Determination of coupling patterns by parallel searches for $\mu^- \to e^+$ and $\mu^- \to e^-$ in muonic atoms

SATO, Joe
Yokohama National University

Based on
J. S, K.Sugawara, Y. Uesaka, M. Yamanaka,
Physics Letters B 836, 2023, 137617

素粒子現象論研究会２０２２大阪公立大学３・１７

A trial on how large Branching ratio for μ^- → e^+ in muonic atoms we can derive

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→ 10^{-18} order

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- Benchmark Model
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  2. Several Bounds on the Model
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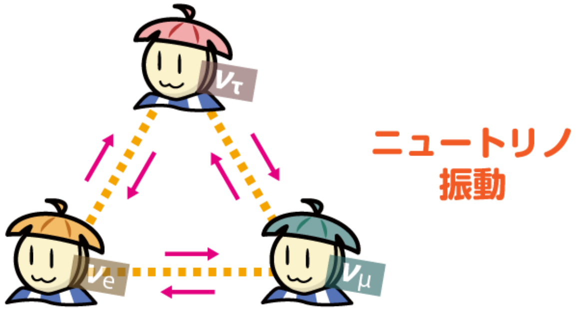
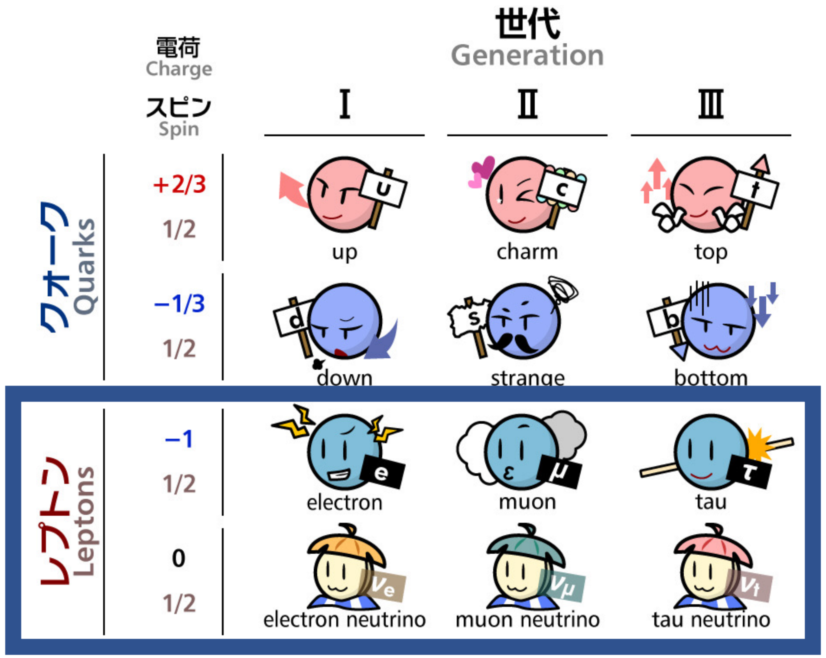
Introduction





- Neutrino oscillation(1998)

  ⇒ Lepton flavor is not conserved !

Lepton Flavor in SM

Conserved "Charge" resulting from massless neutrinos

Electron, muon, tau number

L_e L_μ L_τ

	e^-	ν_e	μ^-	ν_μ	τ^-	ν_τ
L_e	1	1				
L_μ			1	1		
L_τ					1	1

Opposite (−1) for anti particles

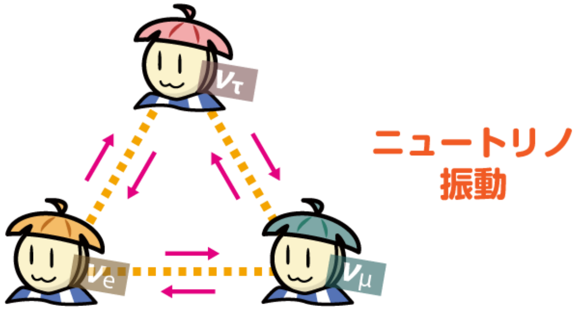
Same charge for Neutrino and charged lepton

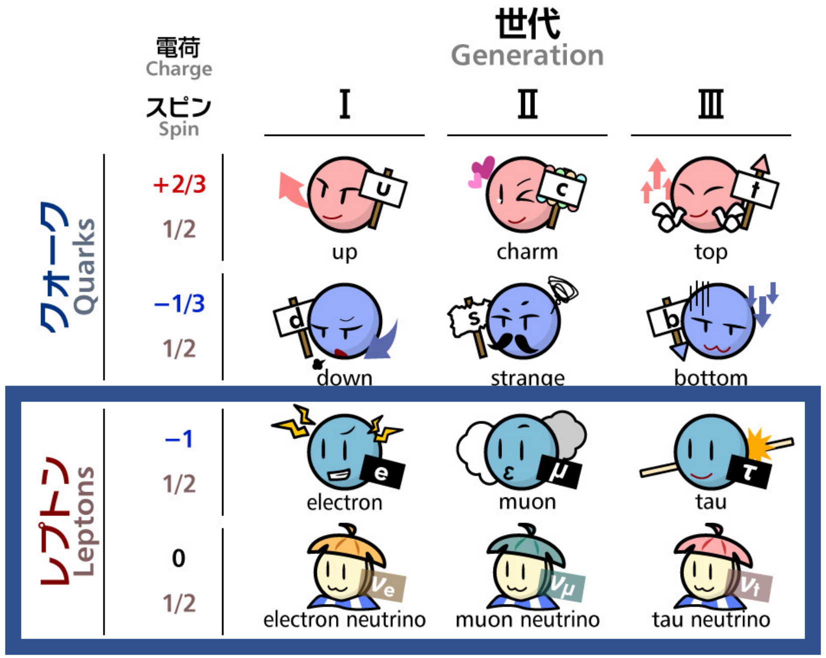
Example of conservation

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

$$L_\mu \quad 0 = 1 + (-1)$$

Introduction





- Neutrino oscillation(1998)

  ⇒We need to build a model that explains neutrino mass and lepton mixing.

  ＝ introduction of a seed for lapton flavor violation

- SU(2) doublet (e_l, ν_l)

  ⇒ charged Lepton Flavor Violation is inevitable.

  How large?

$\mu^- \to e^-$ conversion (cLFV)

○A test for CLFV

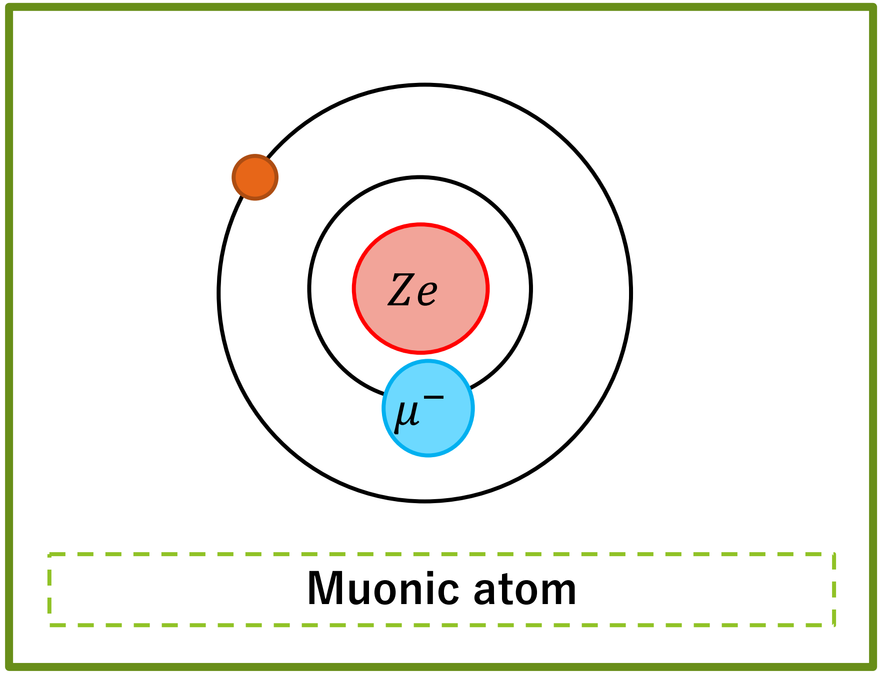
- The initial state is a muonic atom
- cLFV
- The energy of electron

$$m_\mu - (\text{Binding Energy}) - (Q\ \text{value})$$

- The the current experimental constraint is

$$\mathrm{B}(\mu^- \to e^-) = \frac{\Gamma(\mu^- \mathrm{Au} \to e^- \mathrm{Au})}{\Gamma_{\mu-\mathrm{capture}}} < 7\times10^{-13}$$

(P. A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) no.8, 083C01.)



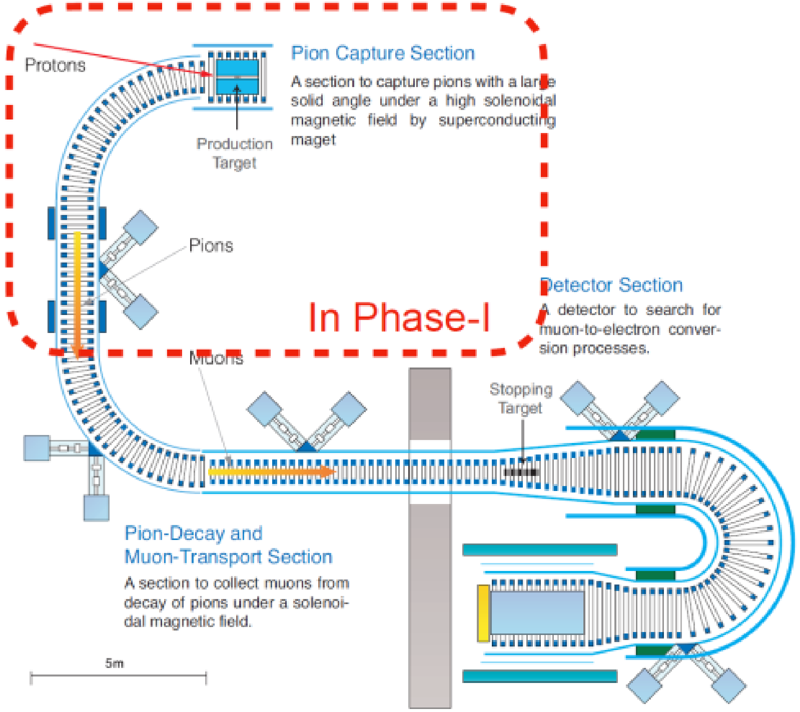
- Several experiments are planned to search for the $\mu^- \to e^-$ conversion.

  e.g. COMET, Mu2e, DeeMe, (PRISM/PRIME)…

COMET Phase-I and -II

COMET (Coherent Muon-to-Electron Transition) Experiment

- **μ-e conversion** ($\mu^- + Al \rightarrow e^- + Al$):
  charged lepton flavour violation process
  - Strongly suppressed by the SM including the neutrino oscillation,
  - Branching ratio ~ $O(10^{-15})$ predicted by several BSM physics models.
- COMET Phase-I (-II) will search for it with a sensitivity of $O(10^{-15})$ ($O(10^{-17})$).
  - @ Hadron Experimental Hall
  - J-PARC's high-intensity proton beam at 8 GeV (suppressing anti-p generation)
- **The facility and beamline are being constructed.**
  - Backward production of π/μ
    - Dominantly generate low-momentum π/μ.
  - Charge and momentum selection with the bent Transport Solenoid.



To explain neutrino oscillation we need to violate Lepton Flavor but

It does not mean the violation of **lepton number = particle number for leptons**

$$L_e + L_\mu + L_\tau = L$$

$$\mu^- \to e^- \gamma$$ can happen

$$L \quad 1 = 1 + 0$$

Lepton number = A part of particle number

= (particle =1 & antiparticle=-1)

Neutrino oscillation indicates neutrinos are massive. If its mass is Majorana mass term, it leads lepton number (in general particle number) violation ➡ neutrinoless double beta decay

$\mu^- \to e^+$ in muonic atom

Due to Simultaneous violation of lepton flavor and particle number

Particle Number

Fermion Operator ψ annihilates particle or creates anti-particle

$$- \quad (+1) \quad / + (-1) \quad = -1$$

Therefore Dirac mass term

$\bar{\psi}\psi$ annihilates particle and then creates **particle**

$$- \quad (+1) \quad + (+1) \quad = 0$$ particle #conservation

On the contrary Majorana mass term

$\psi\psi$ annihilates particle and then creates **anti-particle**

$$- \quad (+1) \quad + (-1) \quad = -2$$ particle #NONconservation

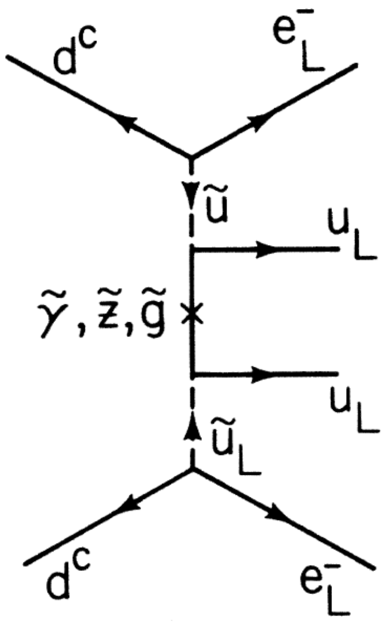
leading particle # violating process, say $0\nu2\beta$

Particle Number

Majorana mass term

$m_{\alpha\beta}\nu_\alpha\nu_\beta$ Neutrino mass term, directly leading as it contains lepton $0\nu2\beta$

$M\tilde{g}\tilde{g}$ Gaugino mass term, which can lead with a connection to leptons $0\nu2\beta$



Mohapatra 1986

Particle Number

- For scalars we can assign a particle number.
- We can introduce correspondence of majorana mass term for scalars
  - Another source for LNV process

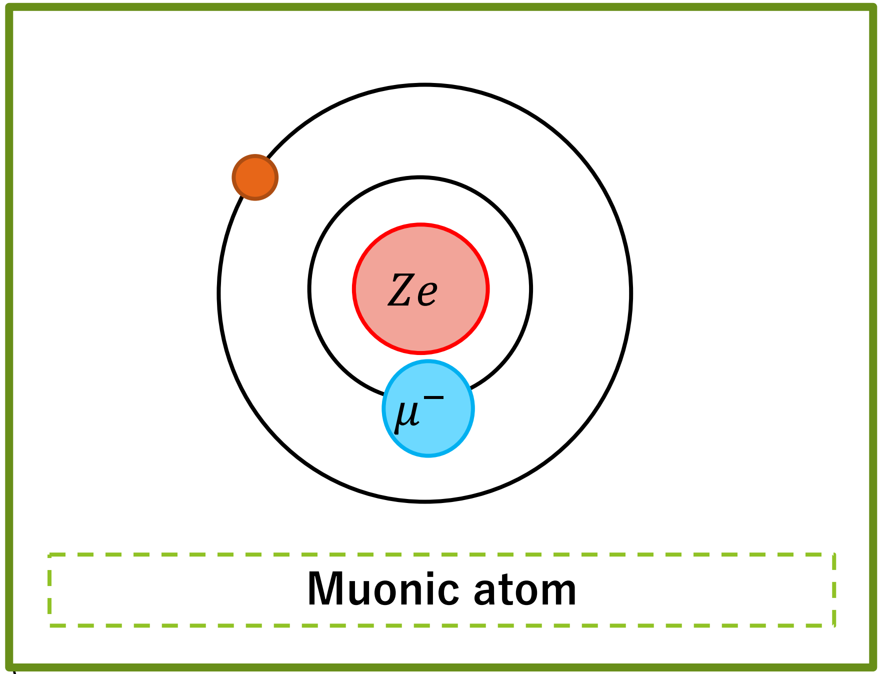
$\mu^- \to e^+$ conversion (LNV)

○ $\mu^- \to e^+$ conversion

- The initial state is a muonic atom
- LNV
- The nucleus changes: $(A, Z) \to (A, Z-2)$
- $m_\mu - (\text{Binding Energy}) - (Q\ \text{value})$
- The current experimental constraint is

$$\mathrm{B}(\mu^- \to e^+) = \frac{\Gamma(\mu^- \mathrm{Ti} \to e^+ \mathrm{Ca})}{\Gamma_{\mu-\mathrm{capture}}} < 3.6 \times 10^{-11}$$

(ref：P. A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) no.8, 083C01.)



