素粒子現象論研究会2022

2023/03/16

S^1/Z_2オービフォールド上の $SU(N)$ ゲージ理論における同値類の網羅的分析

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Based on: KT, T. Inagaki, arXiv:2301.12938 [hep-th]

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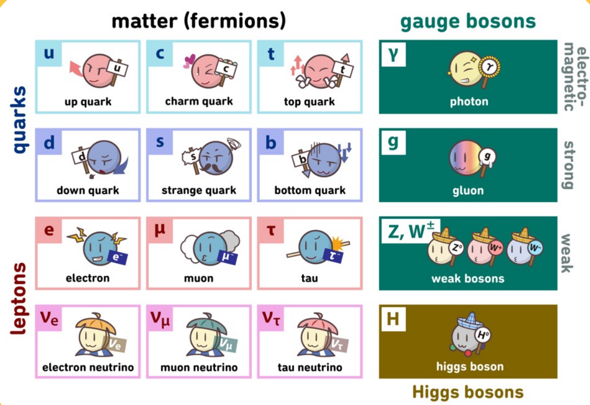
01

Introduction

Extra dimensions

higgstan.com



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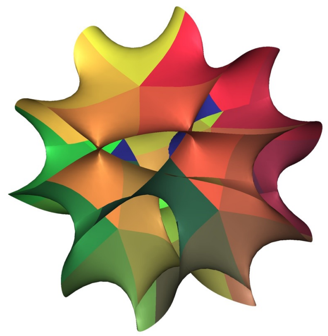


SM: 4-dimension

String Theory: 10-dimension

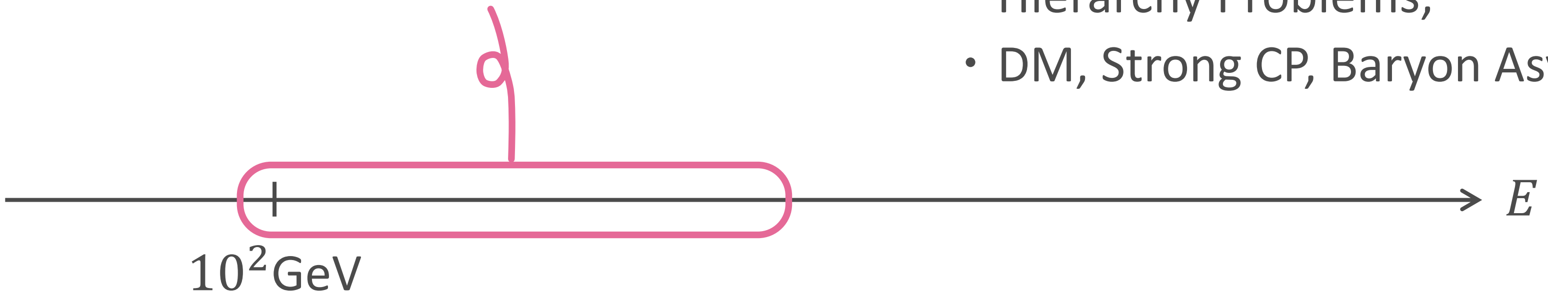
E

10^2GeV

10^{19}GeV

Extra dimensions

Scenario beyond the SM:
4-dimension + Extra-Dimension?



models

- Gauge Higgs Unification (GHU),
- Large Extra Dimension (LED),
- Randall Sundrum (RS), etc.

approach

- Hierarchy Problems,
- DM, Strong CP, Baryon Asym, etc.

Gauge Higgs Unification

Fairlie (1979)
Manton (1979)
Hosotani (1983)

Embedding Higgs in extra-dimensional components of gauge fields

$$x^M = (x^\mu, y) \quad A_M = (A_\mu, A_y) \qquad (\mu = 0,1,2,3)$$

$$\mathcal{L}_{SM}^{4D} = gauge + fermion + Higgs + Yukawa$$

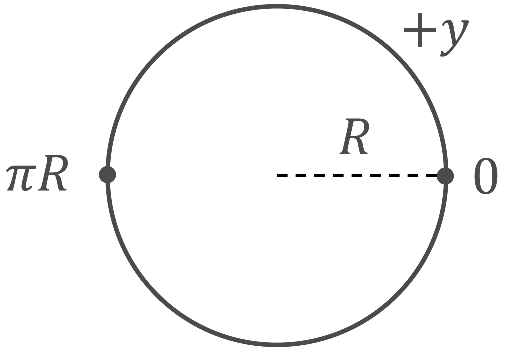
⇩

$$\mathcal{L}_{GHU}^{5D} = gauge + fermion$$

- Reduced parameters and high predictability
- Naturally solve Hierarchy Problem

Boundary Conditions (BCs)

S^1 compactification



S^1: $y \sim y + 2\pi R$

infinite interval: $y \in (-\infty, +\infty)$

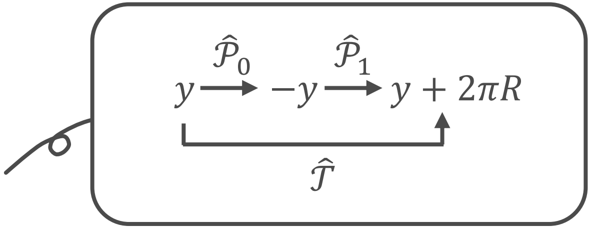
$$\phi(y = \pm\infty) = \partial\phi(y = \pm\infty) = 0$$

finite interval: $y \in [\,0, 2\pi R)$

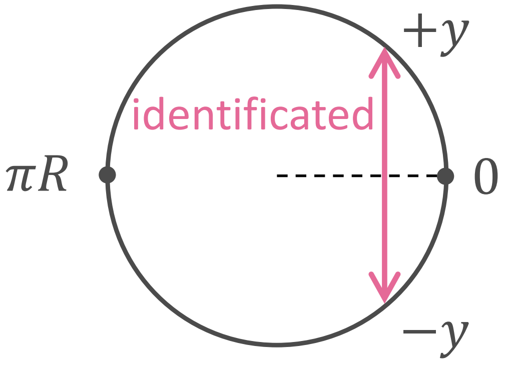
$$\phi(y = 0) = +\phi(y = 2\pi R)?$$
$$\phi(y = 0) = -\phi(y = 2\pi R)?$$
$$\phi(y = 0) = e^{i\theta}\phi(y = 2\pi R)?$$

There are many candidates for BCs.

