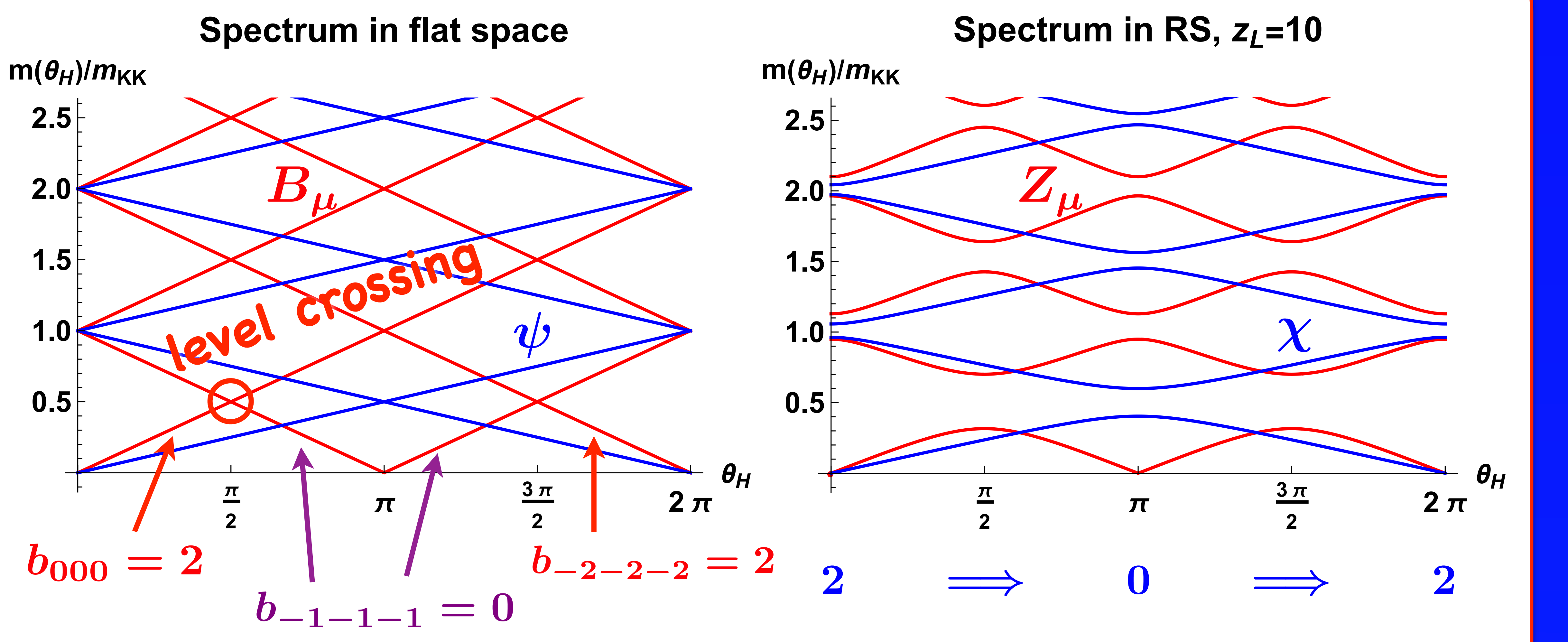
$$b_{n_1 n_2 n_3} = \sum_{m,\ell,p} \Big\{ s^R_{n_1 m\ell} s^R_{n_2 \ell p} s^R_{n_3 pm} + s^L_{n_1 m\ell} s^L_{n_2 \ell p} s^L_{n_3 pm} \Big\}$$

$$= \begin{cases} 2 \\ 0 \end{cases} \text{for } n_1 + n_2 + n_3 = \begin{cases} \text{even} \\ \text{odd} \end{cases}$$

θ_H-independent ?



Anomaly flow



Anomaly must flow with θ_H !