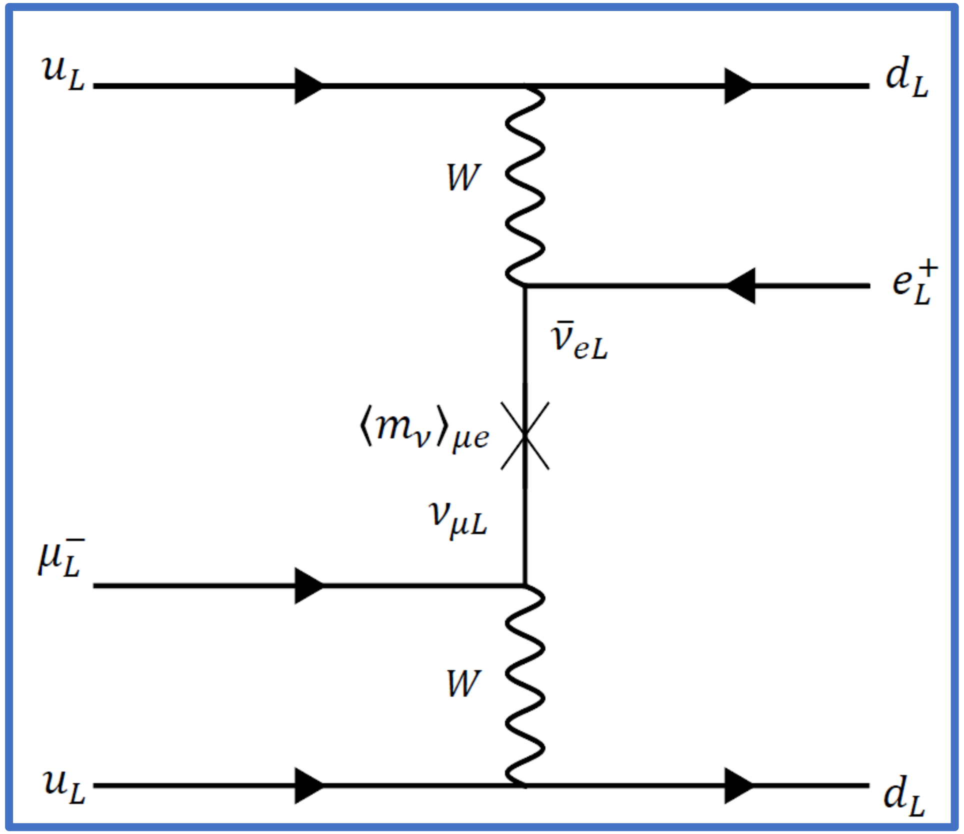
- Majorana mass term for neutrino can lead this process.

- $\mu^- + (A,Z) \rightarrow e^+ + (A,Z-2)$ conversion (majorana mass)

$$\mathrm{B}^{\mathrm{Majorana}}(\mu^- \rightarrow e^+) \sim 10^{-40}$$

（Pavol Domin, Sergey Kovalenko, Amand Faessler, and Fedor Simkovic. Phys. Rev. C, Vol. 70, p. 065501, 2004.）

It is too tiny to be ovserbed.



Introduction

- Is LNV necessarily too small to observe?

⇒ No!

Coupling of doublet and singlet types of leptoquarks can be a source of large LNV.

(K. S. Babu, R. N. Mohapatra, Phys.Rev.Lett. 75 (1995) 2276-2279)

- A model that naturally introduces leptoquarks of the doublet and singlet types and considers their coupling

⇒ Minimal Supersymmetric Standard Model (MSSM) with R-parity Violating (RPV) interaction.

Leptoquark ⇒ squark

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Benchmark Model

- The Lagrangian that causes $\mu^- \to e^+$ conversion is

$$\mathcal{L}_{\mu^- \to e^+} = -\lambda'_{113}\,\overline{(e_L)^C}\,u_L \widetilde{b_R^*} + \lambda'_{231}\widetilde{b_L}\,\overline{d_R}\nu_\mu - \lambda'_{213}\overline{(\mu_L)^C}u_L \widetilde{b_R^*} + \lambda'_{131}\widetilde{b_L}\,\overline{d_R}\nu_e - m_{LR}^2 \tilde{b}_L^* \tilde{b}_R + \mathrm{h.c.}$$

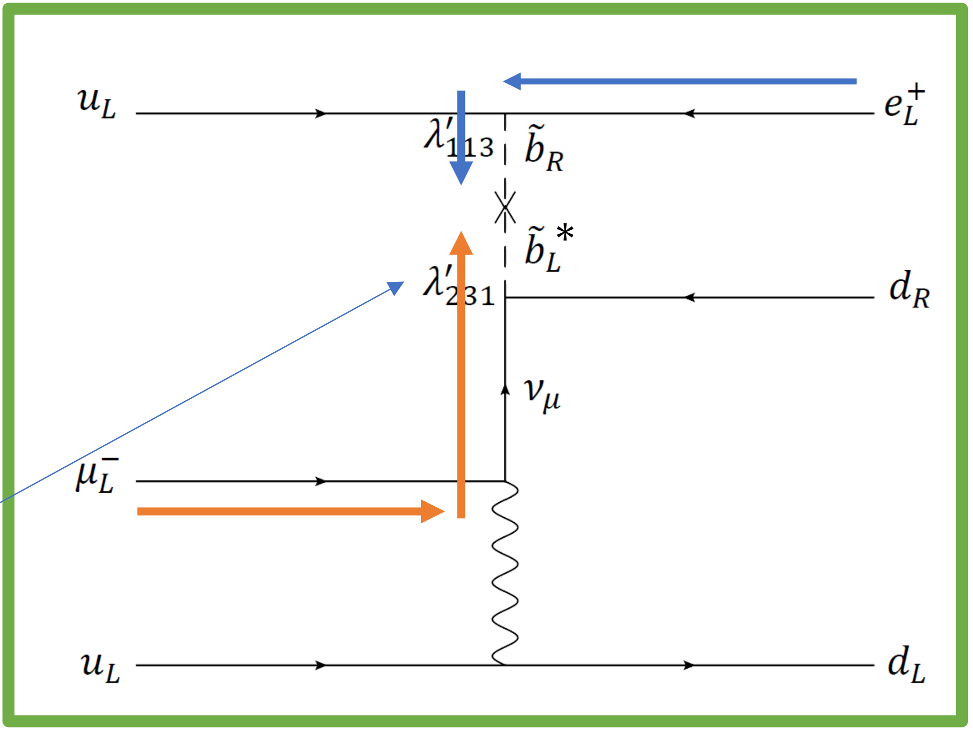
Benchmark Model

- The Lagrangian that causes $\mu^- \to e^+$ conversion is

$$\mathcal{L}_{\mu^-\to e^+} = -\lambda'_{113}\,\overline{(e_L)^C}\,u_L\widetilde{b}_R^* + \lambda'_{231}\widetilde{b_L}\,\overline{d_R}\nu_\mu - \lambda'_{213}\overline{(\mu_L)^C}u_L\widetilde{b}_R^* + \lambda'_{131}\widetilde{b_L}\,\overline{d_R}\nu_e - m_{LR}^2\tilde{b}_L^*\tilde{b}_R + \mathrm{h.c.}$$

Particle	Lepton Number
e_L^+	-1
$\widetilde{b_R}$	+1
ν_μ	+1
$\widetilde{b_L}$	-1

Particle numbers flow in same place
Mass mixing connects them!



Benchmark Model

- Lagrangian of RPV interaction is

$$\mathcal{L}_{\mu^- \to e^+} = -\lambda'_{113}\,\overline{(e_L)^C}\,u_L\widetilde{b_R^*} + \lambda'_{231}\widetilde{b_L}\,\overline{d_R}\nu_\mu - \lambda'_{213}\overline{(\mu_L)^C}u_L\widetilde{b_R^*} + \lambda'_{131}\widetilde{b_L}\,\overline{d_R}\nu_e - m_{LR}^2\tilde{b}_L^*\tilde{b}_R + \mathrm{h.c.}$$

where the other squarks which is so heavy to be ignored.

- we will consider the eigenstates of the mass of sbottom.

$$(\tilde{b}_L \quad \tilde{b}_R)\begin{pmatrix} m_L^2 & m_{LR}^2 \\ m_{LR}^2 & m_R^2 \end{pmatrix}\begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} = (\tilde{b}_1 \quad \tilde{b}_2)\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

Source for Particle # violation

$$\begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

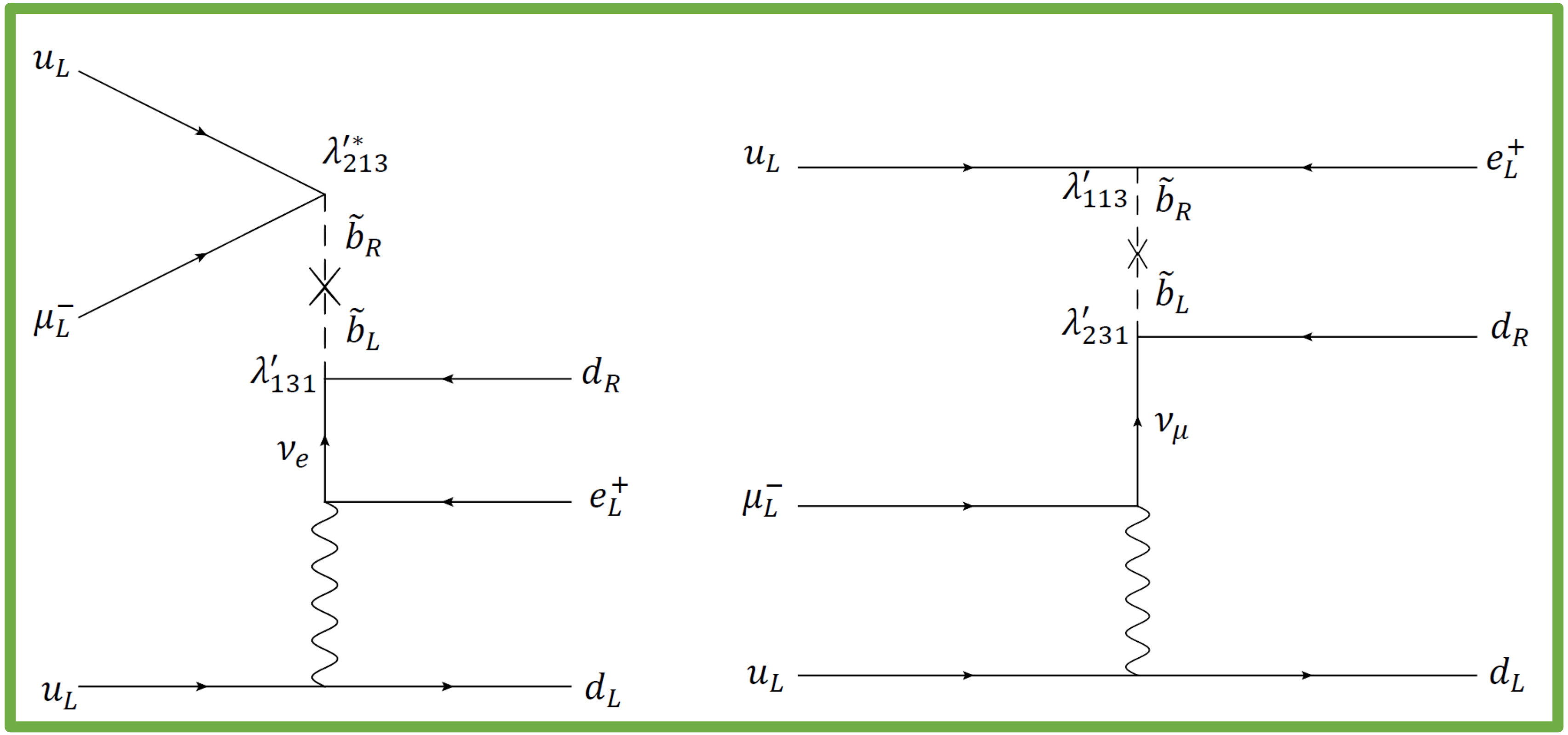
θ is the mixing angle of sbottom.

We assume $m_1 \ll m_2$.

Benchmark Model

- The Lagrangian that causes $\mu^- \to e^+$ conversion is

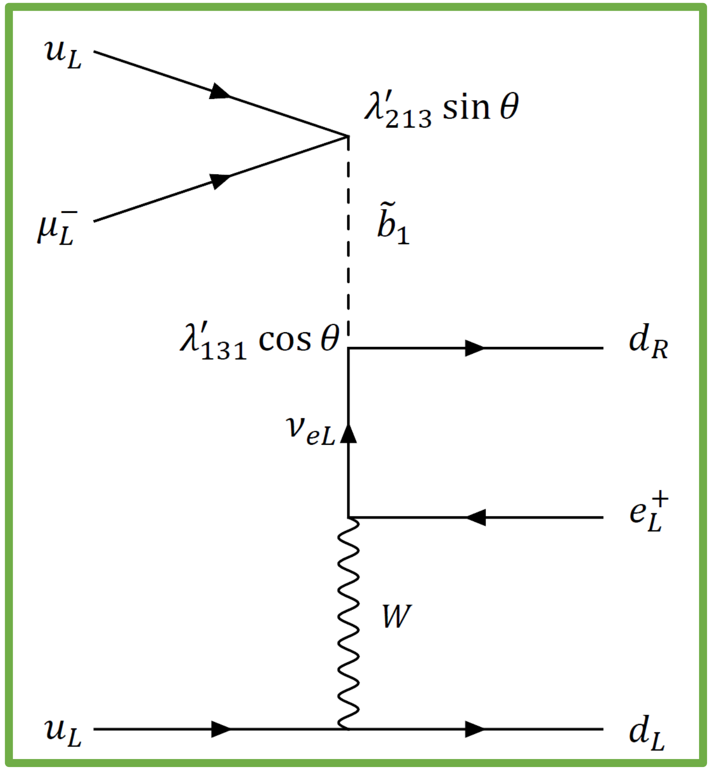
$$\mathcal{L}_{\mu^- \to e^+} = -\lambda'_{113}\,\overline{(e_L)^C}\,u_L\widetilde{b_R^*} + \lambda'_{231}\widetilde{b_L}\,\overline{d_R}\nu_\mu - \lambda'_{213}\overline{(\mu_L)^C}u_L\widetilde{b_R^*} + \lambda'_{131}\widetilde{b_L}\,\overline{d_R}\nu_e - m_{LR}^2\tilde{b}_L^*\tilde{b}_R + \mathrm{h.c.}$$

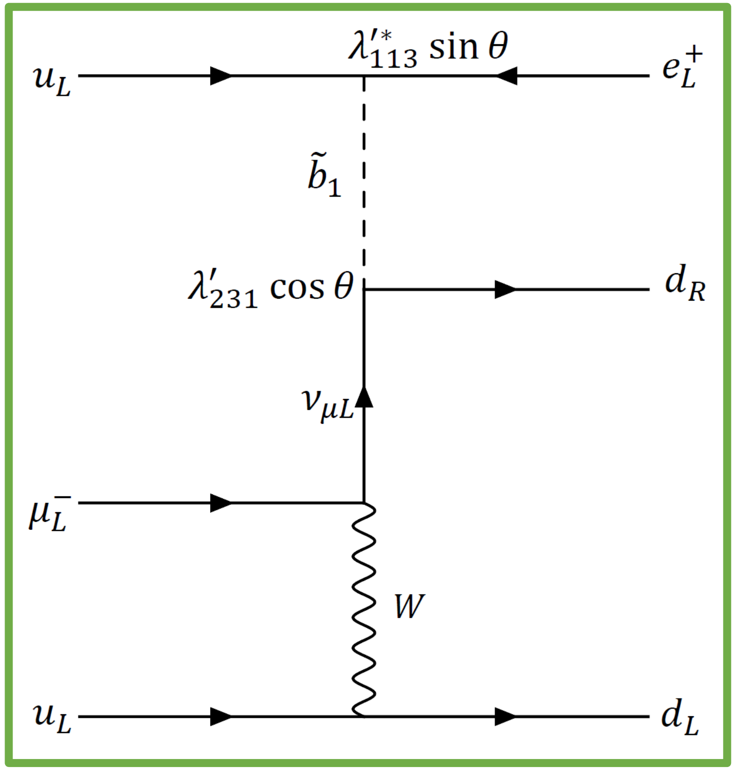


Benchmark Model

- In terms of mass eigenstates the Lagrangian that causes $\mu^- \to e^+$ conversion is

$$\mathcal{L}_{\mu^-\to e^+} = -\lambda'_{113}\sin\theta\ \overline{(e_L)^C}\, u_L \widetilde{b_1^*} + \lambda'_{231}\cos\theta\, \widetilde{b_1}\, \overline{d_R}\nu_\mu$$
$$-\lambda'_{213}\sin\theta\, \overline{(\mu_L)^C} u_L \widetilde{b_1^*} + \lambda'_{131}\cos\theta\, \widetilde{b_1}\, \overline{d_R}\nu_e + \text{h.c.}$$





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The bounds from current experiments

- Bounds are …

Bounds on the RPV couplings applied in Sec. 4.2. Here $m_1 = 200$ GeV. $\beta = \Gamma_e^{\mathrm{SM}}/\Gamma_\mu^{\mathrm{SM}}$, ϵ_e, and ϵ_μ are given in Eqs. (11) and (12).