S^1/Z_2 Orbifold



S^1/Z_2 compactification



S^1: $y \sim y + 2\pi R$

Z_2: $y \sim -y$

$SU(N)$ gauge theory with S^1/Z_2 orbifold

BCs: two parity transformations

$$\begin{cases} A_\mu(x, 0 - y) = P_0 A_\mu(x, 0 + y) P_0 \\ A_\mu(x, \pi R - y) = P_1 A_\mu(x, \pi R + y) P_1 \end{cases}$$

P_i $(i = 0{,}1)$ are any $N \times N$ matrices satisfying,

$$P_i^2 = 1, \quad P_i^\dagger = P_i$$

There are many candidates for BCs.

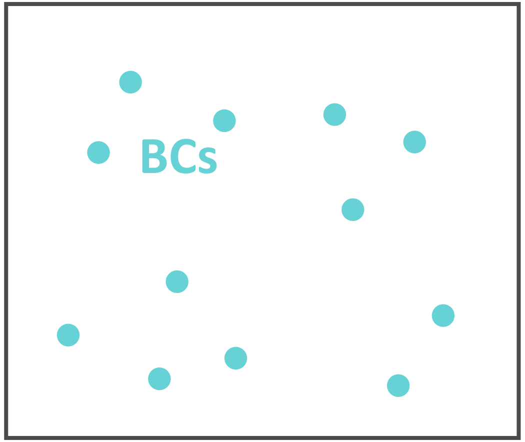
the Arbitrariness Problem of BCs

Different BCs produce different symmetry.

e.g.) SU(3)model

	P_0	P_1	symmetry
	$(+1,+1,+1)$	$(+1,+1,+1)$	$SU(3)$
diag	$(+1,+1,-1)$	$(+1,+1,-1)$	$SU(2)\times U(1)$
	$(+1,+1,-1)$	$(+1,-1,+1)$	$U(1)\times U(1)$
	$(+1,+1,-1)$	$(+1,\sigma_2)$	$U(1)$

Which $(\boldsymbol{P_0},\boldsymbol{P_1})$?



Candidates:

Which BCs should be selected? :

the Arbitrariness Problem of BCs

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y),$$
$$\partial_M P_i' = 0, \quad P_i'^\dagger = P_i'$$

Equivalence Classes (ECs)

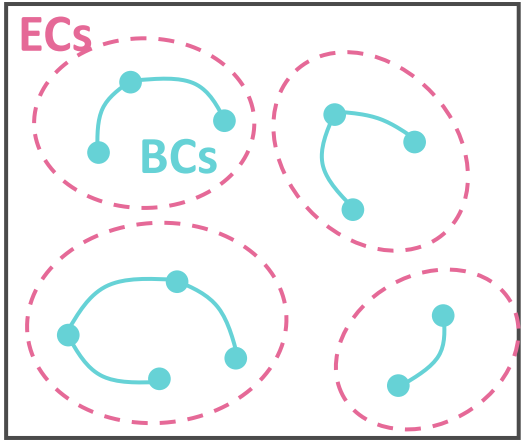
Hosotani(1989)
Haba et al.(2003)
Haba et al.(2004)

Some of BCs are connected by a particular gauge transformation:

→ Equivalence Classes (ECs)

$$(P_0, P_1) \sim (P_0', P_1')$$

Which ECs?



The number of ECs in $SU(N)$ on S^1/Z_2: $(N+1)^2$

$$_{N+3}C_3 - \sum_{k=0}^{N-2} {}_{k+1}C_1 \, {}_{N-k-1}C_1 = (N+1)^2$$

The remaining arbitrariness:

Candidates:

Which of the $(N+1)^2$ ECs should be selected?

Our Work

KT, T. Inagaki, arXiv:2301.12938 [hep-th]

$$\partial_M P_i' = 0, \;\; P_i'^{\dagger} = P_i'$$

Are there really $(N+1)^2$ a lot of ECs?

$$(P_0, P_1) \sim (P_0', P_1')$$

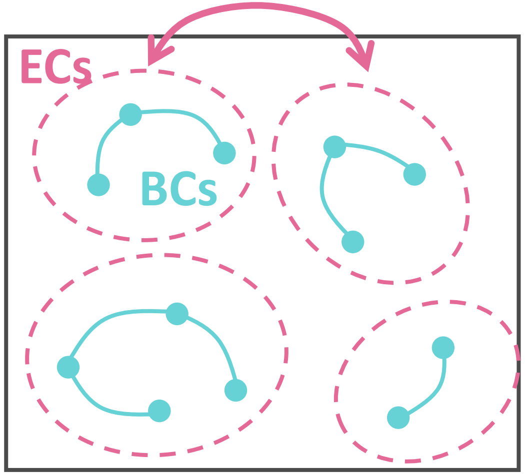
Previous:

$$P_i' = \Omega(x, y_i - y) P_i \Omega^{\dagger}(x, y_i + y), \quad y_0 = 0, \;\; y_1 = \pi R$$

Gauge transf. $\Omega(y) = \exp\left[i y c^a T^a\right]$

T^a: generators
c^a : constants

Not connected more?



Candidates:

→ Analyzing comprehensively by: $\Omega(y) = \exp\left[i f^a(y) T^a\right]$

02

Techniques for Analysis

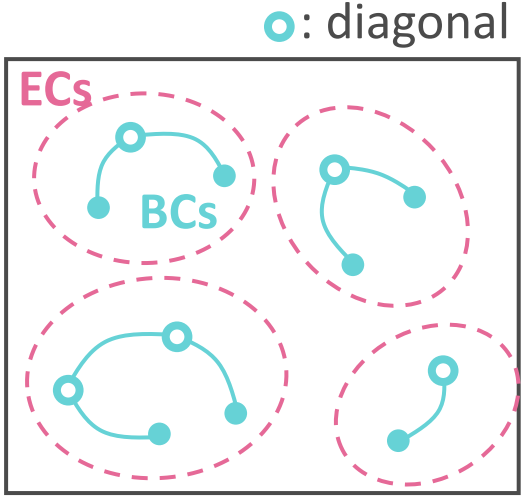
Haba et al.(2004)
Kawamura et al.(2020)

Simultaneous Diagonalizable

Tech. 1

Always simultaneous diagonalizable on S^1/Z_2

$$(P_0, P_1) \sim (P_0^{\rm diag}, P_1^{\rm diag})$$



$$(P_0, P_1)^{\rm diag} \xrightarrow{?} (P_0', P_1')^{\rm diag}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y),$$

$$\Omega(y) = \exp\left[i f^a(y) T^a\right]$$

Just examine the one from diagonal to diagonal

$y_0 = 0, \quad y_1 = \pi R$

Kawamura et al.(2020)