Couplings in RS

$$\frac{g_4}{2}\sum_{n,\ell,m=0}^{\infty} Z_\mu^{(n)}\Big\{t^R_{n\ell m}\,\chi_R^{(\ell)\dagger}\bar{\sigma}^\mu\chi_R^{(m)} + t^L_{n\ell m}\,\chi_L^{(\ell)\dagger}\sigma^\mu\chi_L^{(m)}\Big\}$$

$$t^R_{n\ell m} = \frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\Big\{h_n\big(f^*_{R\ell}g_{Rm} + g^*_{R\ell}f_{Rm}\big) + k_n\big(f^*_{R\ell}f_{Rm} - g^*_{R\ell}g_{Rm}\big)\Big\}$$

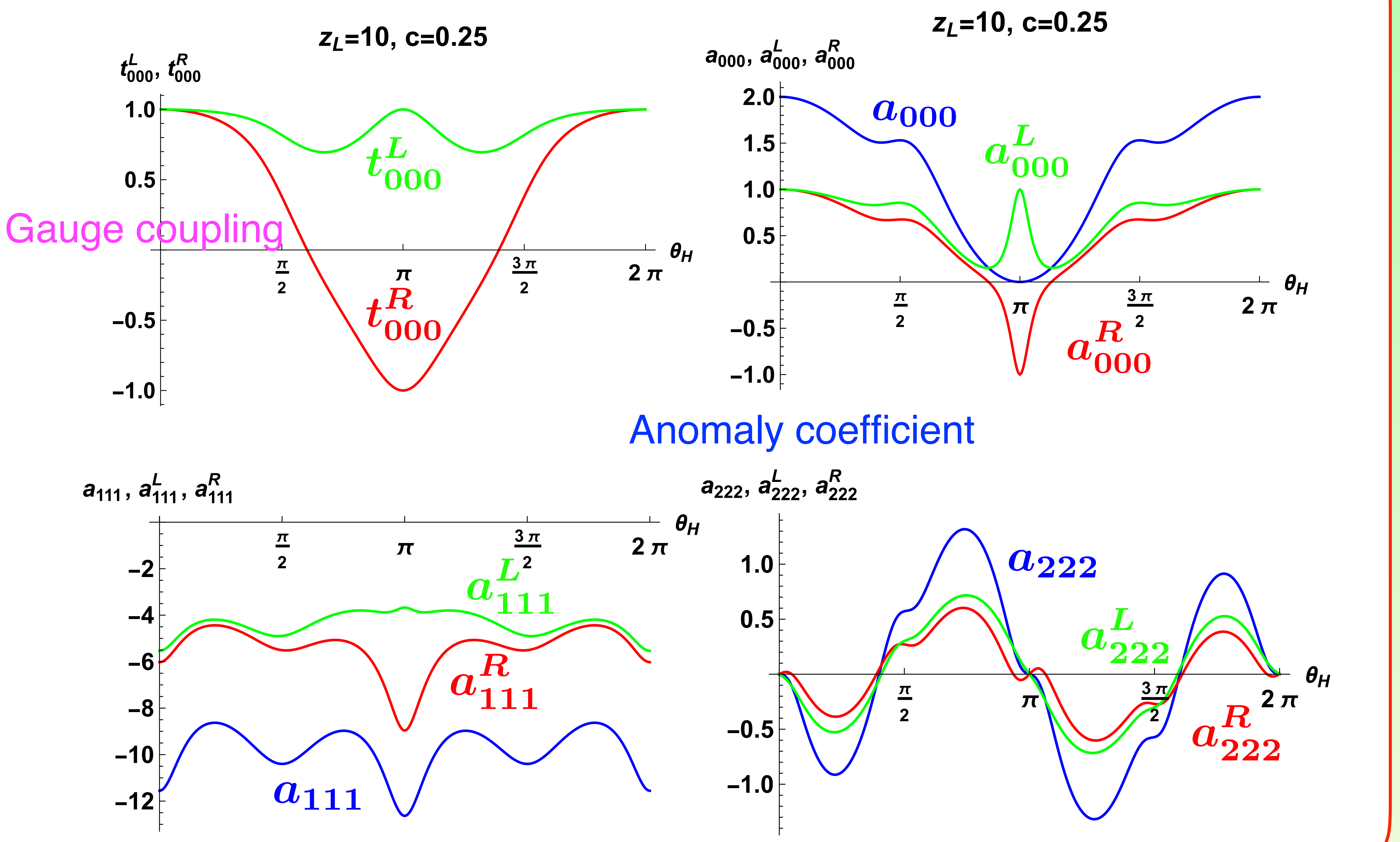
$$t^L_{n\ell m} = -\frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\Big\{\ f_{Rm}\to f_{Lm}\ ,\ g_{Rm}\to g_{Lm}\ \Big\}$$