Observables	Bound	Section
Direct sbottom search	$\tilde{\lambda}'_{i13} \leq 5 \times 10^{-3}$ $(i = 1, 2)$	2.1
APV and PVES	$\tilde{\lambda}'_{131} \leq 0.69$	2.2
$\nu_\mu d_R \to \nu_\mu d_R$	$\tilde{\lambda}'_{231} \leq 0.36$	2.3
LFU of $\pi^\pm$ decays	$-7 \times 10^{-7} \leq \beta\left(\epsilon_e - \epsilon_\mu\right) \leq 2 \times 10^{-7}$	2.4
$0\nu 2\beta$	$\tilde{\lambda}'_{113}\tilde{\lambda}'_{131} \leq 8.3 \times 10^{-10}$	2.5
$\mu^- \to e^-$ conversion	$\tilde{\lambda}'_{213}\tilde{\lambda}'_{113} \leq 1.6 \times 10^{-7}$	3.1

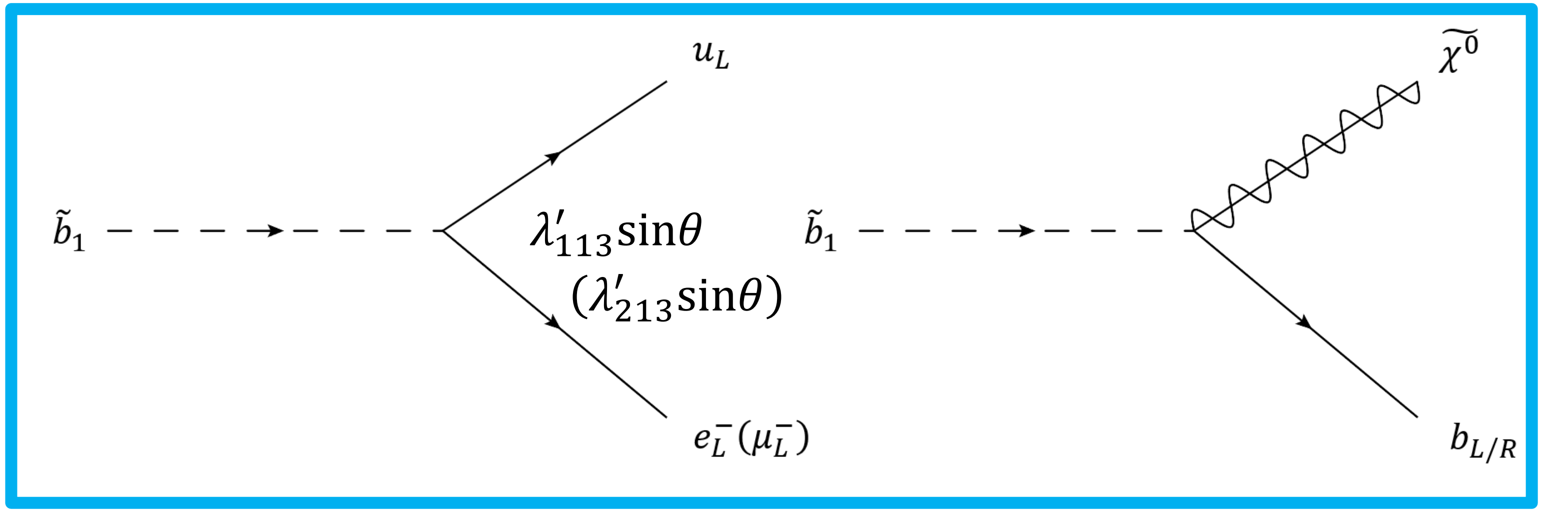
Direct sbottom search

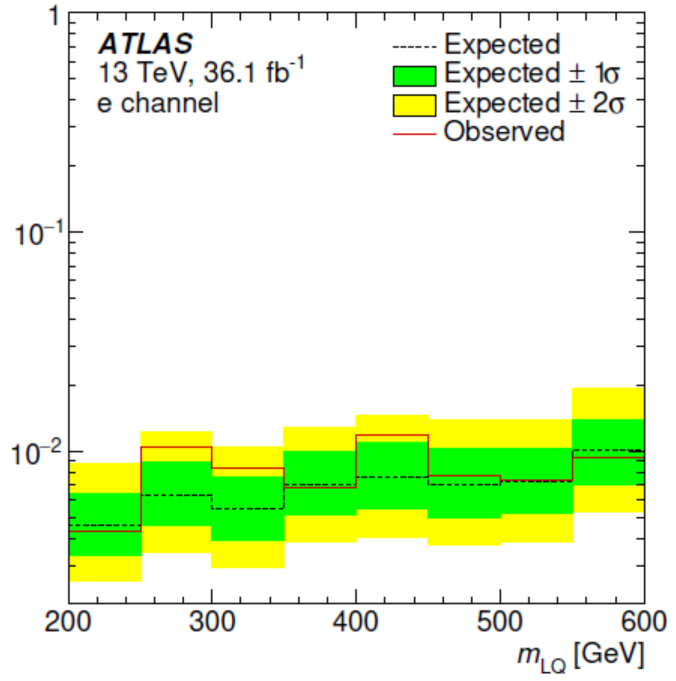
- The ATLAS experimental constraint is

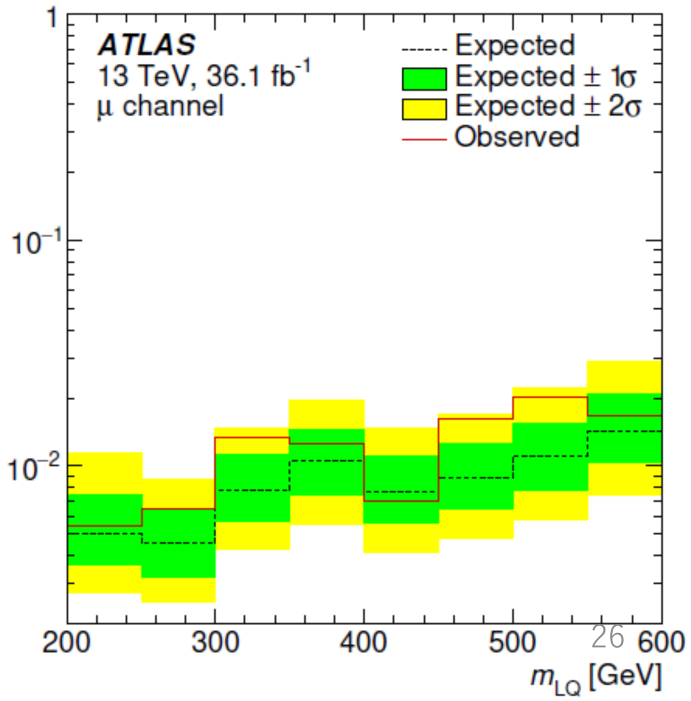
$$m_1 = 200(\text{GeV}) \Rightarrow \beta = \frac{\Gamma_{(\tilde{b}\to l\nu)}}{\Gamma_{(\tilde{b}\to\widetilde{\chi}^0 b)}} < 0.01$$

(M. Aaboud et al. (ATLAS), Eur. Phys. J. C79, 733 (2019), 1902.00377)

- The diagram of the sbottom decay







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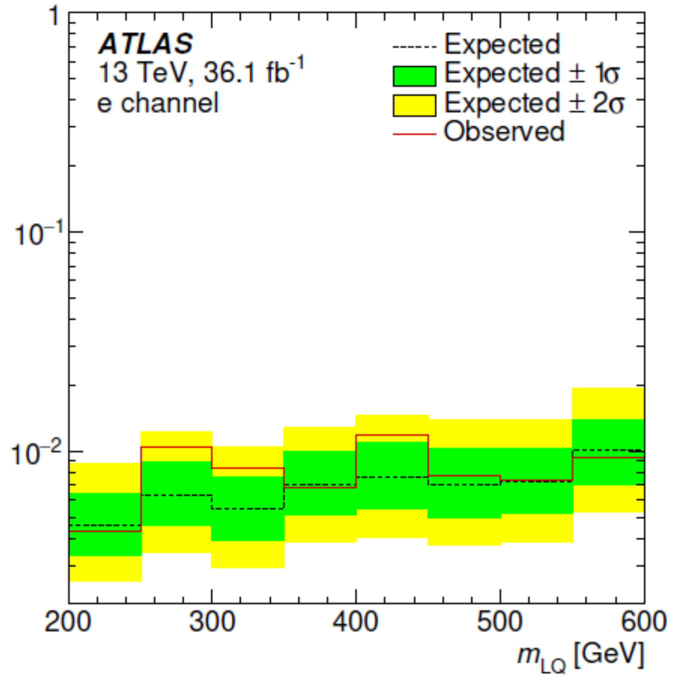
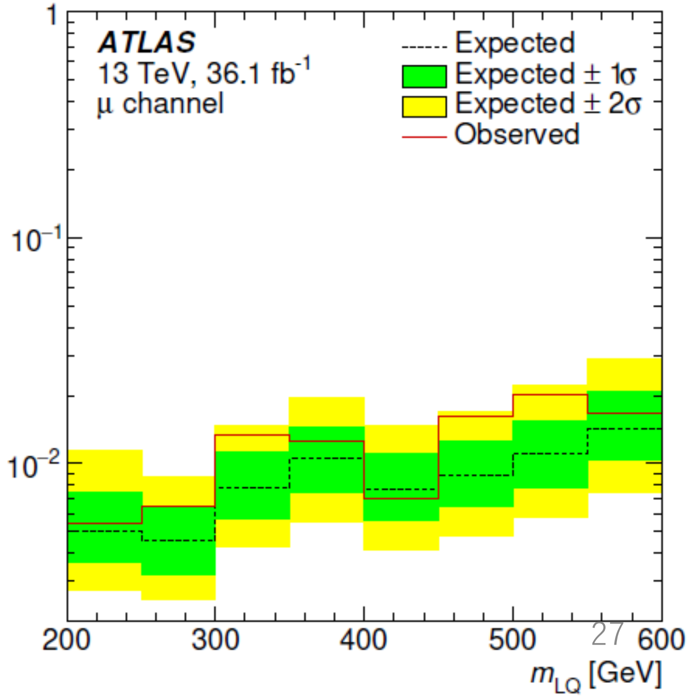
(M. Aaboud et al. (ATLAS), Eur. Phys. J. C79, 733 (2019), 1902.00377)

- The bound of the direct sbottom search is

$$\lambda'_{k13}\sin\theta \leq 5.0\times10^{-3} \quad (k=1,2)$$

$$m_1 = 200\,\mathrm{GeV} \text{ and } m_{\tilde{\chi}^0} = 160\,\mathrm{GeV}$$





Bound from lepton flavor universality of π decay

- The lepton universality

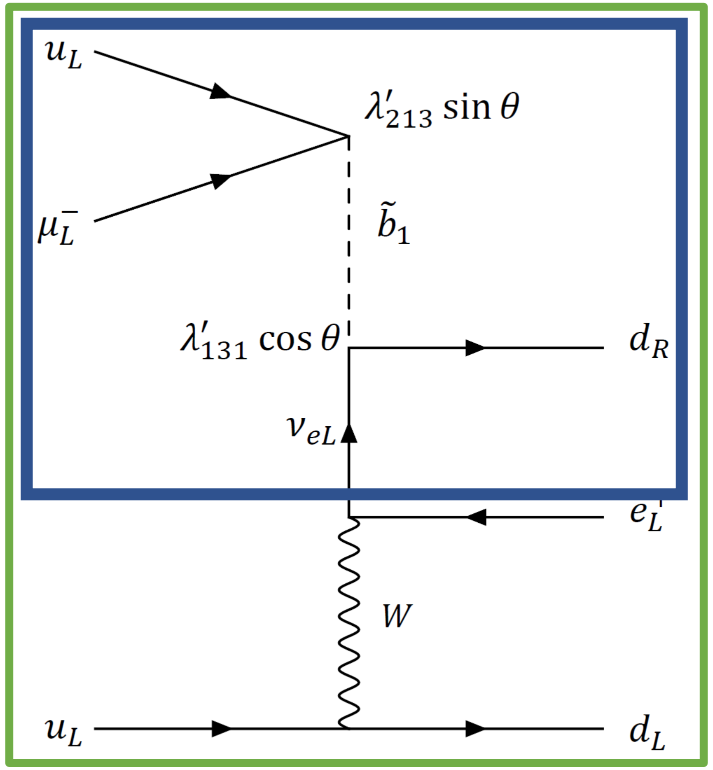
$$\beta_{theory} = \left|\frac{\Gamma_e^{\mathrm{SM}}}{\Gamma_\mu^{\mathrm{SM}}}\right| = 1.2352\times10^{-4}$$

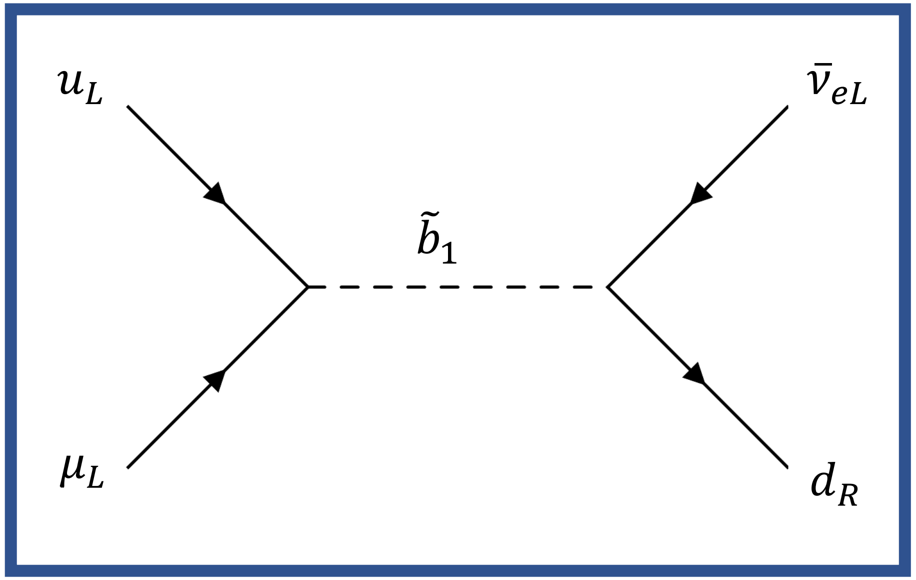
- The constraint of the lepton universality

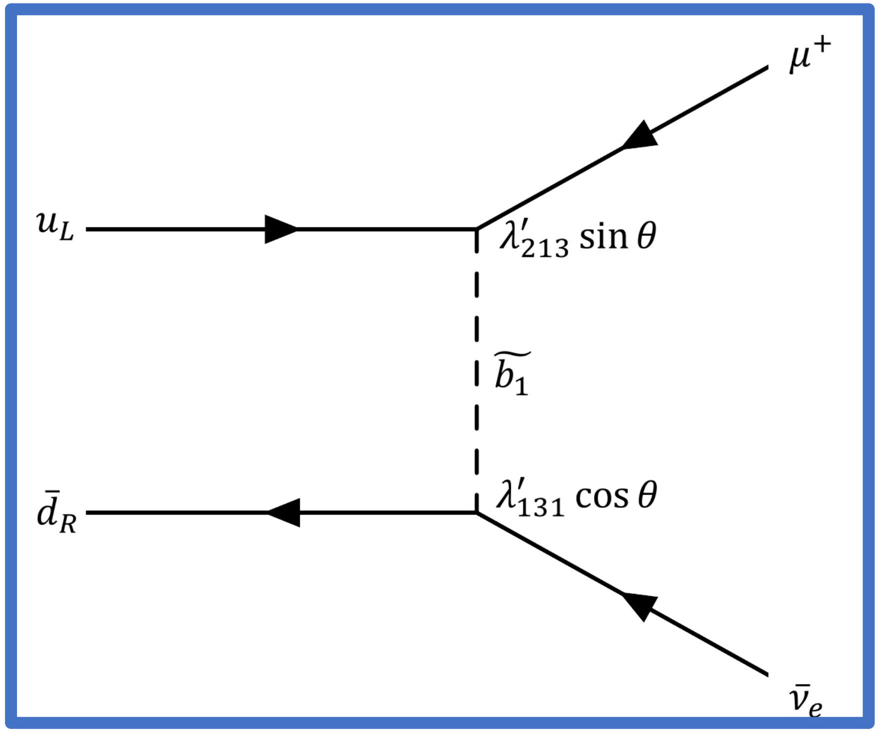
$$\beta_{exp} \pm Err = 1.2327(46)\times10^{-4}$$

Relation between $\mu^- \to e^+$ conversion and π decay

- The bound from π decay is strongly correlated with $\mu^- \to e^+$ conversion.

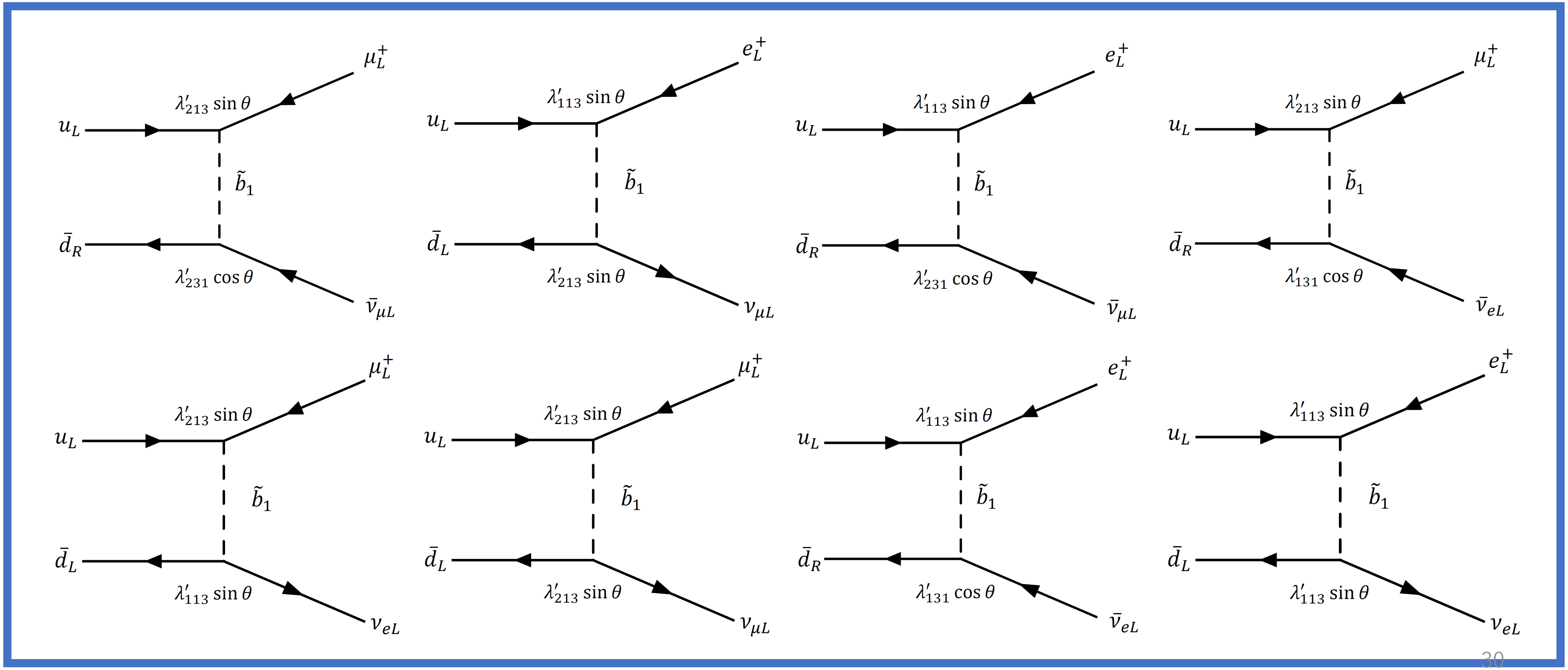






The contribution from sbottom to π decay

- The contribution to the pion decay : the following eight diagrams.