Classification of Generators

Tech. 2

Classify $U(N)$ generators into 4-types

$$\{T^a\} = \{T^{a++}\} \oplus \{T^{a+-}\} \oplus \{T^{a-+}\} \oplus \{T^{a--}\}$$

Exchangeability with P_i :

P_0 \ P_1	commute	anti-commute
commute	$T^{a_{++}}$	$T^{a_{+-}}$
anti-commute	$T^{a_{-+}}$	$T^{a_{--}}$

$$(P_0, P_1)^{\rm diag} \overset{?}{\to} (P_0', P_1')^{\rm diag}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y),$$

$$\Omega(y) = \exp[i f^a(y) T^a]$$

Requirements for building ECs:

commutative : $f^a(y_i + y) - f^a(y_i - y) = \text{const.}$

anti-commutative: $f^a(y_i + y) + f^a(y_i - y) = \text{const.}$

Just examine, $P_i' = e^{-i(f^a \mp f^a)T^a} P_i$

03

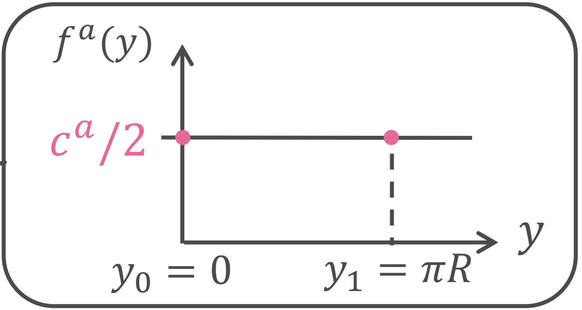
Results

Result:

for Global parameters

$$P' = e^{-i(f^a \mp f^a)T^a} P$$

$$f^a(y) = c^a/2 \quad c^a\text{: constant}$$



com. $f^a(y_i + y) - f^a(y_i - y) = 0$

anti. $f^a(y_i + y) + f^a(y_i - y) = c^a$

$(y_0 = 0, y_1 = \pi R)$

$T^{+-} = c^{a_{+-}} T^{a_{+-}}$

$T^{--} = c^{a_{--}} T^{a_{--}}$

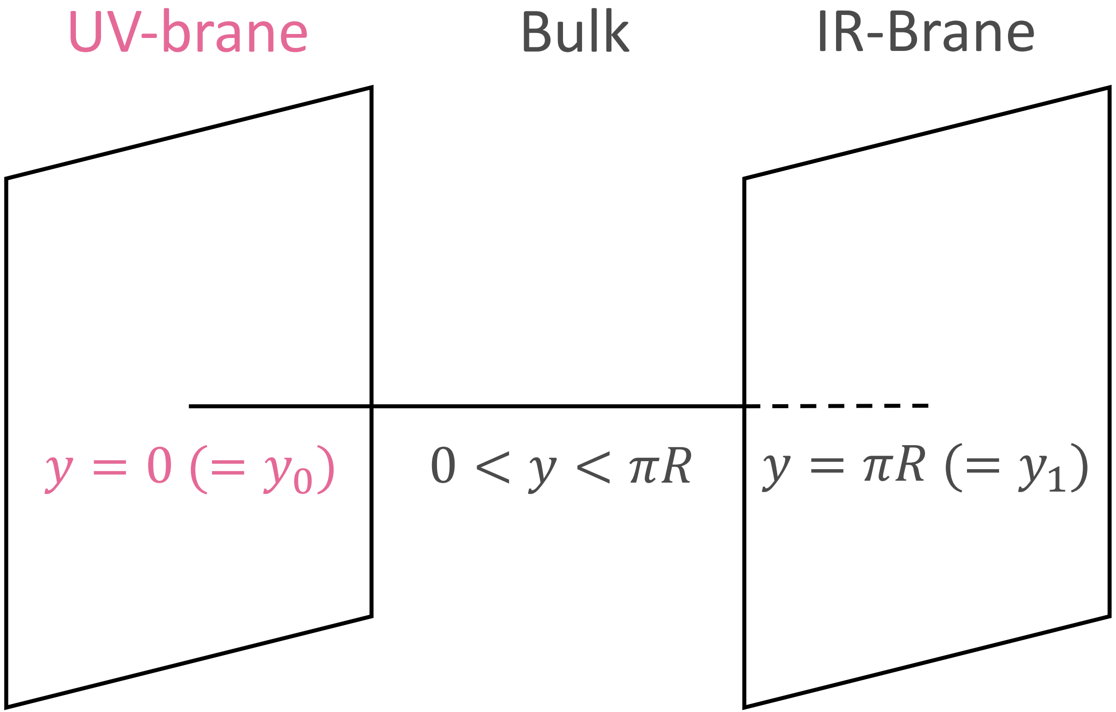
Generator $\{T^a\}$	(P'_0, P'_1)
Commute: $\{T^{a_{++}}\}$	(P_0, P_1)
Mixed: $\{T^{a_{+-}}\}$	$(P_0, e^{iT^{+-}} P_1)$
anti-commute: $\{T^{a_{--}}\}$	$(e^{iT^{--}} P_0, e^{iT^{--}} P_1)$

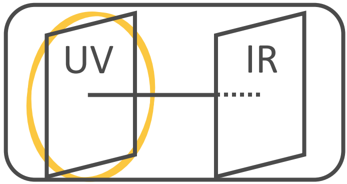
▷ connect nothing

▷ connect nothing

▷ connect nothing

for Local parameters



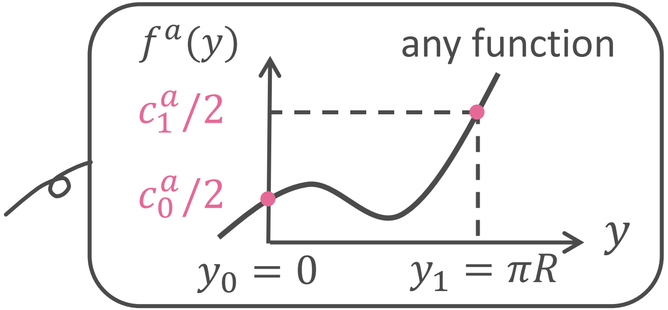


Result:

for Local on UV-brane

$$P' = e^{-i(f^a \mp f^a)T^a} P$$

UV limit: $y \to 0$



- com. $f^a(y_i + y) - f^a(y_i - y) \to 0$
- anti. $f^a(y_i + y) + f^a(y_i - y) \to c_i^a$