Anomaly coefficient

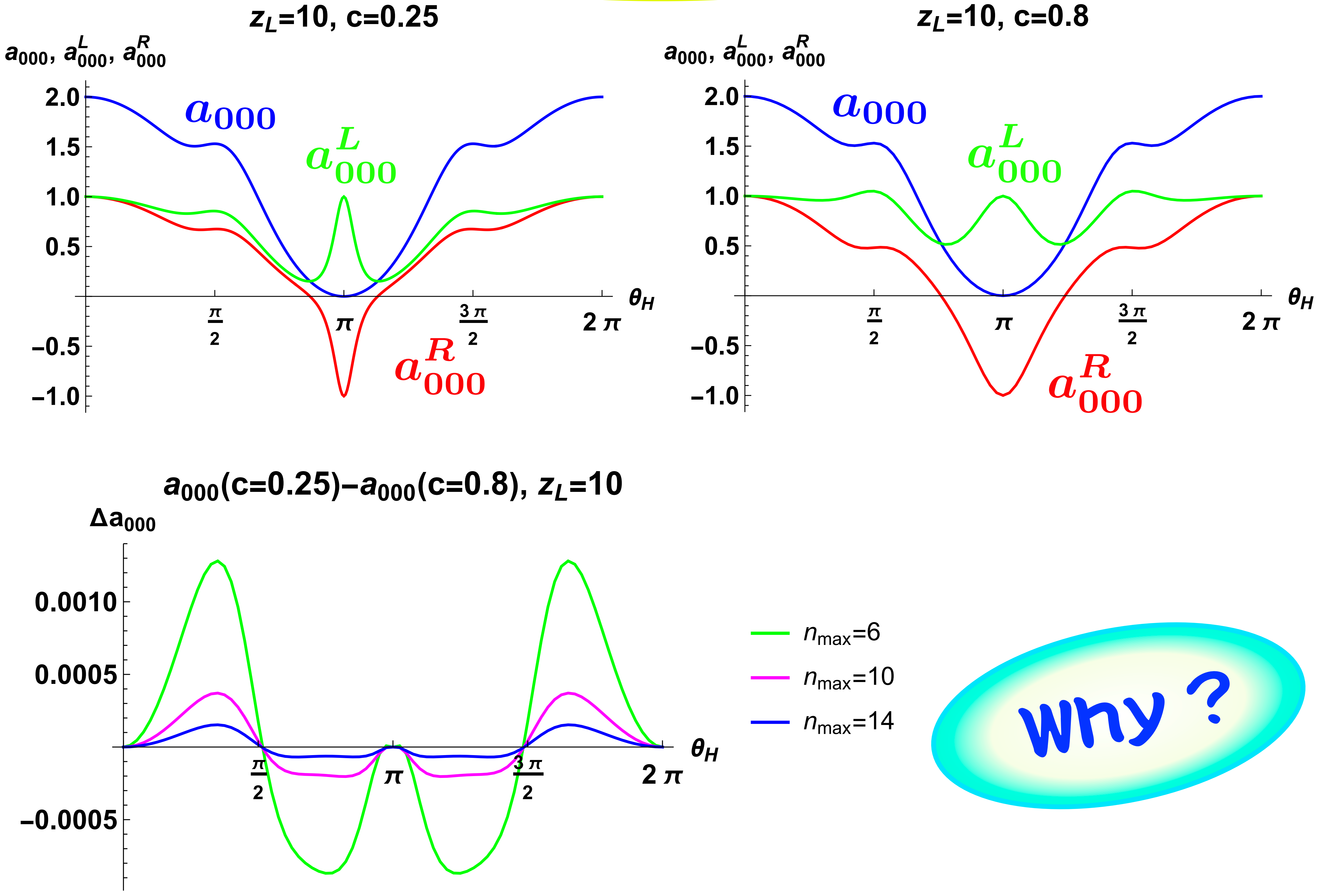
$$a_{n_1n_2n_3} = \sum_{m,\ell,p}\Big\{t^R_{n_1m\ell}t^R_{n_2\ell p}t^R_{n_3pm} + t^L_{n_1m\ell}t^L_{n_2\ell p}t^L_{n_3pm}\Big\}$$

Note: $t^{R/L}_{n\ell m} = t^{R/L}_{n\ell m}(\theta_H, z_L, c)$

Anomaly flows !