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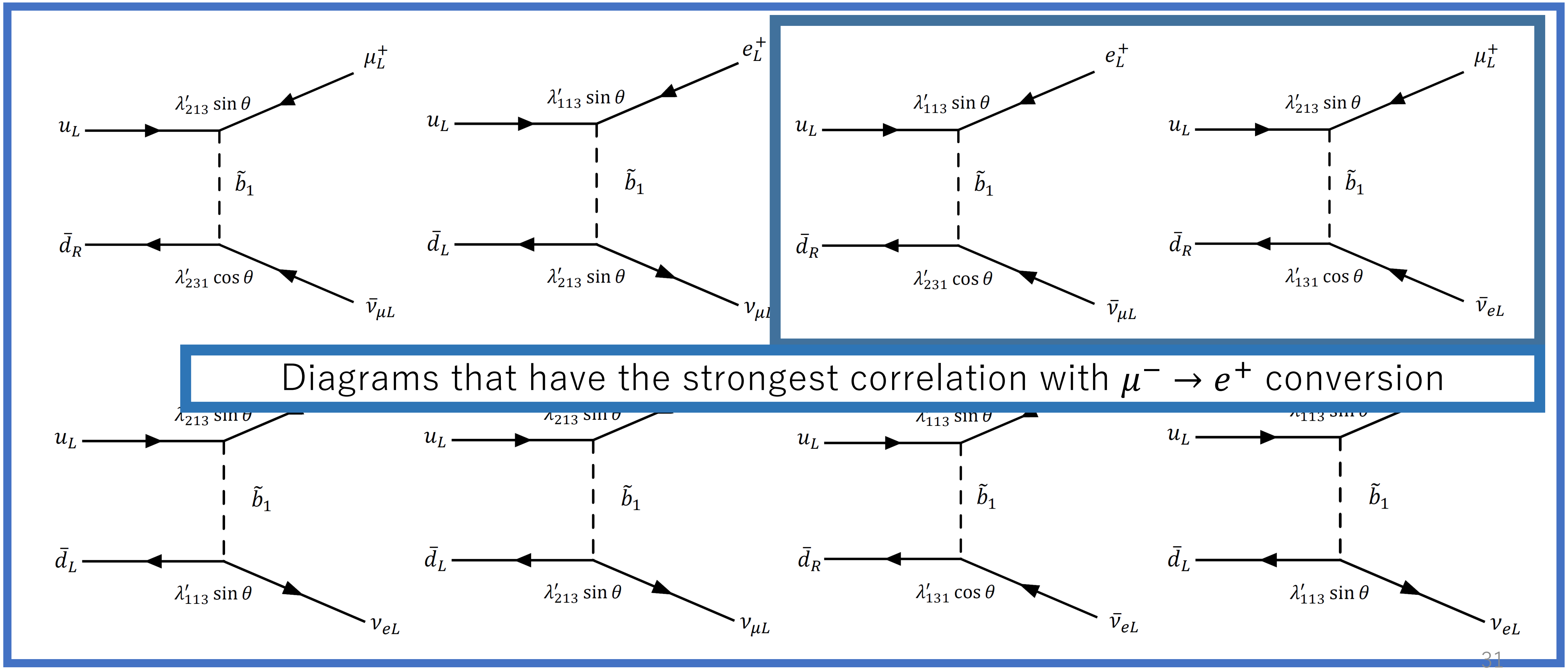


The chiral enhancement

- The part of $\langle 0|(\overline{u_L} d_R)|\pi\rangle$

  By the equation of motion of the quark,

$$\partial_\mu(\bar{u}_L\gamma^\mu d_L) = -m_u(\bar{u}_R d_L) + m_d(\bar{u}_L d_R) = -\frac{(m_u+m_d)}{2}(\bar{u}\gamma^5 d) + \cdots$$

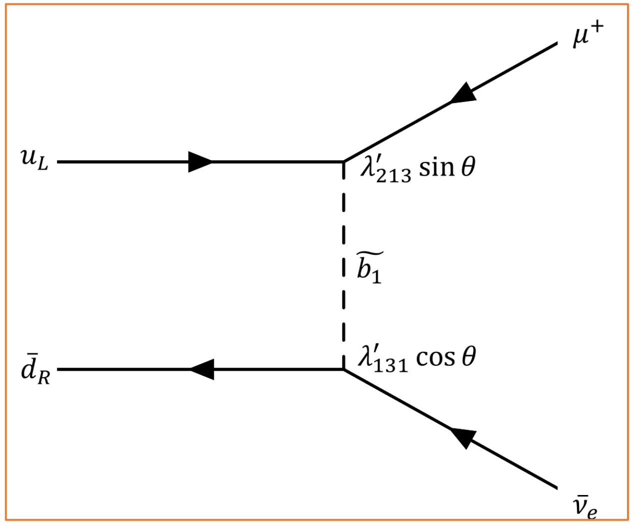
$$\langle 0|(\overline{u_L} d_R)|\pi\rangle \approx -\frac{1}{(m_u+m_d)}\langle 0|\partial_\mu(\overline{u_L}\gamma^\mu d_L)|\pi^+\rangle$$

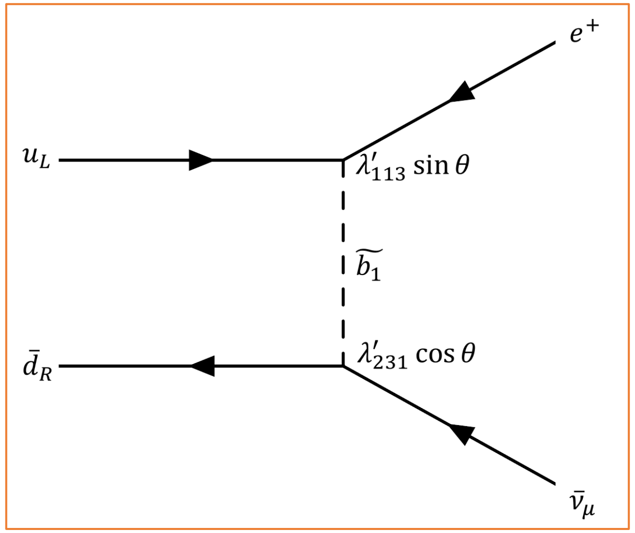
$$= -\frac{1}{(m_u+m_d)} k_{\pi^+\mu}\langle 0|(\overline{u_L}\gamma^\mu d_L)|\pi^+\rangle$$

$$= -\frac{1}{(m_u+m_d)} k_{\pi^+\mu}\left(k^\mu_{\pi^+} f_\pi\right) \qquad \left(f_\pi \approx 130(\mathrm{MeV})\right)$$

$$= -\frac{m_\pi^2 f_\pi}{(m_u+m_d)} \simeq -20 m_\pi f_\pi$$

<u>It is about **20-30** times chiral enhancement at amplitude.</u>





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- The lepton universality

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- The constraint of the lepton universality

$$\beta_{exp} \pm Err = 1.2327(46)\times 10^{-4}$$

- The contribution of RPV

$$\left|\frac{\Gamma_e^{\mathrm{SM}} + \Gamma_e^{\mathrm{RPV}}}{\Gamma_\mu^{\mathrm{SM}} + \Gamma_\mu^{\mathrm{RPV}}}\right| \simeq \beta_{theory} + \left|\frac{\Gamma_e^{\mathrm{RPV}}}{\Gamma_\mu^{\mathrm{SM}}} - \frac{\Gamma_e^{\mathrm{SM}}}{\left(\Gamma_\mu^{\mathrm{SM}}\right)^2}\Gamma_\mu^{\mathrm{RPV}}\right|$$

$$\beta_{exp} - Err - \beta_{theory} \leq \frac{\Gamma_e^{\mathrm{RPV}}}{\Gamma_\mu^{\mathrm{SM}}} - \frac{\Gamma_e^{\mathrm{SM}}}{\left(\Gamma_\mu^{\mathrm{SM}}\right)^2}\Gamma_\mu^{\mathrm{RPV}} \leq \beta_{exp} + Err - \beta_{theory}$$

$$-7.1\times 10^{-7} \leq \frac{\Gamma_e^{\mathrm{RPV}}}{\Gamma_\mu^{\mathrm{SM}}} - \frac{\Gamma_e^{\mathrm{SM}}}{\left(\Gamma_\mu^{\mathrm{SM}}\right)^2}\Gamma_\mu^{\mathrm{RPV}} \leq 2.1\times 10^{-7}$$

The Bound of $0\nu2\beta$

- The Lagrangian that causes $0\nu2\beta$ is

$$\mathcal{L}_{0\nu2\beta} = -\lambda'_{113}\sin\theta\,\overline{(e_L)^C}\,u_L\widetilde{b_R^*} + \lambda'_{131}\cos\theta\,\widetilde{b_L}\,\overline{d_R}\nu_e + \text{h.c.}$$

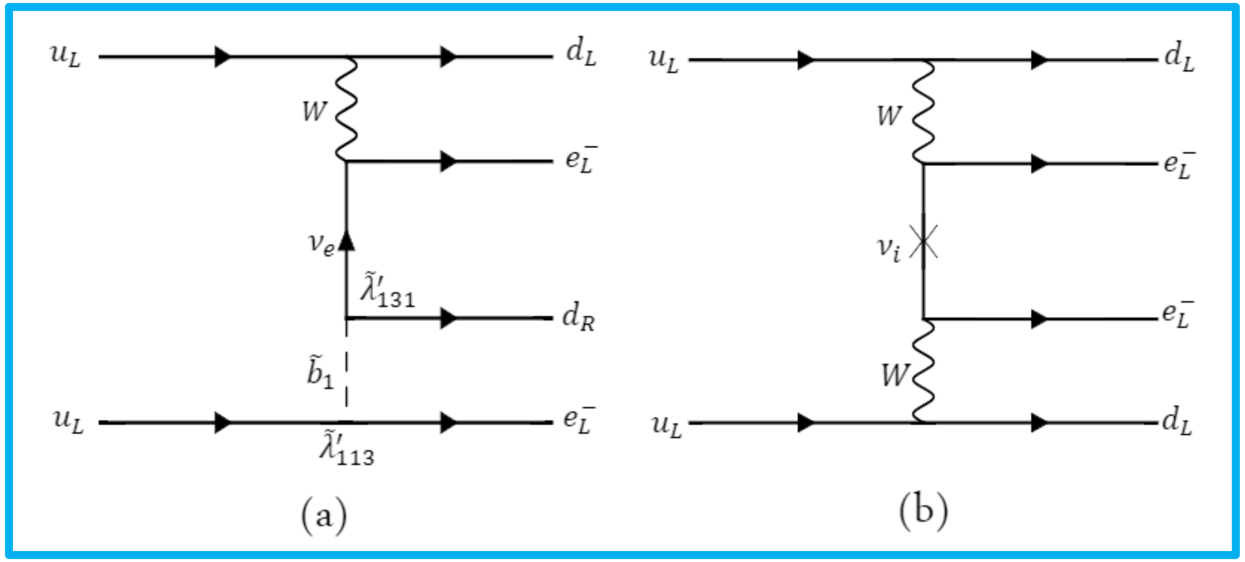
- we can apply the constraints on the Majorana mass to the couplings

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\left[\{\bar{e}\gamma^\alpha(1-\gamma_5)\nu\}\{\bar{u}\gamma_\alpha(1-\gamma_5)d\} + \epsilon_{S+P}\{\bar{e}(1+\gamma_5)\nu\}\{\bar{u}(1+\gamma_5)d\}\right.$$
$$\left.+\epsilon_{TR}\{\bar{e}\sigma^{\alpha\beta}(1+\gamma_5)\nu\}\{\bar{u}\sigma_{\alpha\beta}(1+\gamma_5)d\}\right],$$

$$|\epsilon_{S+P}| = \left|\frac{\sqrt{2}\tilde{\lambda}'_{131}\tilde{\lambda}'_{113}}{2G_F m_1^2}\right|, \qquad |\epsilon_{TR}| = \left|\frac{\sqrt{2}\tilde{\lambda}'_{131}\tilde{\lambda}'_{113}}{8G_F m_1^2}\right|,$$

- The bound of $0\nu2\beta$

$$\lambda'_{113}\sin\theta\,\lambda'_{131}\cos\theta \le 8.3\times10^{-10}\left(\frac{m_1}{200(\text{GeV})}\right)^2$$

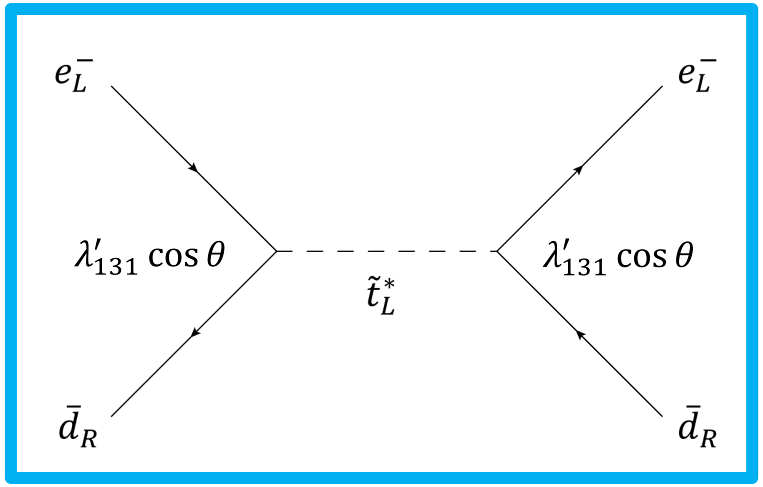


The other bounds

- $\lambda'_{131}\cos\theta$ ： Atomic Parity Violation(APV) and Parity Violating Electron Scattering(PVES)

  P. A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) no.8, 083C01.

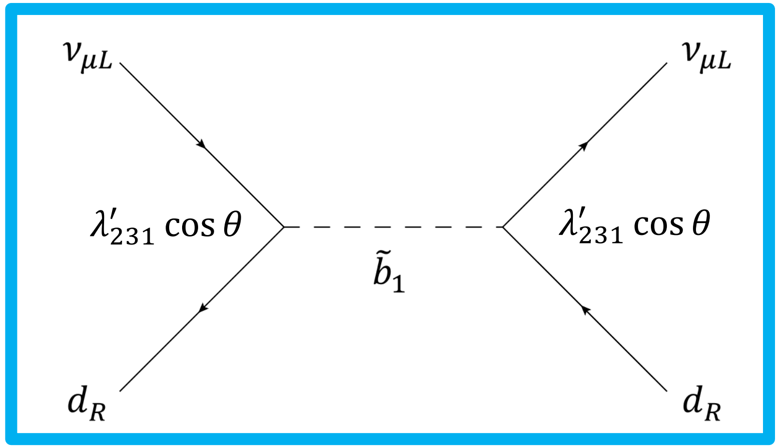
$$|\lambda'_{131}\cos\theta|\left(\frac{1.0(\mathrm{TeV})}{m_{\tilde{t}_L}}\right)\leq 6.9\times10^{-1}$$



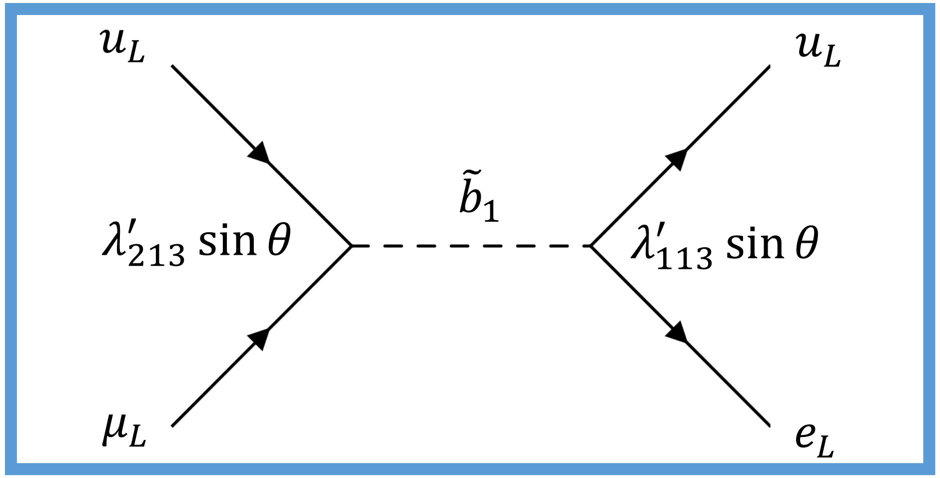
- $\lambda'_{231}\cos\theta$ ： Deep Inerastic Scattering(DIS)

  V. D. Barger, G. F. Giudice and T. Han, Phys. Rev. D 40 (1989), 2987

$$|\lambda'_{231}\cos\theta|\left(\frac{200(\mathrm{GeV})}{m_1}\right)\leq 3.6\times10^{-1}$$



$\mu^- \to e^-$ conversion



- The upper limit for $\mu^- \to e^-$ conversion is

$$B(\mu^- \to e^-;\ \mathrm{Au}) < 7\times10^{-13}$$

- The contribution of RPV

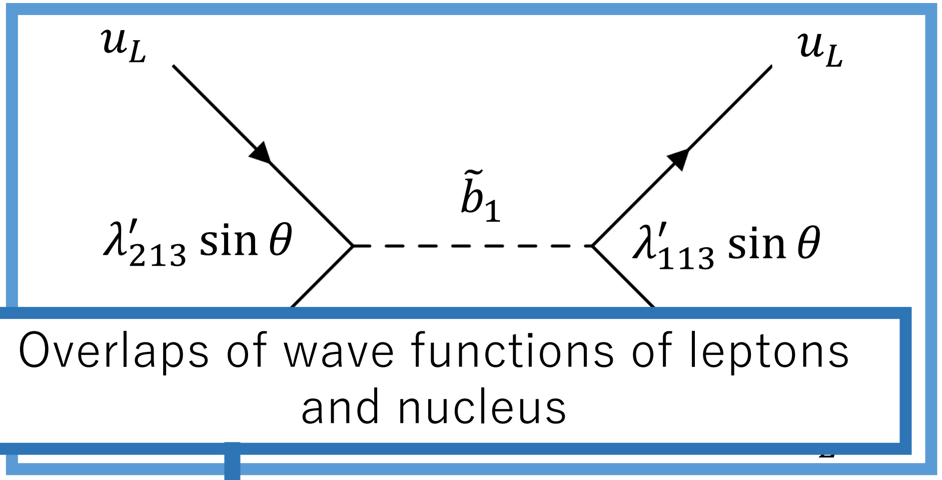
$$B(\mu^- \to e^-) \simeq \tilde{\tau}_\mu \frac{|\lambda'_{213} \sin\theta\, \lambda'_{113} \sin\theta|^2}{4m_1^4} m_\mu^5 \left(2V^{(p)} + V^{(n)}\right)^2$$

- $\tilde{\tau}_\mu$ is the mean lifetime of muon in muonic atom.