$(y_0 = 0, y_1 = \pi R)$

Generator $\{T^a\}$	(P_0', P_1')
Commute: $\{T^{a++}\}$	(P_0, P_1)
Mixed: $\{T^{a+-}\}$	$(P_0, e^{iT^{+-}} P_1)$
anti-commute: $\{T^{a--}\}$	$(e^{iT_0^{--}} P_0, e^{iT_1^{--}} P_1)$

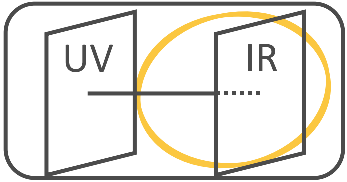
$$T_0^{--} = c_0^{a--} T^{a--}$$

$$T_1^{--} = c_1^{a--} T^{a--}$$

- ▷ connect nothing
- ▷ connect nothing
- ▷ produce $(N + 1)^2$

Result:

for Local on Bulk & IR-brane

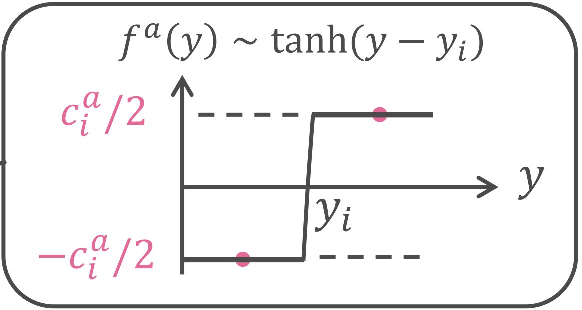


$$P' = e^{-i(f^a \mp f^a)T^a} P$$

Bulk & IR: $0 < y \leq \pi R$

com. $f^a(y_i + y) - f^a(y_i - y) = c_i^a \ (\neq 0)$

achieved by parameters with a kink!



$T_0^{++} = c_0^{a++} T^{a++}$

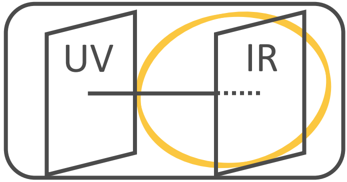
$T_1^{++} = c_1^{a++} T^{a++}$

Generator $\{T^a\}$	(P_0', P_1')
Commute: $\{T^{a++}\}$	$(e^{iT_0^{++}} P_0,\ P_1)$, $(P_0,\ e^{iT_1^{++}} P_1)$
Mixed: $\{T^{a+-}\}$	$(P_0, e^{iT^{+-}} P_1)$
anti-commute: $\{T^{a--}\}$	$(e^{iT_0^{--}} P_0, e^{iT_1^{--}} P_1)$

▷ ?

▷ connect nothing

▷ connect $(N+1)^2$



Result:

for Local on Bulk & IR-brane

$P' = e^{-i(f^a \mp f^a)T^a} P$

Bulk & IR: $0 < y \leq \pi R$

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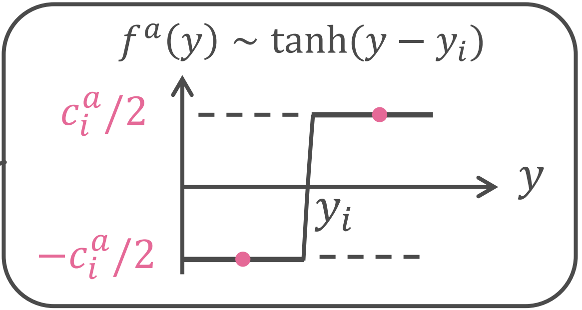


Generator $\{T^a\}$	(P_0', P_1')
Commute: $\{T^{a++}\}$	$(e^{iT_0^{++}}P_0,\ P_1)$, $(P_0,\ e^{iT_1^{++}}P_1)$
Mixed: $\{T^{a+-}\}$	$(P_0, e^{iT^{+-}}P_1)$
anti-commute: $\{T^{a--}\}$	$(e^{iT_0^{--}}P_0, e^{iT_1^{--}}P_1)$

$T_0^{++} = c_0^{a++}T^{a++}$

$T_1^{++} = c_1^{a++}T^{a++}$

▷ connect all set of BCs!

▷ connect nothing

▷ connect $(N+1)^2$

On the Bulk and IR-brane

$$(P'_0, P'_1) = (P_0,\ e^{iT_1^{++}} P_1)$$

$$T_1^{++} = \begin{pmatrix} a_1 & & & 0 \\ & a_2 & & \\ & & \ddots & \\ 0 & & & a_N \end{pmatrix}$$

$$e^{iT_1^{++}} = \mathrm{diag}\,(e^{ia_1}, e^{ia_2}, \cdots, e^{ia_N})$$

$$= \tilde{I}$$: diagonal matrix with ±1 $\quad ({P'}_1^{\dagger} = P'_1)$

$$P_1'^{\,\mathrm{diag}} = \tilde{I}\, P_1^{\,\mathrm{diag}} = $$ { All patterns of P_1 }

$$P_1 = \begin{pmatrix} + & & & & \\ & + & & & \\ & & + & & \\ & & & + & \\ & & & & + \end{pmatrix}$$

↓

$$P_1 = \begin{pmatrix} + & & & & \\ & + & & & \\ & & + & & \\ & & & - & \\ & & & & + \end{pmatrix}$$

freely flip!

→ All set of the BCs are connected!

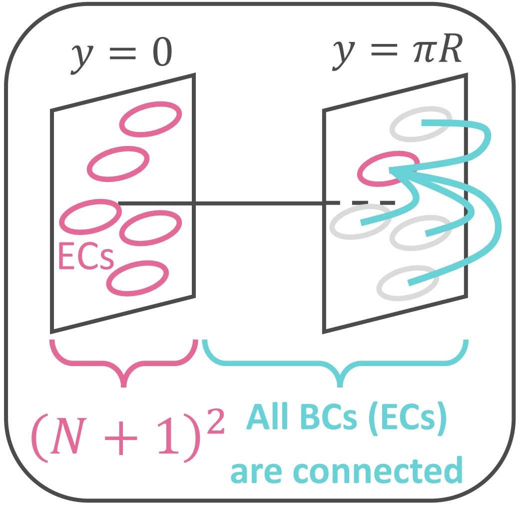
04

Summary

Summary

We have comprehensively investigated gauge connected BCs in $SU(N)$ on S^1/Z_2.