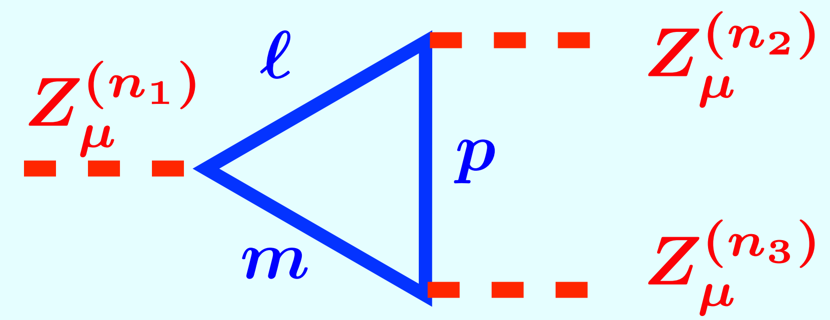
c-independence



Why ?

Holography in anomaly flow

Anomaly coefficient



$$a_{n_1 n_2 n_3} = \sum_{m,\ell,p} \left\{ t^R_{n_1 m\ell} t^R_{n_2 \ell p} t^R_{n_3 pm} + t^L_{n_1 m\ell} t^L_{n_2 \ell p} t^L_{n_3 pm} \right\}$$

Method 1 (i) Evaluate $t^{R/L}_{nm\ell}$. (ii) Do $\sum_{m,\ell,p}$.

Method 2 (i) Do $\sum_{m,\ell,p}$. (ii) Do $\int dy_1 dy_2 dy_3$.

Anomaly formula

$$a_{n\ell m}(\theta_H, z_L) = Q_0 k_n(0) k_\ell(0) k_m(0) + Q_1 k_n(L) k_\ell(L) k_m(L)$$

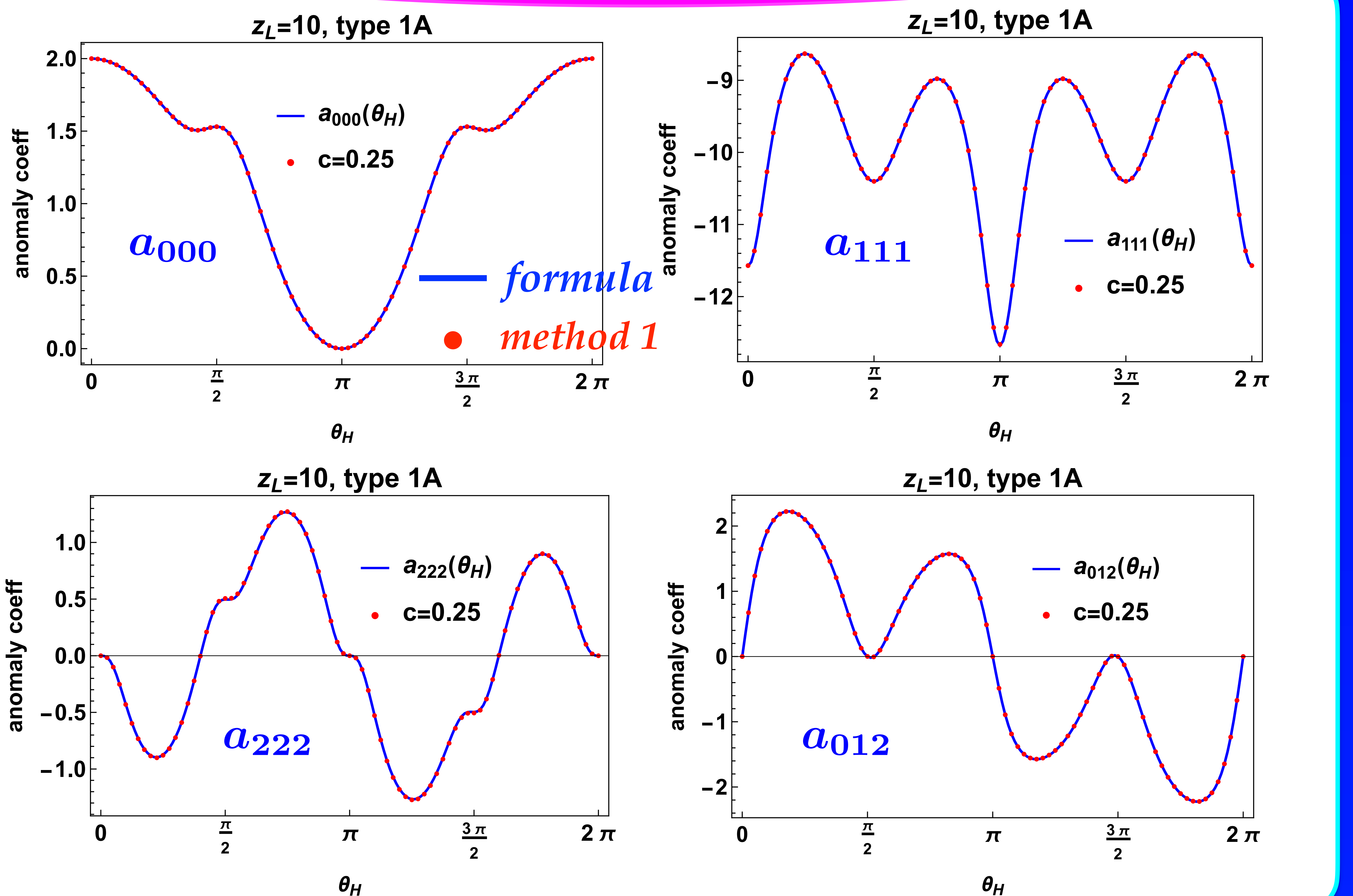
$$(Q_0, Q_1) = \begin{cases} (+1,+1) & \text{for type 1A} \\ (-1,-1) & \text{for type 1B} \\ (+1,-1) & \text{for type 2A} \\ (-1,+1) & \text{for type 2B} \end{cases}$$