- The bound from $\mu^- \to e^-$ conversion is

$$\lambda'_{213} \sin\theta\, \lambda'_{113} \sin\theta < 1.6\times10^{-7} \left(\frac{m_1}{200(\mathrm{GeV})}\right)^2$$

$\mu^- \to e^-$ conversion



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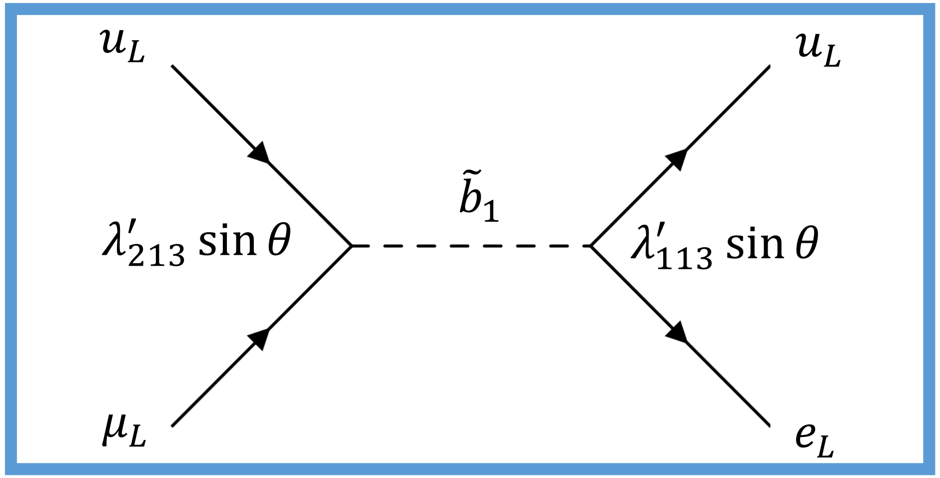

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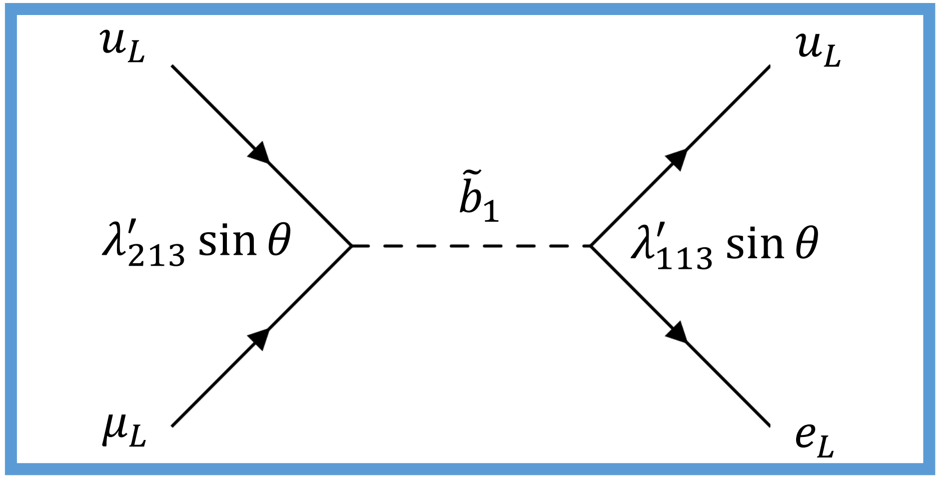
$\mu^- \to e^-$ conversion



- The formula to estimate $\mu^- \to e^-$ conversion

$$B(\mu^- \to e^-) \simeq \tilde{\tau}_\mu \frac{|\lambda'_{213}\sin\theta\,\lambda'_{113}\sin\theta|^2}{4m_1^4} m_\mu^5 \left(2V^{(p)} + V^{(n)}\right)$$

- $\tilde{\tau}_\mu$ is the mean lifetime of muon in muonic atom.

- We used aluminum(Al) to estimate the $\mu^- \to e^-$ conversion. (COMET)

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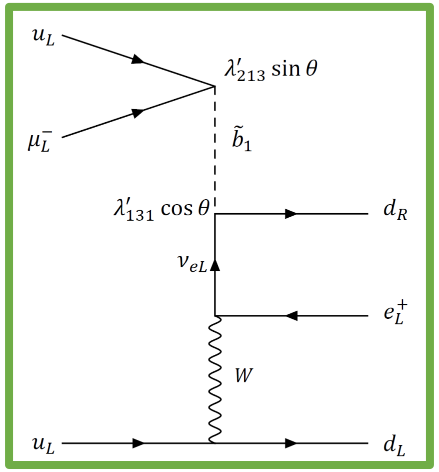
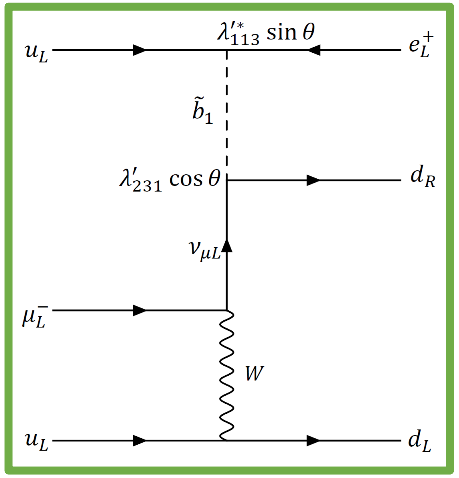
$\mu^- \to e^+$ conversion





- The formula to estimate $\mu^- \to e^+$ conversion

$$B(\mu^- \to e^+) \simeq \tilde{\tau}_\mu \left(\frac{\lambda'_{231}\cos\theta\, \lambda'_{113}\sin\theta + \lambda'_{131}\cos\theta\, \lambda'_{213}\sin\theta}{m_1^2}\right)^2 \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{Q'^8}{q^2} \left(\frac{m_\mu Z_{eff}\alpha}{\pi^{1/3}}\right)^3 \times \left[Z_{eff}\left(1 - 3.125\left(\frac{A-Z}{2A}\right)\right)\right]^2$$

- $\tilde{\tau}_\mu$ is the mean lifetime of muon in muonic atom.
- Q' is typical energy scale including phase space.
- q is the momentum the neutrino.
- Muon capture happens twice

$$\Gamma_{\rm cap} \simeq Z_{\rm eff}^4 X_1 \left(1 - X_2 \frac{A-Z}{2A}\right)$$

(Jeffrey M. Berryman, Andr´e de Gouvˆea, Kevin J. Kelly, and Andrew Kobach, Phys. Rev. D, Vol. 95, No. 11,p. 115010, 2017)

$\mu^- \to e^+$ conversion

The muon wave function $|\psi_\mu(0)|$

- The formula to estimate $\mu^- \to e^+$ conversion

$$B(\mu^- \to e^+) \simeq \tilde{\tau}_\mu \left(\frac{\lambda'_{231}\cos\theta\,\lambda'_{113}\sin\theta + \lambda'_{131}\cos\theta\,\lambda'_{213}\sin\theta}{m_1^2}\right)^2 \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{Q'^8}{q^2}\left(\frac{m_\mu Z_{eff}\alpha}{\pi}\right)^3 \times \left[Z_{eff}\left(1-3.125\left(\frac{A-Z}{2A}\right)\right)\right]^2$$

- $\tilde{\tau}_\mu$ is the mean lifetime of muon in muonic atom.
- Q' is typical energy scale including phase space.
- q is the momentum the neutrino.

$\mu^- \to e^+$ conversion

Pauli exclusion principle for neutron transitions
$\mu^- + (A, Z) \to e^+ + (A, Z-2)$

- The formula to estimate $\mu^- \to e^+$ conversion

$$B(\mu^- \to e^+) \simeq \tilde{\tau}_\mu \left(\frac{\lambda'_{231} \cos\theta \, \lambda'_{113} \sin\theta + \lambda'_{131} \cos\theta \, \lambda'_{213} \sin\theta}{m_1^2} \right)^2 \left(\frac{G_F}{\sqrt{2}} \right)^2 \frac{Q'^8}{q^2} \left(\frac{m_\mu Z_{eff} \alpha}{\pi} \right)^2 \times \left[Z_{eff} \left(1 - 3.125 \left(\frac{A - Z}{2A} \right) \right) \right]^2$$

- $\tilde{\tau}_\mu$ is the mean lifetime of muon in muonic atom.
- Q' is typical energy scale including phase space. ～muon mass
- q is the momentum the neutrino.

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Result (Pattern I)

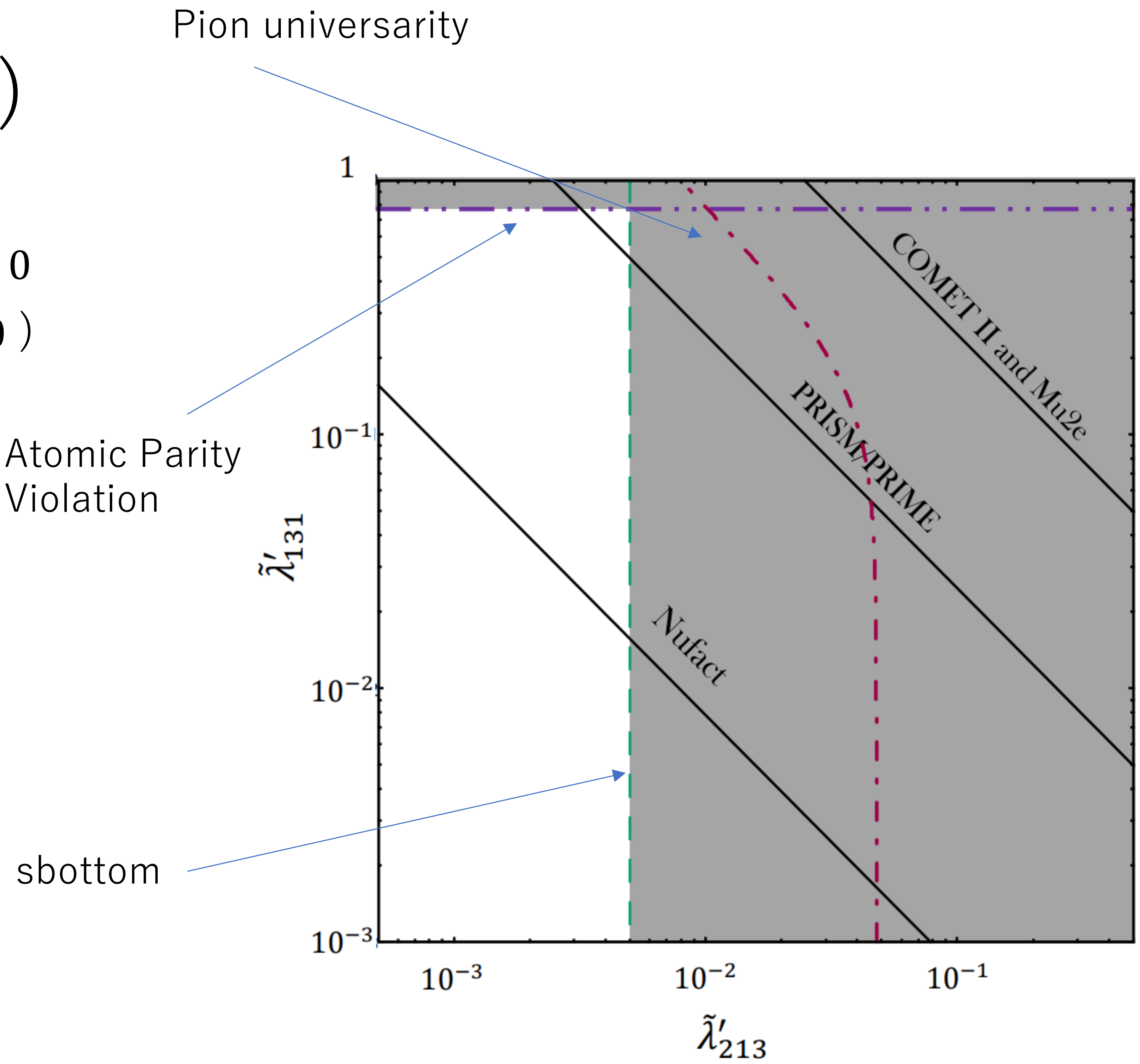
- $\lambda'_{213}\sin\theta \neq 0, \lambda'_{131}\cos\theta \neq 0 \ (\lambda'_{213}\sin\theta = \lambda'_{131}\cos\theta = 0\ )$

experiment	bounds
$Q_{\rm weak}$ collaboration : APV, PVES	$\lambda'_{131}\cos\theta_1 \leq 0.69\ (\ m_{\tilde{t}_L}/1.0(\mathrm{TeV})\ )$
DIS	$\lambda'_{231}\cos\theta_1 \leq 0.36\ (\ m_1/200(\mathrm{GeV})\ )$
ATLAS : sbottom direct search	$\lambda'_{213}\sin\theta_1 \leq 5.0\times10^{-3}$
	$\lambda'_{113}\sin\theta_1 \leq 5.0\times10^{-3}$
$0\nu\beta\beta$	$\lambda'_{113}\sin\theta_1\lambda'_{131}\cos\theta_1 \leq 2.65\times10^{-9}\ (\ m_1/200(\mathrm{GeV})\ )^2$
$\mu^- \to e^-$ conversion	$\lambda'_{213}\sin\theta_1\lambda'_{113}\sin\theta_1 \leq 1.63\times10^{-7}\ (\ m_1/200(\mathrm{GeV})\ )^2$

and the bound of π decay

Result (Pattern I)

- $\lambda'_{213} \sin\theta \neq 0, \lambda'_{131} \cos\theta \neq 0$
  $(\lambda'_{213} \sin\theta = \lambda'_{131} \cos\theta = 0)$
- $\mathrm{B}(\mu^- \to e^-; \mathrm{Al}) = 0$
- $\mathrm{B}(\mu^- \to e^+; \mathrm{Ca}) \sim 10^{-18}$
  ⇒ PRISM/PRIME



Result (Pattern II)

- $\lambda'_{113}\sin\theta \neq 0, \lambda'_{231}\cos\theta \neq 0$ $(\lambda'_{113}\sin\theta = \lambda'_{231}\cos\theta = 0)$

experiment	bounds
$Q_{\rm weak}$ collaboration : APV, PVES	$\lambda'_{131}\cos\theta_1 \leq 0.69\,(\,m_{\tilde{t}_L}/1.0({\rm TeV})\,)$
DIS	$\lambda'_{231}\cos\theta_1 \leq 0.36\,(\,m_1/200({\rm GeV})\,)$
ATLAS : sbottom direct search	$\lambda'_{213}\sin\theta_1 \leq 5.0\times10^{-3}$
	$\lambda'_{113}\sin\theta_1 \leq 5.0\times10^{-3}$
$0\nu\beta\beta$	$\lambda'_{113}\sin\theta_1\lambda'_{131}\cos\theta_1 \leq 2.65\times10^{-9}\,(\,m_1/200({\rm GeV})\,)^2$
$\mu^-\to e^-$ conversion	$\lambda'_{213}\sin\theta_1\lambda'_{113}\sin\theta_1 \leq 1.63\times10^{-7}\,(\,m_1/200({\rm GeV})\,)^2$