- There are only $(N+1)^2$ Equivalence Classes (ECs) on UV-brane: ($y=0$).
- The arbitrariness of boundary conditions is completely resolved on Bulk and IR-brane: ($0 < y \leq \pi R$).



Future Work

We can freely choose BCs on Bulk & IR

→ Apply to existing S^1/Z_2 models

ECs are unrelated on UV, but related on Bulk & IR

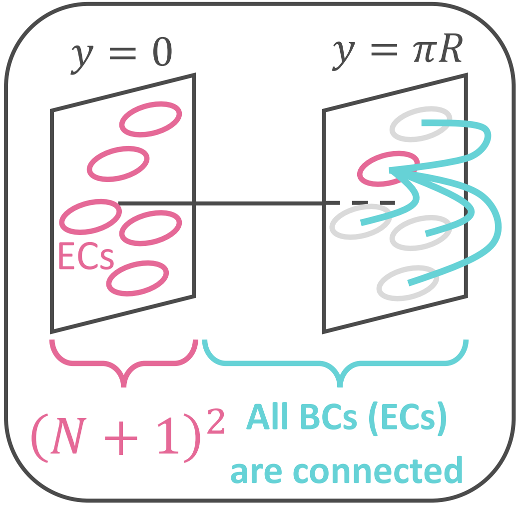
→ Construction of a mechanism to transition between ECs on UV

Thank you for listening

Summary

We have comprehensively investigated gauge connected BCs in $SU(N)$ on S^1/Z_2.

- There are only $(N+1)^2$ Equivalence Classes (ECs) on UV-brane: $(y=0)$.
- The arbitrariness of boundary conditions is completely resolved on Bulk and IR-brane: $(0 < y \leq \pi R)$.



Future Work

We can freely choose BCs on Bulk & IR

→ Apply to existing S^1/Z_2 models

ECs are unrelated on UV, but related on Bulk & IR

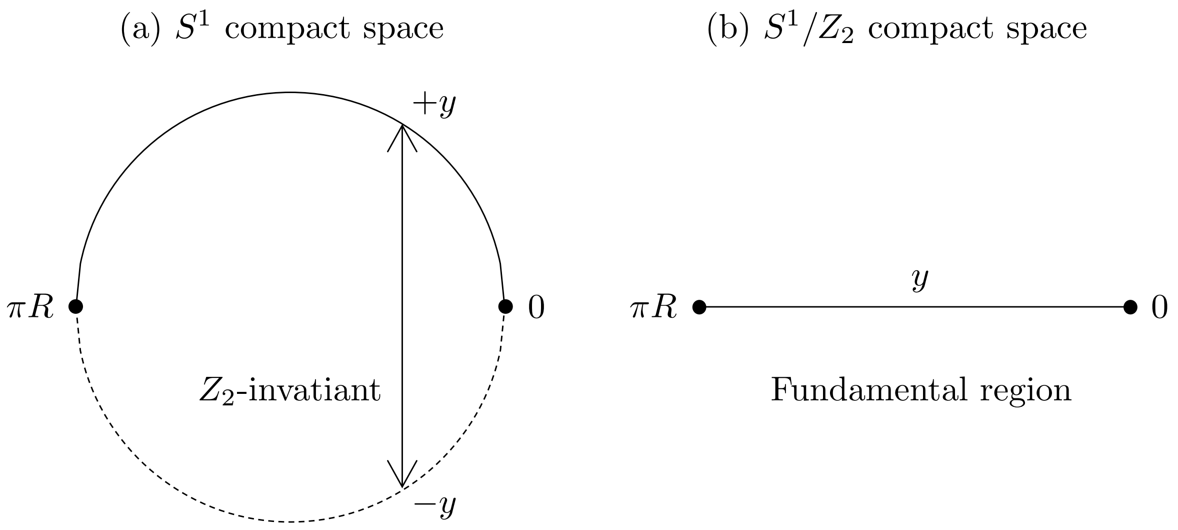
→ Construction of a mechanism to transition between ECs on UV

05

Follow-up

Two pictures for S^1/Z_2 Orbifold

The field are treated in two equivalent pictures on S^1/Z_2.

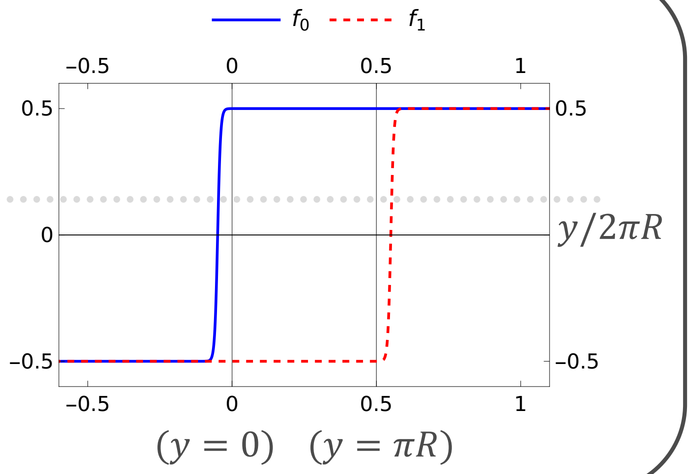


Physical fields are defined only on the fundamental region: $0 \leq y \leq \pi R$

Step parameters

Parameters with a kink:

$$f_0(y) = \frac{1}{2}\tanh\left[\lambda\left(y-(0-\epsilon)\right)\right]$$



The transformed gauge field and the BCs:

$$A'_y(x,y) = \Omega(x,y)A_y(x,y)\Omega^{-1}(x,y) - \frac{i}{g}\Omega(x,y)\partial_y\Omega^{-1}(x,y)$$

$$A'_y(x,y_i-y) = -P'_i A'_y(x,y_i+y)P'^{\dagger}_i - \frac{i}{g}P'_i\left(-\partial_y\right)P'^{\dagger}_i$$

Well-defined
in the fundamental region

Redefine Bulk & IR-brane area: $1/\lambda \ll \epsilon < y \leq \pi R$

$$\Rightarrow \left| \begin{array}{l} f_0(0+y) - f_0(0-y) = 1 \\ f_0(\pi R+y) - f_0(\pi R-y) = 0 \end{array} \right.$$