(P_0, P_1) for Ψ_R

Determined by
wave functions of gauge fields at y=0, L,
and BC of fermions.

Holography

Anomaly flow and holography



Anomaly cancellation

$$\partial_\mu j^\mu_{(n)} \Rightarrow -\left(\frac{g_4}{2}\right)^3 \sum_\ell \sum_m \frac{a_{n\ell m}}{16\pi^2} Z^{(\ell)}_{\mu\nu} \widetilde{Z}^{(m)\mu\nu}$$

$$a_{n\ell m}(\theta_H, z_L) = Q_0 k_n(0) k_\ell(0) k_m(0) + Q_1 k_n(L) k_\ell(L) k_m(L)$$

$$(Q_0, Q_1) = \begin{cases} (+1,+1) & \text{for type 1A} \\ (-1,-1) & \text{for type 1B} \\ (+1,-1) & \text{for type 2A} \\ (-1,+1) & \text{for type 2B} \end{cases}$$

All anomalies are cancelled at once if

$$n_{1A} = n_{1B} \ , \ n_{2A} = n_{2B}$$

independent of θ_H

More generally if

$$(n_{1A} - n_{1B}) + (n_{2A} - n_{2B}) + (\hat{n}_R - \hat{n}_L) = 0 \ ,$$

$$(n_{1A} - n_{1B}) - (n_{2A} - n_{2B}) = 0$$

↑ brane fermions at $y = 0$

Summary

Chiral anomaly flows by θ_H

In RS, no level crossing,

smooth variation of $a_{n\ell m}(\theta_H)$

Chiral fermion - vectorlike fermion transition

Holography : anomaly formula

$$a_{n\ell m}(\theta_H, z_L)$$
$$= Q_0 k_n(0) k_\ell(0) k_m(0) + Q_1 k_n(L) k_\ell(L) k_m(L)$$