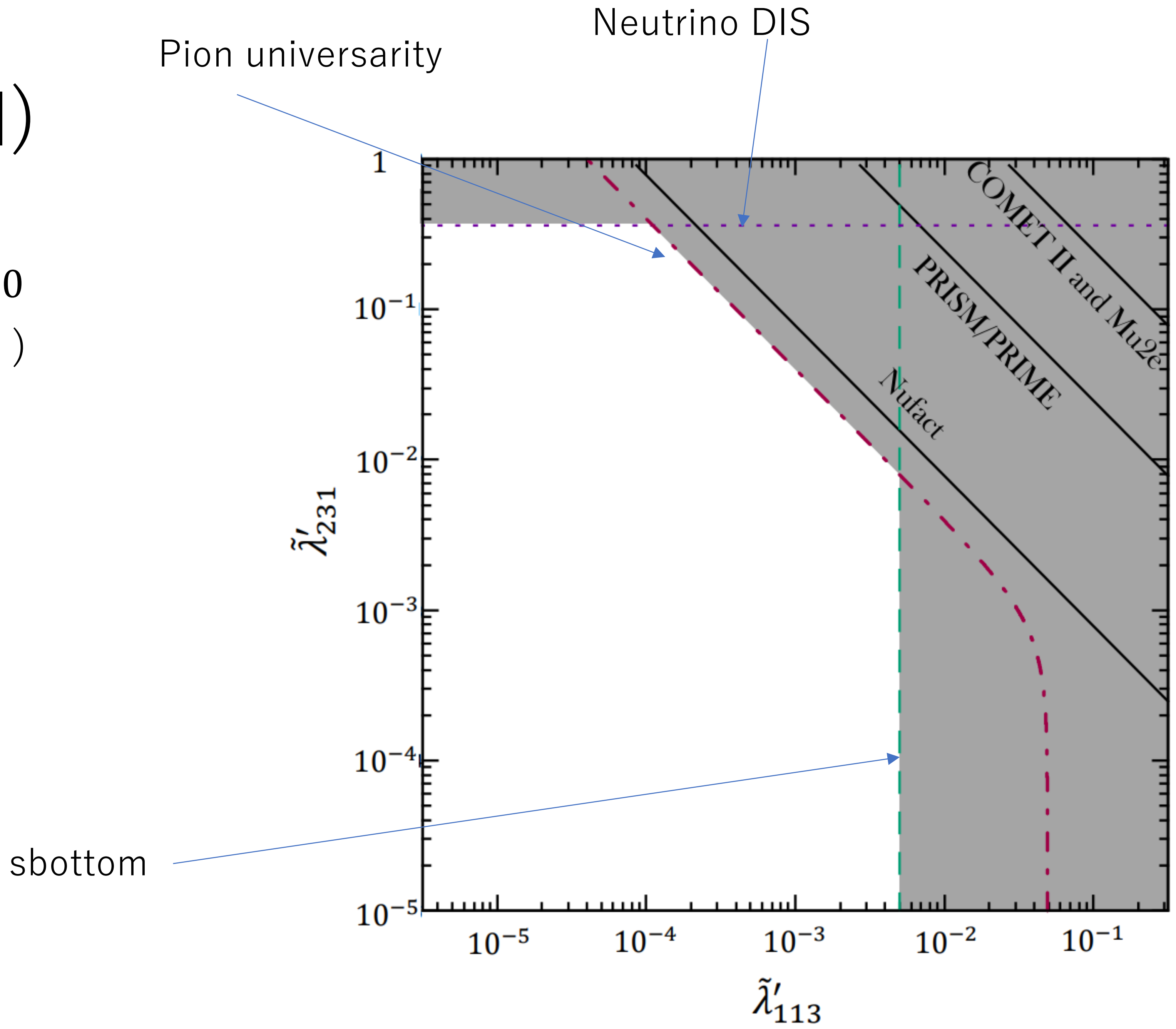
and the bound of π decay

Result (Pattern II)

- $\lambda'_{113}\sin\theta \neq 0, \lambda'_{231}\cos\theta \neq 0$
  $(\lambda'_{113}\sin\theta = \lambda'_{231}\cos\theta = 0\ )$

- $\mathrm{B}(\mu^- \to e^-; \mathrm{Al}) = 0$

- $\mathrm{B}(\mu^- \to e^+; \mathrm{Ca}) \sim 10^{-22}$



The expectation for future experiments

- Resultss are …

Nucleus	Z_{eff}	$\tilde{\tau}_\mu$ [ns]	$\tilde{B}$ (Pattern I)	$\tilde{B}$ (Pattern II)
^{27}Al	11.48	864	7.0×10^{-19}	9.2×10^{-23}
^{32}S	13.64	540	1.4×10^{-18}	1.8×10^{-22}
^{40}Ca	16.15	333	2.0×10^{-18}	2.6×10^{-22}
^{48}Ti	17.38	330	1.4×10^{-18}	1.8×10^{-22}
^{65}Zn	21.61	161	2.2×10^{-18}	2.8×10^{-22}
^{73}Ge	22.43	167.4	1.6×10^{-18}	2.1×10^{-22}

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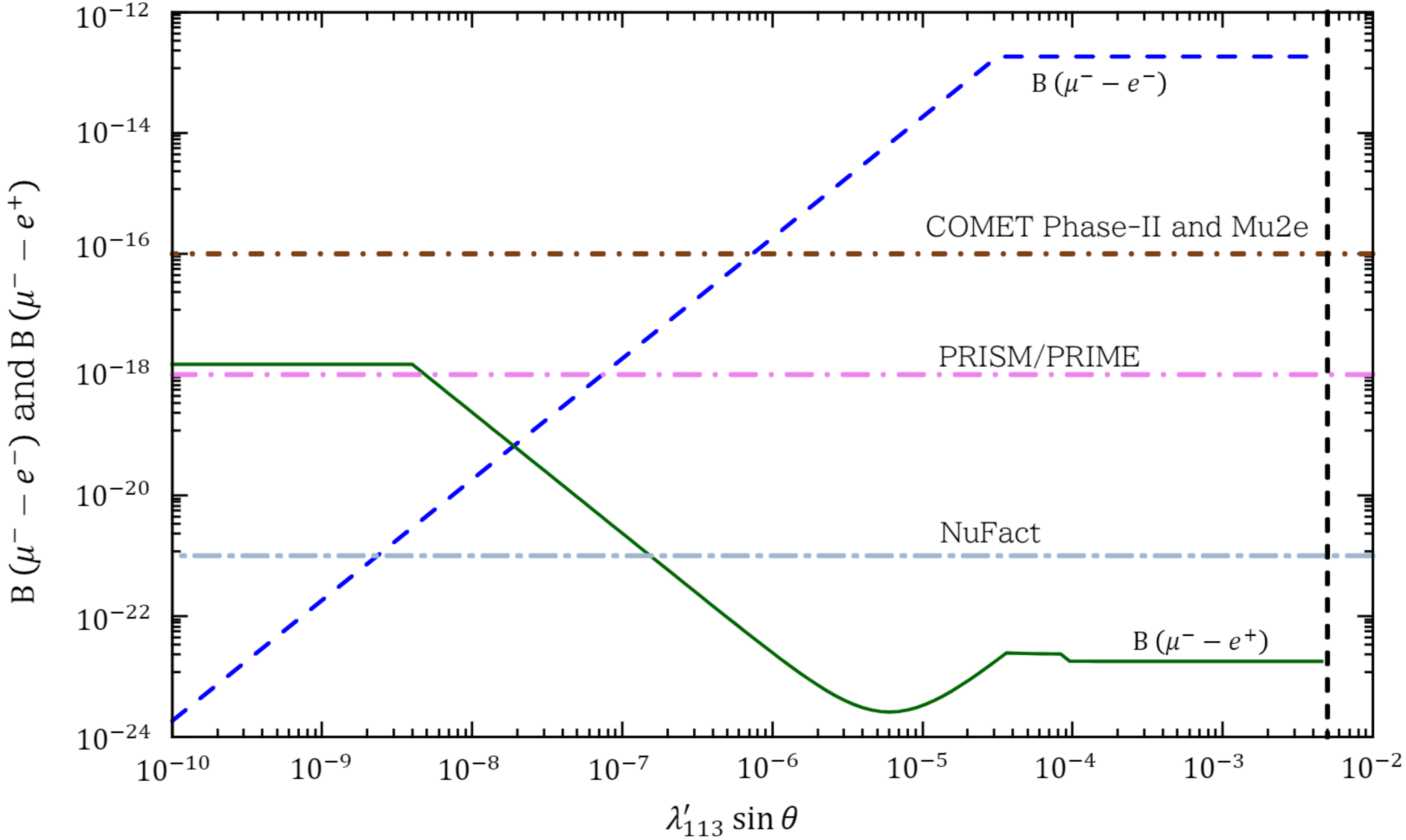
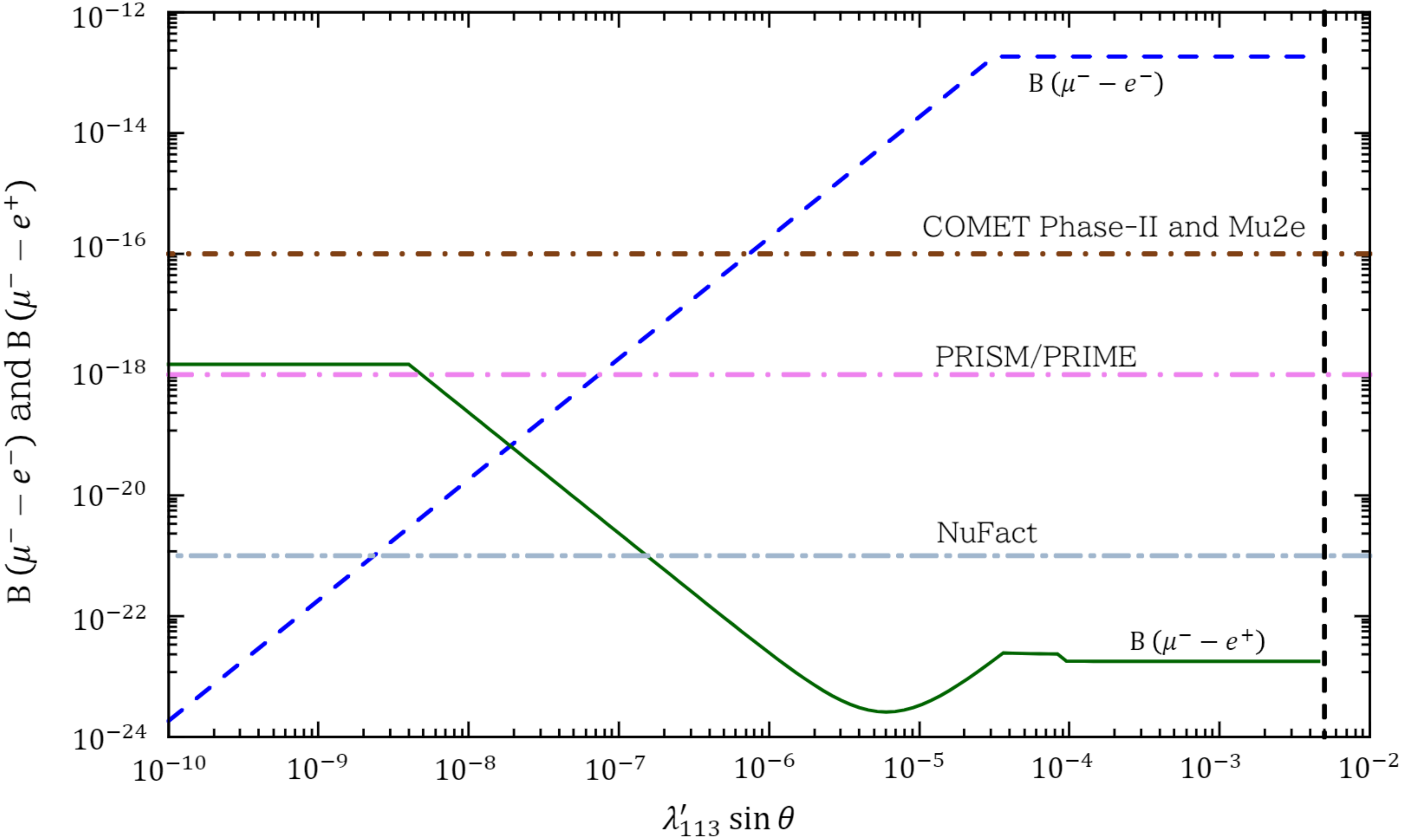
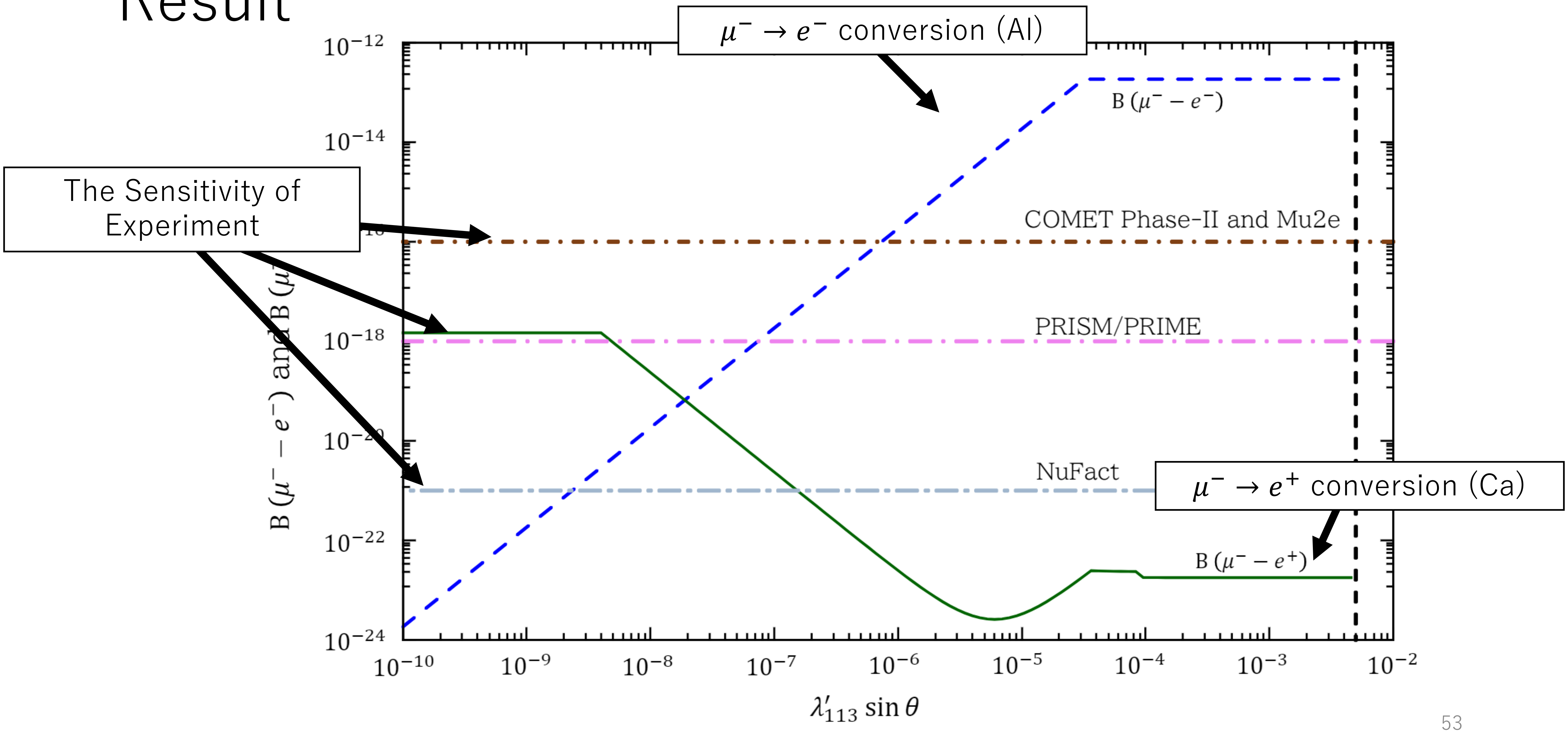
Result (all coupling)



Result (all coupling)