→ Commutative type produces the phase factor $e^{iT_{++}}$

by Multiple Generators

It is expected that they cannot produce non-trivial diagonal matrix.

generators	c-number components	sub-matrix components
two (ex. $e^{T_{+-}+T_{--}}$)	Done!	Done!
three (ex. $e^{T_{+-}+T_{-+}+T_{--}}$)	Done!	Not yet.
full ($e^{T_{++}+T_{+-}+T_{-+}+T_{--}}$)	Not yet.	Not yet.

$$T^{a++} = \begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix},$$

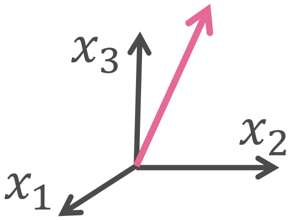
$$T^{a+-} = \begin{pmatrix} 0 & \star & 0 & 0 \\ \star & 0 & 0 & 0 \\ 0 & 0 & 0 & \star \\ 0 & 0 & \star & 0 \end{pmatrix},$$

$$T^{a-+} = \begin{pmatrix} 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \\ \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \end{pmatrix},$$

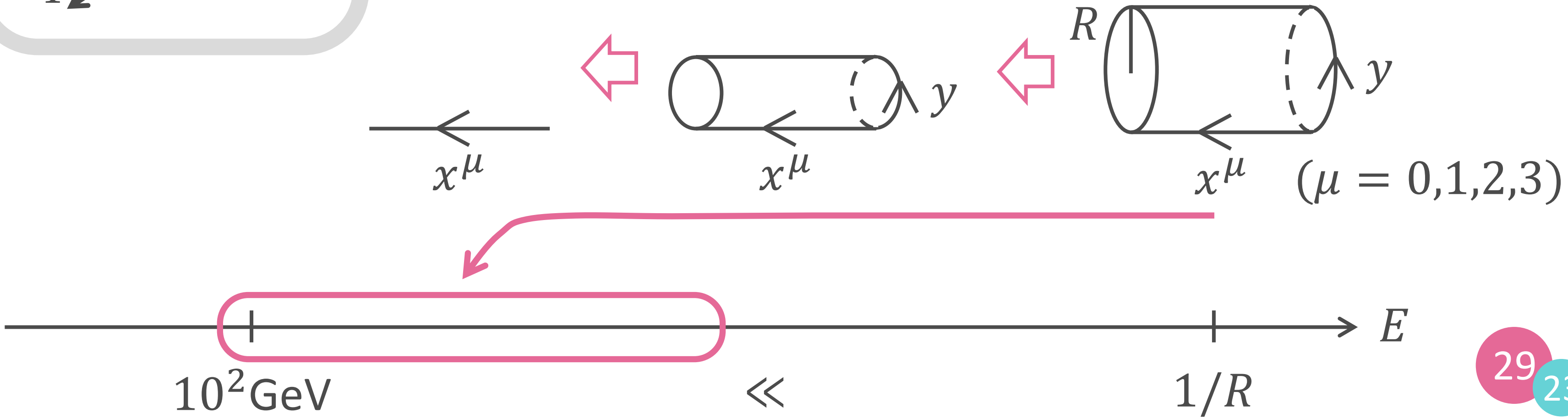
$$T^{a--} = \begin{pmatrix} 0 & 0 & 0 & \star \\ 0 & 0 & \star & 0 \\ 0 & \star & 0 & 0 \\ \star & 0 & 0 & 0 \end{pmatrix},$$

Compact space

Where??



Extra space is curled up with small radius!



$SU(N)$ model on S^1/Z_2 orbifold

$$
\begin{cases}
\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN}(x,y) + \bar{\Psi}i\Gamma^M D_M \Psi(x,y) \\
\Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \\
A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \qquad (i = 0, 1) \\
A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i
\end{cases}
$$

$$\Gamma^M = (\gamma^\mu, i\gamma^5)$$

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} I_{4\times 4}$$

$$D_M = \partial_M - igA_M$$

$$F_{MN} = \frac{i}{g}[D_M, D_N]$$

$$y_0 = 0,\ y_1 = \pi R$$

Unitary + Parity → Hermite

$$P_i^{-1} = P_i^\dagger \qquad P_i^2 = 1 \qquad P_i^\dagger = P_i = P_i^{-1}$$

P_i : Hermitian N×N matrices with ±1 eigenvalues

Symmetry Breaking by BCs

Different BCs generally produce different symmetry.

e.g.) SU(3)model

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
A_μ^0	$\begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \star \\ 0 & \star & 0 \end{pmatrix}$
generators	$\{T^1, T^2, T^3, T^8\}$	$\{T^3, T^8\}$	$\{\frac{\sqrt{3}}{2}T^3 + \frac{1}{2}T^8\}$
symmetry	SU(2)×U(1)	U(1)×U(1)	U(1)

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A_\mu'(x, y_i - y) = P_i' A_\mu'(x, y_i + y) P_i'^\dagger - P_i' \partial_\mu P_i'^\dagger \\ A_y'(x, y_i - y) = -P_i' A_y'(x, y_i + y) P_i'^\dagger - P_i' (-\partial_y) P_i'^\dagger \end{cases}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

Not invariant!

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A'_\mu(x, y_i - y) = P_i' A'_\mu(x, y_i + y) P_i'^\dagger - P_i' \partial_\mu P_i'^\dagger \\ A'_y(x, y_i - y) = -P_i' A'_y(x, y_i + y) P_i'^\dagger - P_i' (-\partial_y) P_i'^\dagger \end{cases}$$

$P_i'^\dagger = P_i$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

$P_i' = P_i$ (i=0,1)

$\Rightarrow$ gauge invariant

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A'_\mu(x, y_i - y) = P_i' A'_\mu(x, y_i + y) P_i'^\dagger - \frac{i}{g} P_i' \partial_\mu P_i'^\dagger \\ A'_y(x, y_i - y) = -P_i' A'_y(x, y_i + y) P_i'^\dagger - \frac{i}{g} P_i' (-\partial_y) P_i'^\dagger \end{cases}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

$$\begin{cases} \partial_M P_i' = 0 \\ P_i'^\dagger = P_i' \end{cases} \quad \text{(i=0,1)}$$

$\Rightarrow$ not invariant, but P_i' become other BCs !