Anomaly cancellation in GHU

Flat space limit

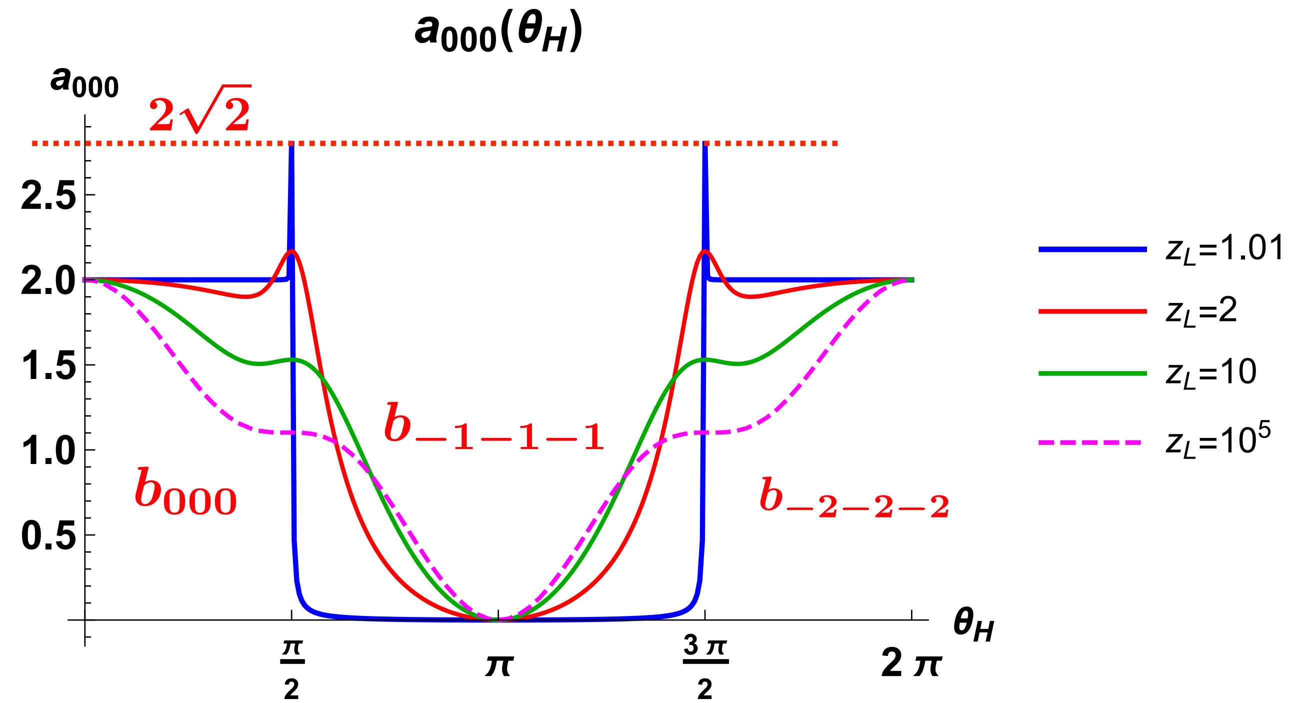
flat $M^4 \times (S^1/Z_2)$ $\qquad 0 \le y \le L = \pi R$

$$\text{RS:}\ \ ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + \frac{dz^2}{k^2}\right) \quad (1 \le z \le z_L = e^{kL})$$

$$z = e^{ky}, \quad \text{AdS curvature} = -6k^2 \qquad m_{\rm KK} = \frac{\pi k}{z_L - 1} \quad \Longrightarrow \quad \frac{1}{R}$$

$$b_{n\ell m} = Q_0 k_n^{\rm flat}(0) k_\ell^{\rm flat}(0) k_m^{\rm flat}(0) + Q_1 k_n^{\rm flat}(L) k_\ell^{\rm flat}(L) k_m^{\rm flat}(L)$$

$$k_n^{\rm flat}(y) = \cos\frac{n\pi y}{L} \quad \Rightarrow \quad b_{n\ell m} = Q_0 + (-1)^{n+\ell+m} Q_1$$



$$a_{n_1 n_2 n_3} = \left(\frac{k}{2}\right)^3 \int\int\int_{-a}^{2L-a} dy_1 dy_2 dy_3 \, e^{\sigma(y_1)+\sigma(y_2)+\sigma(y_3)}$$

$$\times\Big[\, k_1 k_2 k_3 \{ A_R(1,2) A_R(2,3) A_R(3,1) - B_R(1,2) B_R(2,3) B_R(3,1)$$

$$+ B_L(1,2) B_L(2,3) B_L(3,1) - A_L(1,2) A_L(2,3) A_L(3,1) \}$$

$$+ k_1 h_2 h_3 \{ \ \cdots \ \} + h_1 k_2 h_3 \{ \ \cdots \ \} + h_1 h_2 k_3 \{ \ \cdots \ \} \Big]$$

$$k_j = k_{n_j}(y_j) \ , \quad h_j = h_{n_j}(y_j)$$

$$\begin{pmatrix} A_R(j,k) \\ B_R(j,k) \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} f_{Rn}(y_j) f_{Rn}^*(y_k) \\ g_{Rn}(y_j) g_{Rn}^*(y_k) \end{pmatrix}$$

Gauge wave functions $k_n(0), k_n(L)$