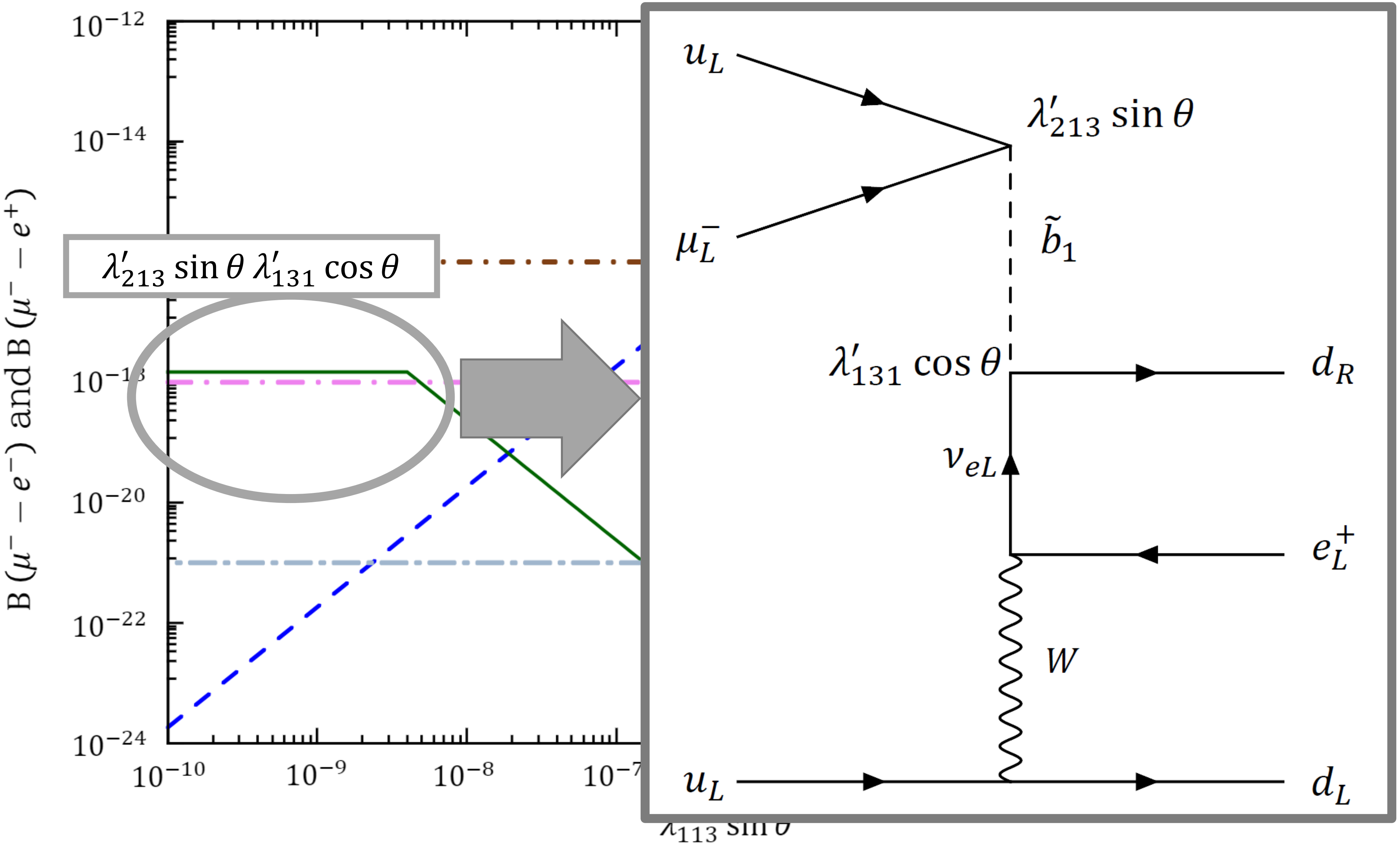


Result



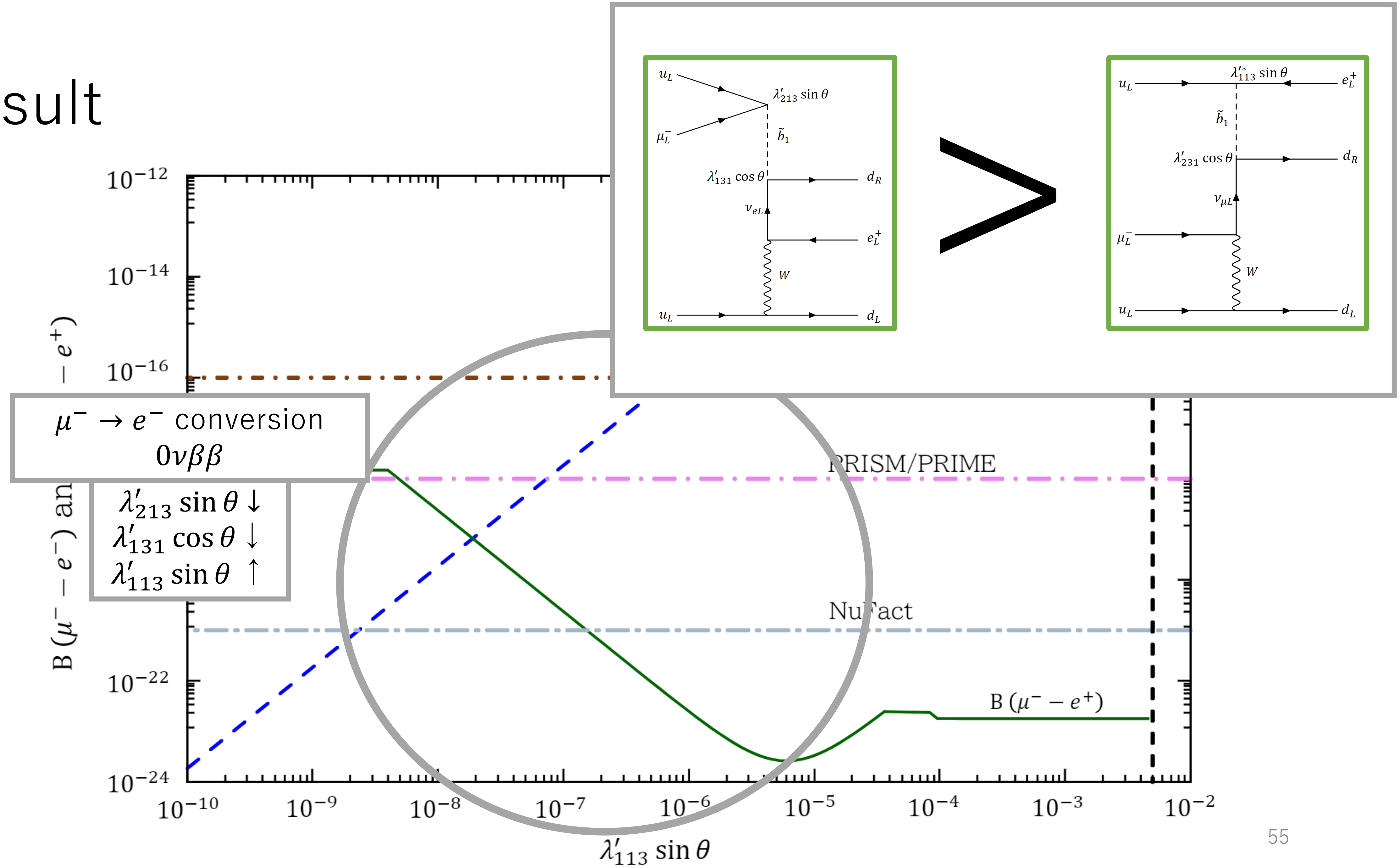
$\mu^- \rightarrow e^-$ conversion (Al)
The Sensitivity of Experiment
$\mu^- \rightarrow e^+$ conversion (Ca)
B $(\mu^- - e^-)$
COMET Phase-II and Mu2e
PRISM/PRIME
NuFact
B $(\mu^- - e^+)$
B $(\mu^- - e^-)$ and B $(\mu^- - e^+)$
$\lambda'_{113} \sin\theta$
10^{-12}
10^{-14}
10^{-18}
10^{-22}
10^{-24}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

Result



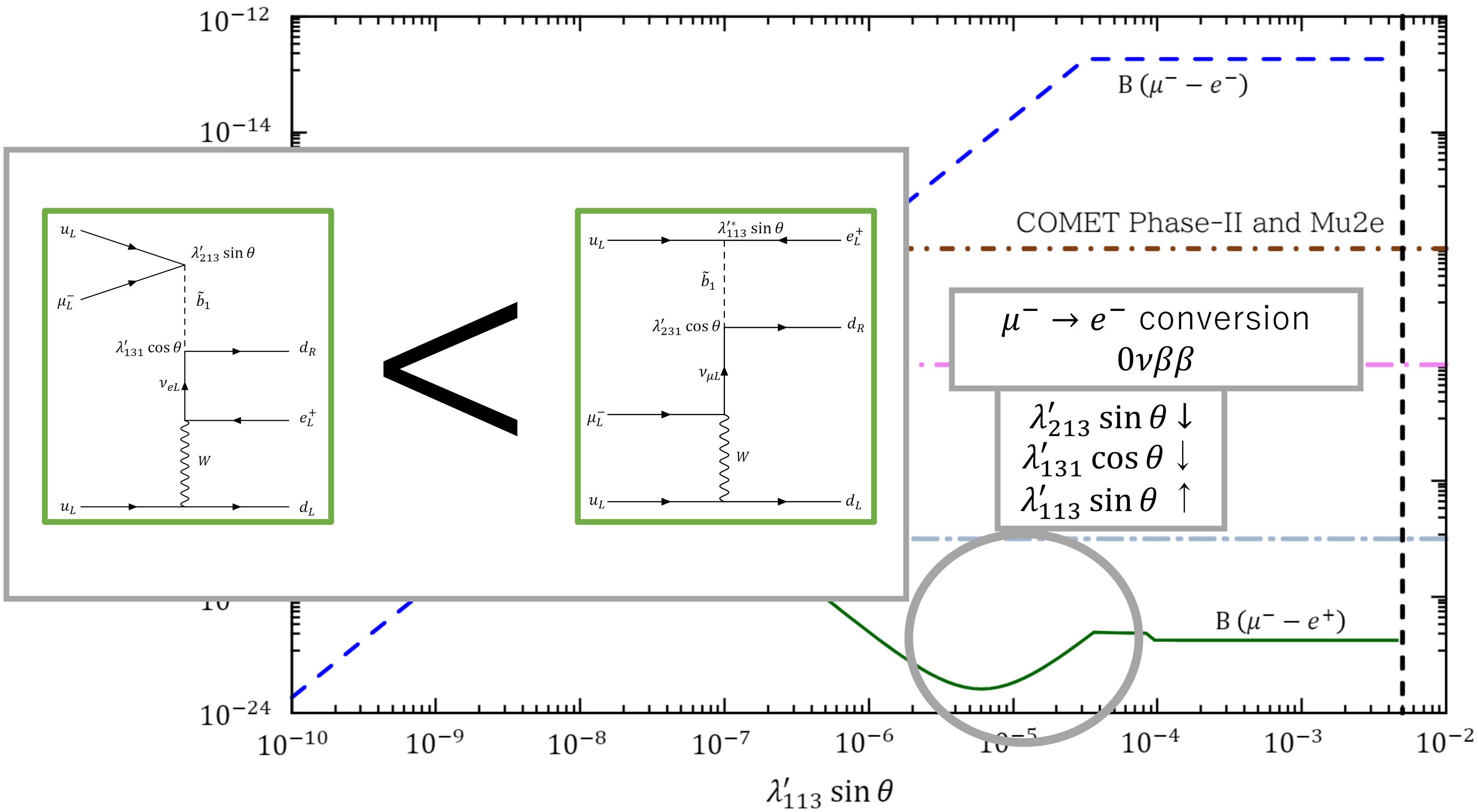
10^{-12}
10^{-14}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
B $(\mu^- - e^-)$ and B $(\mu^- - e^+)$
10^{-10}
10^{-9}
10^{-8}
$\lambda'_{213} \sin\theta \, \lambda'_{131} \cos\theta$
u_L
μ_L^-
$\lambda'_{213} \sin\theta$
$\tilde{b}_1$
$\lambda'_{131} \cos\theta$
d_R
ν_{eL}
e_L^+
W
u_L
d_L

Result



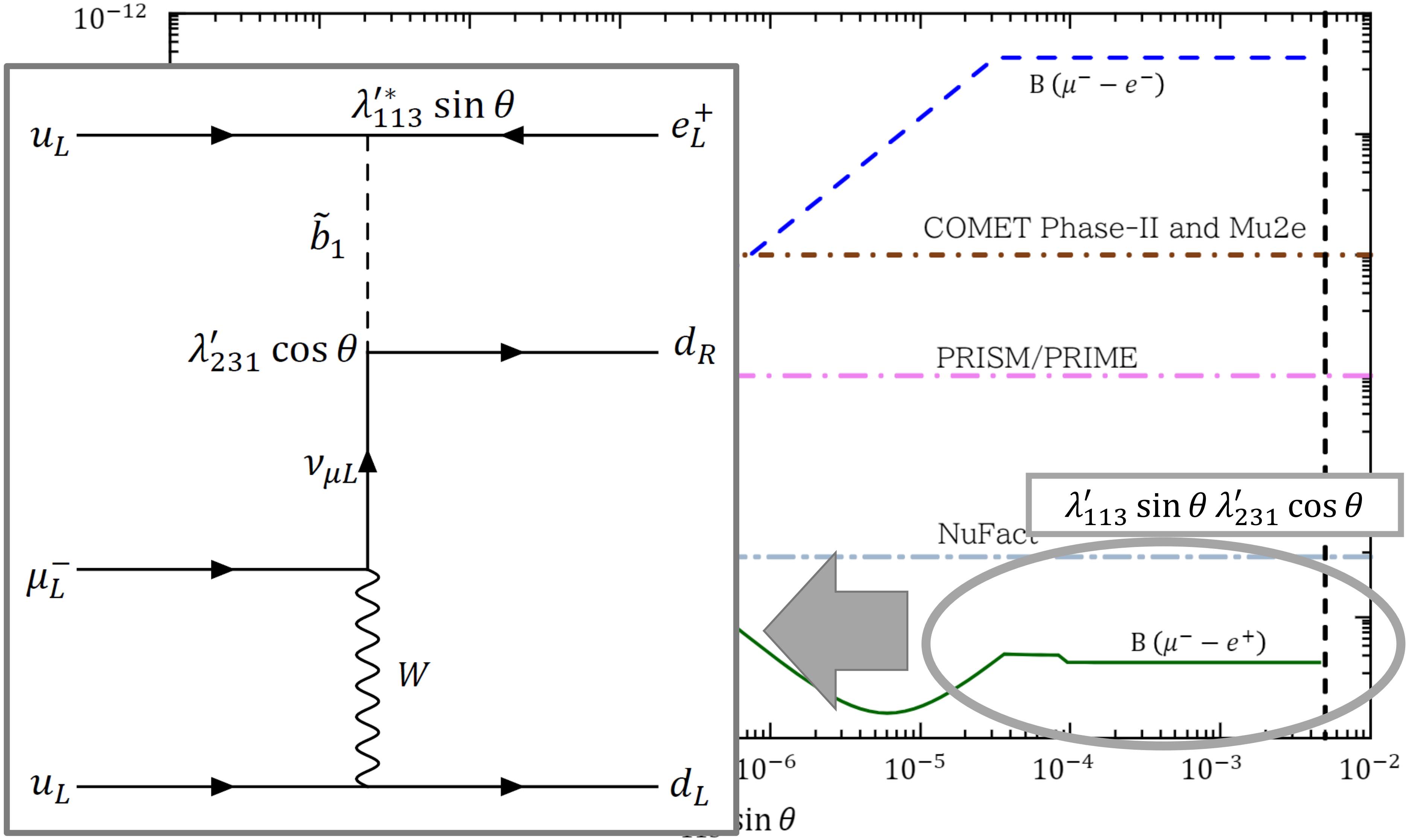
u_L
μ_L^-
$\lambda'_{213}\sin\theta$
$\tilde{b}_1$
$\lambda'_{131}\cos\theta$
d_R
ν_{eL}
e_L^+
W
u_L
d_L
>
$\lambda'^*_{113}\sin\theta$
u_L
e_L^+
$\tilde{b}_1$
$\lambda'_{231}\cos\theta$
d_R
$\nu_{\mu L}$
μ_L^-
W
u_L
d_L
$\mu^- \to e^-$ conversion
$0\nu\beta\beta$
$\lambda'_{213}\sin\theta \downarrow$
$\lambda'_{131}\cos\theta \downarrow$
$\lambda'_{113}\sin\theta \uparrow$
PRISM/PRIME
NuFact
$\mathrm{B}(\mu^- - e^+)$
10^{-12}
10^{-14}
10^{-16}
10^{-22}
10^{-24}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
$\lambda'_{113}\sin\theta$

Result



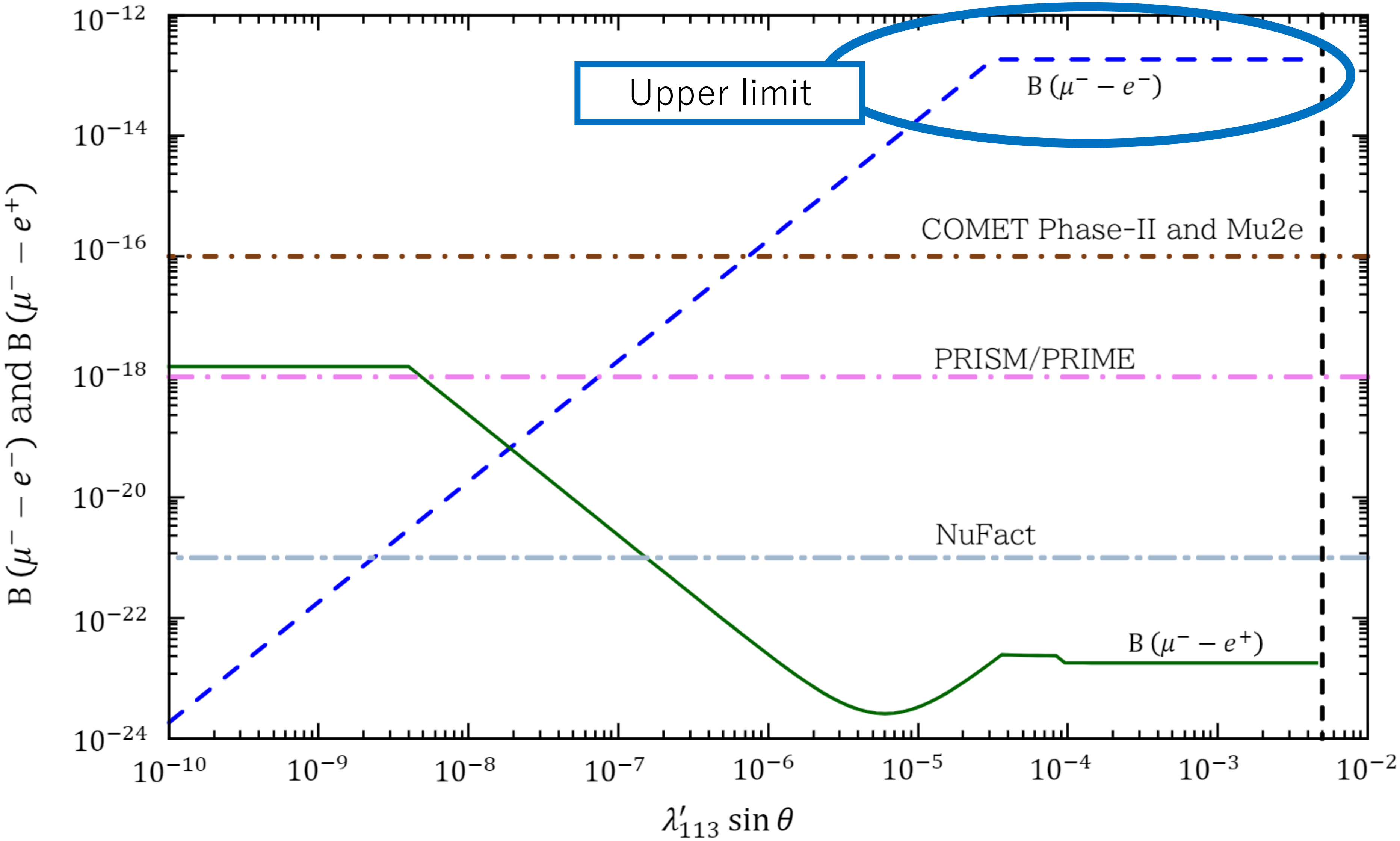
10^{-12}
10^{-14}
10^{-24}
$B(\mu^- - e^-)$
COMET Phase-II and Mu2e
$\mu^- \to e^-$ conversion
$0\nu\beta\beta$
$\lambda'_{213}\sin\theta \downarrow$
$\lambda'_{131}\cos\theta \downarrow$
$\lambda'_{113}\sin\theta \uparrow$
$B(\mu^- - e^+)$
u_L
μ_L^-
$\lambda'_{213}\sin\theta$
$\tilde{b}_1$
$\lambda'_{131}\cos\theta$
ν_{eL}
d_R
e_L^+
W
d_L
$\lambda'^*_{113}\sin\theta$
$\lambda'_{231}\cos\theta$
$\nu_{\mu L}$
10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-4} 10^{-3} 10^{-2}
$\lambda'_{113}\sin\theta$