Physical Symmetry

- Wilson line phase (AB phase)

$$WT \equiv \mathcal{P}\exp\left[ig\int_0^{2\pi R} dy A_y(x,y)\right] T \qquad (T = P_1 P_0)$$

- There is one physics in one EC. Hosotani(1989)

$$V_{eff}(A_M^c; P_0, P_1) = V_{eff}(A_M^{\prime c}; P_0', P_1')$$

- Physical symmetry

$$(A_y^c\ ; P_o, P_1) \sim (A_y^{\prime c} = 0\ ; P_o^{phys}, P_1^{phys})$$

$$\mathcal{H}^{phys} = \left\{T^a \in \mathcal{G} \,\middle|\, \left[T^a, P_i^{phys}\right] = 0\ (i = 0, 1)\right\}$$

ECs in $SU(N)$ on S^1/Z_2

- Rearrangement

$$P_0 = \mathrm{diag}\,\overbrace{(+1,\dots,+1,+1,\dots,+1,-1,\dots,-1,-1,\dots,-1)}^{N},$$
$$P_1 = \mathrm{diag}(\underbrace{+1,\dots,+1}_{p},\underbrace{-1,\dots,-1}_{q},\underbrace{+1,\dots,+1}_{r},\underbrace{-1,\dots,-1}_{s=N-p-q-r}),$$

$$(P_0, P_1) = [p, q, r, s]$$

- Well-known ECs gauge transformations on S^1/Z_2

$${}_{N+3}C_3 - \sum_{k=0}^{N-2} {}_{k+1}C_1\, {}_{N-k-1}C_1 = (N+1)^2$$

$$\begin{aligned}[p,q,r,s] &\sim [p-1, q+1, r+1, s-1] \\ &\sim [p+1, q-1, r-1, s+1]\end{aligned}$$

Simple Example

e.g.) mixed type for $U(2)$ matrix

$$T^{+-} = \begin{pmatrix} 0 & c \\ c^* & 0 \end{pmatrix}$$

$$e^{iT^{+-}} = \begin{pmatrix} \cos|c| & \frac{ic}{|c|}\sin|c| \\ \frac{ic^*}{|c|}\sin|c| & \cos|c| \end{pmatrix}$$

$$\mathrm{diag}(e^{iT^{+-}}) = I_2,\ -I_2$$

$$P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1 = \begin{pmatrix} + & \\ & - \end{pmatrix}$$

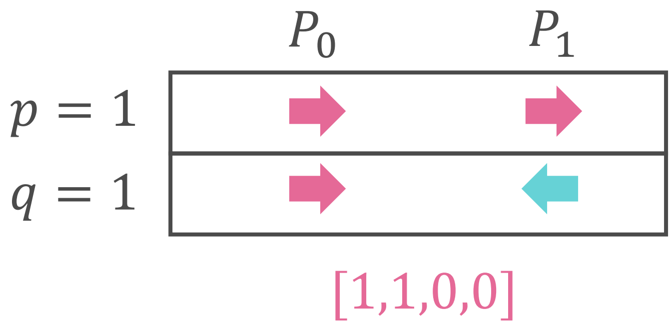
$$\downarrow$$

$$P_0' = P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1' = e^{iT^{+-}} P_1 = \begin{pmatrix} - & \\ & + \end{pmatrix}$$

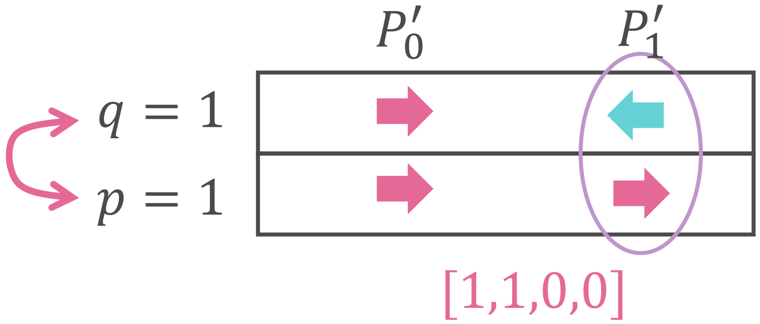
Simple Example

e.g.) mixed type for $U(2)$ matrix

$$P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1 = \begin{pmatrix} + & \\ & - \end{pmatrix} \quad \rightarrow \quad P_0' = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1' = \begin{pmatrix} - & \\ & + \end{pmatrix}$$



$\rightarrow$



A set of BCs $[p, q, r, s]$ are invariant

General Example

e.g.) anti-commutative type for $U(N)$ matrix

$$T^- = \begin{pmatrix} 0 & 0 & 0 & A \\ 0 & 0 & B & 0 \\ 0 & B^\dagger & 0 & 0 \\ A^\dagger & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{diag}(e^{iT^-}) = \begin{pmatrix} \tilde{I}_{p,X} & 0 & 0 & 0 \\ 0 & \tilde{I}_{q,Y} & 0 & 0 \\ 0 & 0 & \tilde{I}_{r,Y} & 0 \\ 0 & 0 & 0 & \tilde{I}_{s,X} \end{pmatrix}$$

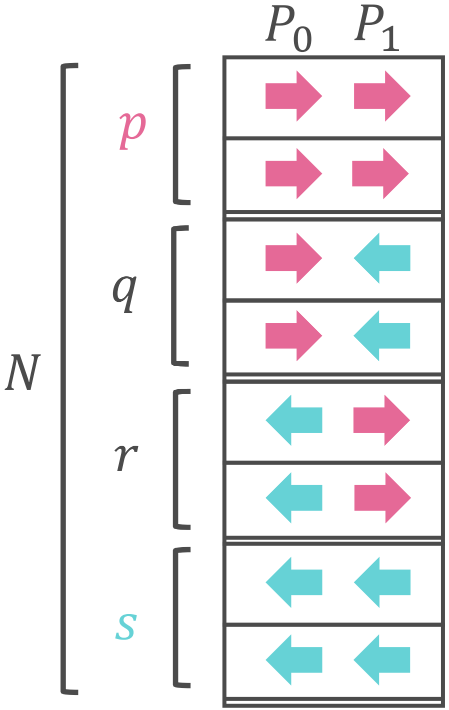
A, B: sub-matrices

$\tilde{I}_{p,X}$: diagonal $(p \times p)$ sub-matrix with ± 1

X : the number of -1 components

General Example

e.g.) anti-commutative type for $U(N)$ matrix

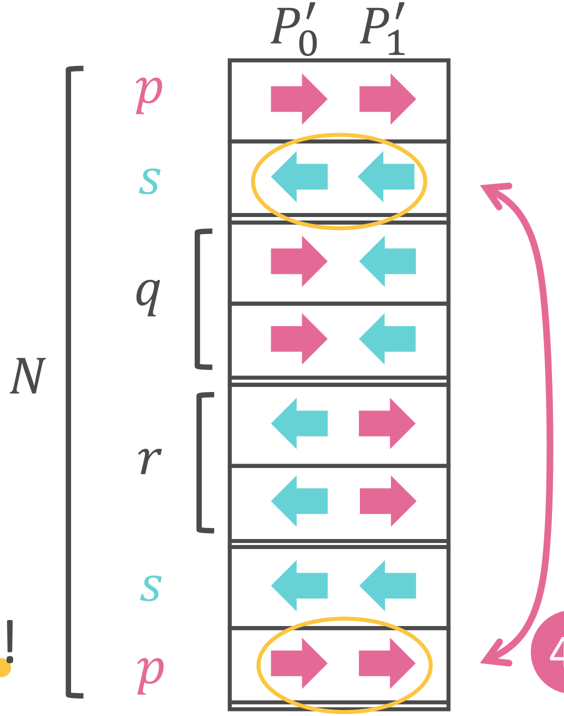


e.g.)

$X = 1$

$Y = 0$

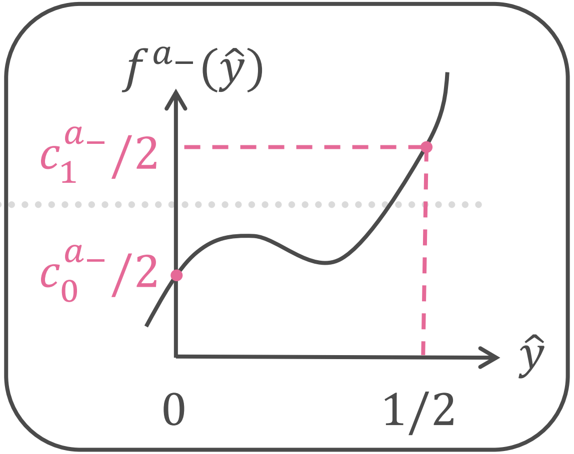
$\rightarrow$



$[p, q, r, s]$ is invariant!

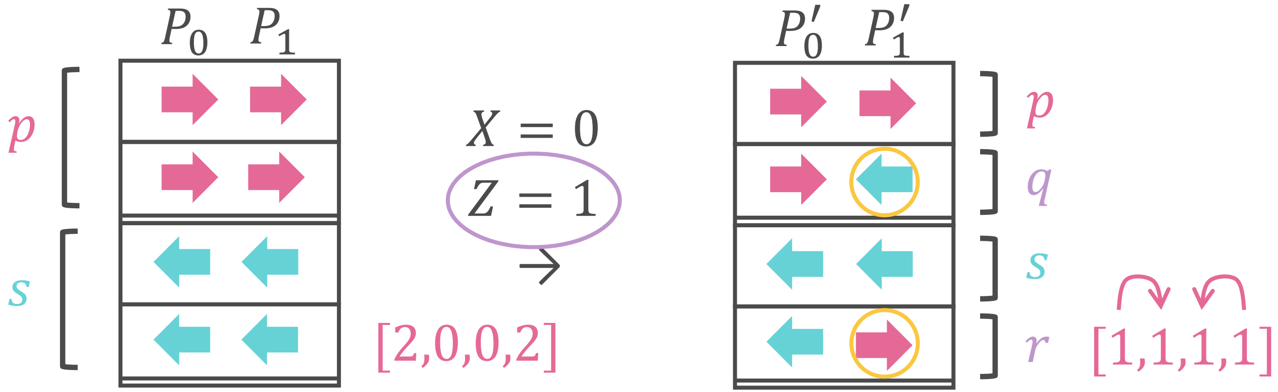
On the UV-brane

e.g.) anti-commutative type: $[p,q,r,s]=[2,0,0,2]$



$$\mathrm{diag}(e^{iT_0^{--}})=\mathrm{diag}(\tilde{I}_{p,X},\tilde{I}_{s,X})$$

$$\mathrm{diag}(e^{iT_1^{--}})=\mathrm{diag}(\tilde{I}_{p,Z},\tilde{I}_{s,Z})$$



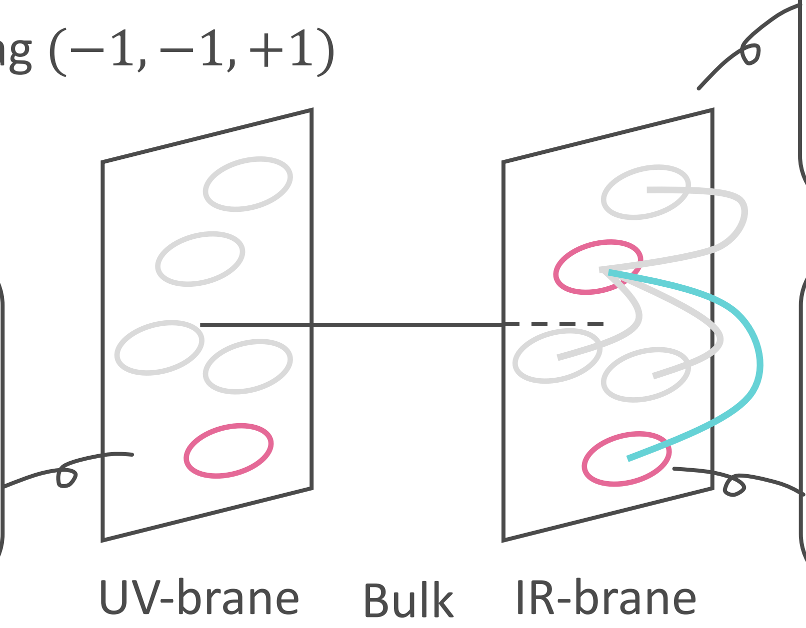
Applications (1/2)

- Apply to phenomenological models

e.g.) $P_0 = P_1 = \mathrm{diag}\,(-1,-1,+1)$

$SU(2)\times U(1)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$$



$SU(3)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{pmatrix}$$

$SU(2)\times U(1)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$$

Applications (2/2)

- Approach to the arbitrariness problem of BCs:

Which type of BCs should be selected
without relying on phenomenological information?