Result



10^{-12}
B ($\mu^- - e^-$)
COMET Phase-II and Mu2e
PRISM/PRIME
NuFact
$\lambda'_{113}\sin\theta\,\lambda'_{231}\cos\theta$
B ($\mu^- - e^+$)
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
$\sin\theta$
u_L
$\lambda'^*_{113}\sin\theta$
e^+_L
$\tilde{b}_1$
$\lambda'_{231}\cos\theta$
d_R
$\nu_{\mu L}$
μ^-_L
W
u_L
d_L

Result

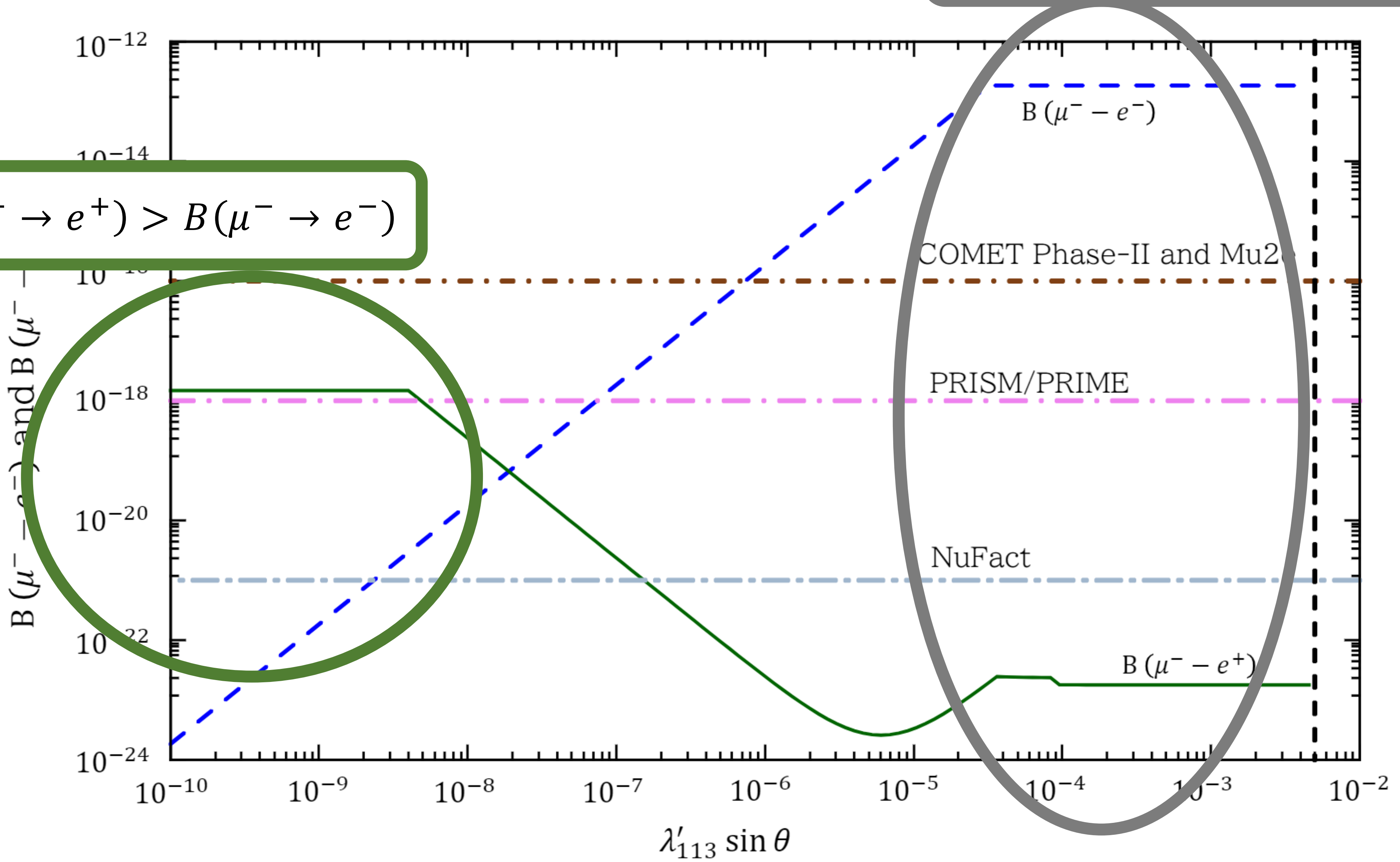


Upper limit
B ($\mu^- - e^-$)
COMET Phase-II and Mu2e
PRISM/PRIME
NuFact
B ($\mu^- - e^+$)
B ($\mu^- - e^-$) and B ($\mu^- - e^+$)
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2}
$\lambda'_{113} \sin\theta$

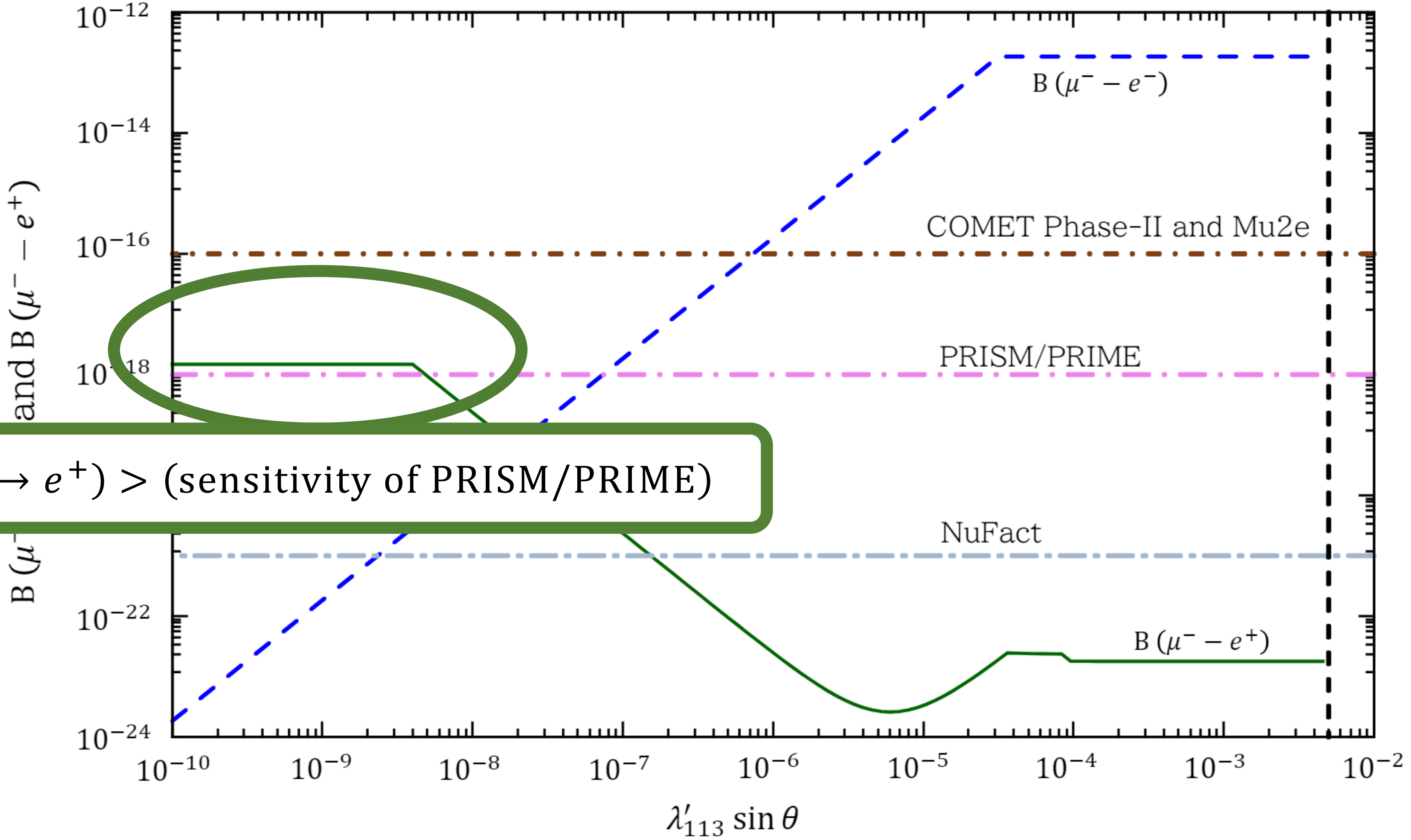
Result

- $B(\mu^- \to e^+) < B(\mu^- \to e^-)$

- $B(\mu^- \to e^+) > B(\mu^- \to e^-)$



Result



- $B(\mu^- \to e^+) >$ (sensitivity of PRISM/PRIME)

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Summary

- $\mu^- \to e^+$ conversion (LNV) is lead by

  1. coupling of doublet and singlet types of leptoquarks (particle #)

  and

  2. RPV interaction.(Lepton Flavor)

- The case where $B(\mu^- \to e^+) > (\mathrm{PRISM/PRIME\ sensitivity})$ can be realized.
- The case where $B(\mu^- \to e^+) > B(\mu^- \to e^-)$ can be realized.

  ⇒**The $\boldsymbol{\mu^- \to e^+}$ conversion is so important!!**

- Complementary verification of the $\mu^- \to e^+$ conversion and the $\mu^- \to e^-$ conversion is very useful for model verification.

Backup

Derivation of lepton flavor charge

Lepton Part Only

$$L_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad e_{Ri}, \quad (i = 1, 2, 3)$$

Kinetic Part

$$\mathcal{L}_k = \bar{L}_i i \not{D}_L L_i + \bar{e}_{Ri} i \not{D}_R e_{Ri}$$

$$D_{L\mu} = \begin{pmatrix} \partial_\mu + \frac{i}{2} g_1 B_\mu - g_2 \frac{i}{2} W^0_\mu, & g_2 \frac{i}{\sqrt{2}} W^+_\mu \\ g_2 \frac{i}{\sqrt{2}} W^-_\mu, & \partial_\mu + \frac{i}{2} g_1 B_\mu + \frac{i}{2} g_2 W^0_\mu \end{pmatrix}$$

$$D_{R\mu} = \partial_\mu + i g_1 B_\mu$$

$$\mathcal{L}_k = \mathcal{L}_{k,diag} + \mathcal{L}_{k,W}$$

$$\mathcal{L}_{k,diag} = \bar{\Phi}_j i \not{D}_j \Phi_j$$

$$D_{\mu j =} \partial_\mu - ieQ_j A_\mu - ig_Z (T_{j3} - Q_j \sin^2\theta_w) Z_\mu \quad 3 \times 3$$

$$\Phi_j = \{\nu_L, e_L, e_R\}$$

Sum of 3 species of Weyl sprinors

Invariant under **3** independent unitary transformation

$$U_l, \quad l = \nu_L,\ e_L,\ e_R, \quad 3 \times 3 \text{ UnitaryMatrix}$$

$$l \;\to\; U_l l \;\; (e_{Li} \to (U_{e_L} e_L)_i \quad) \;\; U_l$$ independent

$$\mathcal{L}_{k,W} = ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_{Li} \gamma_\mu e_{Li} \ + \ h.c.$$

To make it invariant $U_{\nu_L} = U_{e_L}$ Is necessary. Reduction of symmetry

Higgs Part

$$\mathcal{L}_H = Y_{ij} \bar{L}_i e_{Rj} + h.c.$$

Y_{ij} **3 × 3 complex :: diagonalized by 2 unitary matrices**

$$Y_{ij} \longrightarrow Y_{diag} = \mathrm{diag}\{y_e, y_\mu, y_\tau\} = U_L Y_{ij} U_R^\dagger$$

$$L_\alpha \equiv U_{L\alpha i} L_i = \begin{pmatrix} U_{L\alpha i} \nu_{Li} \\ U_{L\alpha i} e_{Li} \end{pmatrix}, \quad e_{R\alpha} \equiv U_{Ri} E_{Ri}, \quad \alpha = e, \mu, \tau$$

$$\begin{aligned} \mathcal{L}_H &= Y_\alpha \bar{L}_\alpha e_{R\alpha} + h.c. \\ &= h^+ \left(y_e \bar{\nu}_{eL} e_R + y_\mu \bar{\nu}_{\mu L} \mu_R + y_\tau \bar{\nu}_{\tau L} \tau_R \right) \\ &\quad + h^0 \left(y_e \bar{e}_L e_R + y_\mu \bar{\mu}_L \mu_R + y_\tau \bar{\tau}_L \tau_R \right) + h.c. \end{aligned}$$

Since $U_{\nu_L} = U_{e_L}$ kinetic term is invariant

$$\mathcal{L}_{k,diag} = \bar{\Phi}_\alpha i \not{D} \Phi_\alpha \qquad \Phi_\alpha = \{\nu_{\alpha L}, e_{\alpha L}, e_{\alpha R}\}$$

$$\mathcal{L}_{k,W} = ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_{\alpha L} \gamma_\mu e_{\alpha L} \ + \ h.c.$$

Kinetic terms under flavor basis !!

Residual symmetry ：：Lepton Flavor

$$\Phi_\alpha = \{\nu_{L\alpha}, e_{L\alpha}, e_{R\alpha}\} \quad \alpha = e, \mu, \tau$$

Paired with same flavor
→Lagrangian is invariant under phase shift of each flavor
→**Lepton flavor conservation**

e.g $$\{e'_L, e'_R, \nu'_{eL}\} = \exp\{-i\theta_e\}\{e_L, e_R, \nu_{eL}\}$$

Phase transformation of electron flavor

$$\begin{aligned}\mathcal{L}'_{k,W} &= ig_2\frac{1}{\sqrt{2}}W^+_\mu \bar{\nu}'_e\gamma_\mu e'_L \ + \ h.c. \\ &= ig_2\frac{1}{\sqrt{2}}W^+_\mu \bar{\nu}_e e^{i\theta}\gamma_\mu e^{-i\theta}e_L + h.c = \mathcal{L}_{k,W}\end{aligned}$$

From Noether's theorem Consered current exists

In each flavor the conserved current is given by

$$j^\mu_\alpha = \bar{\nu}_{L\alpha}\gamma^\mu\nu_{L\alpha} + \bar{e}_{L\alpha}\gamma^\mu e_{L\alpha} + \bar{e}_{R\alpha}\gamma^\mu e_{R\alpha}$$

"Charge" is expressed as follows and it conserve

$$Q_\alpha = \int d^3x j^0_\alpha$$

For example $\alpha = e$, that is, electron flavor charge is given in terms of creation and annihilation operators of electrons

$$Q_e = L_e = \int d^3p \sum_{l=\nu_{eL}, e_L, e_R} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}) - d_l^\dagger(\mathbf{p}) d_l(\mathbf{p})$$

$b^\dagger b$ number operator for particle

$d^\dagger d$ number operator for anti – particle

Electron and electron neutrino $L_e = +1$

Positron and anti-electron neutrino $L_e = -1$

Similarly muon and tau flavor charge $L_\mu,\ L_\tau$ is defined.

Lepton Flavor is conserved under SM

Electron, muon, tau number

L_e L_μ L_τ

	e^-	ν_e	μ^-	ν_μ	τ^-	ν_τ
L_e	1	1				
L_μ			1	1		
L_τ					1	1

Opposite (−1) for anti particles

If SM is correct,
in **all** process, these numbers are **conserved**

Contraposition

If **non-conserved** is found, SM is not correct

With additional particles and hence additional operator in Lagrangian,
in general, under the transformation

$$\{\alpha'_L, \alpha'_R, \nu'_{\alpha L}\} = \exp\{-i\theta_\alpha\}\{\alpha_L, \alpha_R, \nu_{\alpha L}\} \quad \alpha = e, \mu, \tau$$

+ appropriate transformation for extra particles

Lagrangian is not invariant

→Lepton flavor cannot be defined

→Lepton flavor “ charge” defined under SM Lagrangian

cannot be conserved