Inflation, Primordial Black Holes, and Gravitational Wave Background

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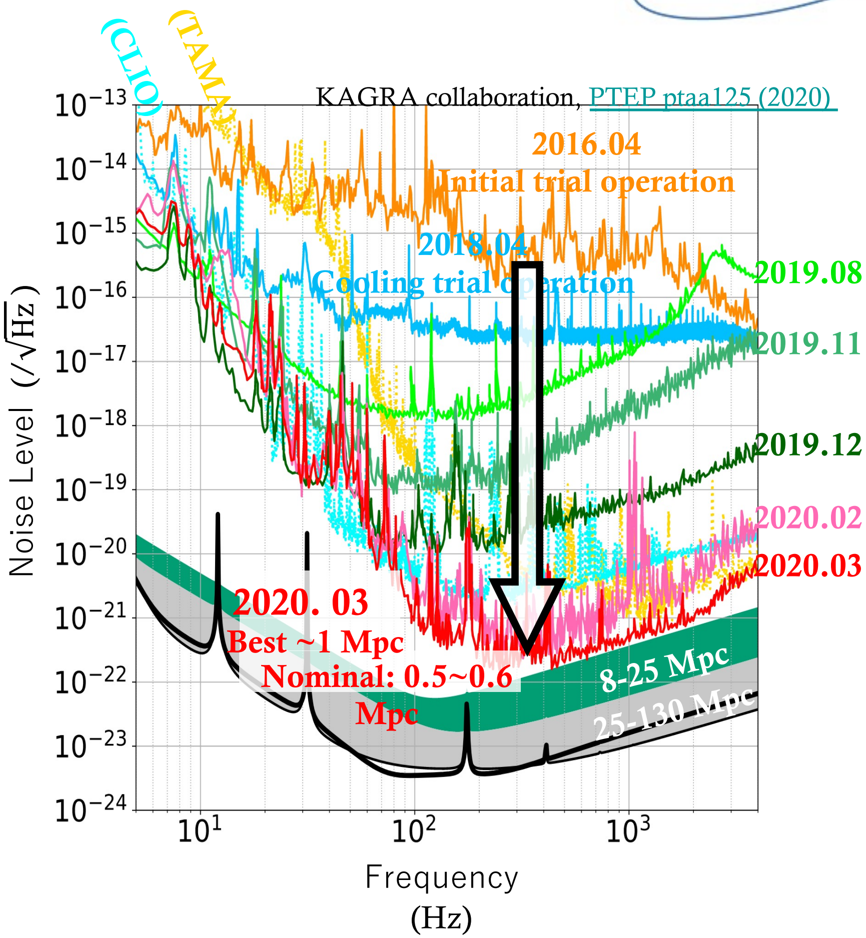
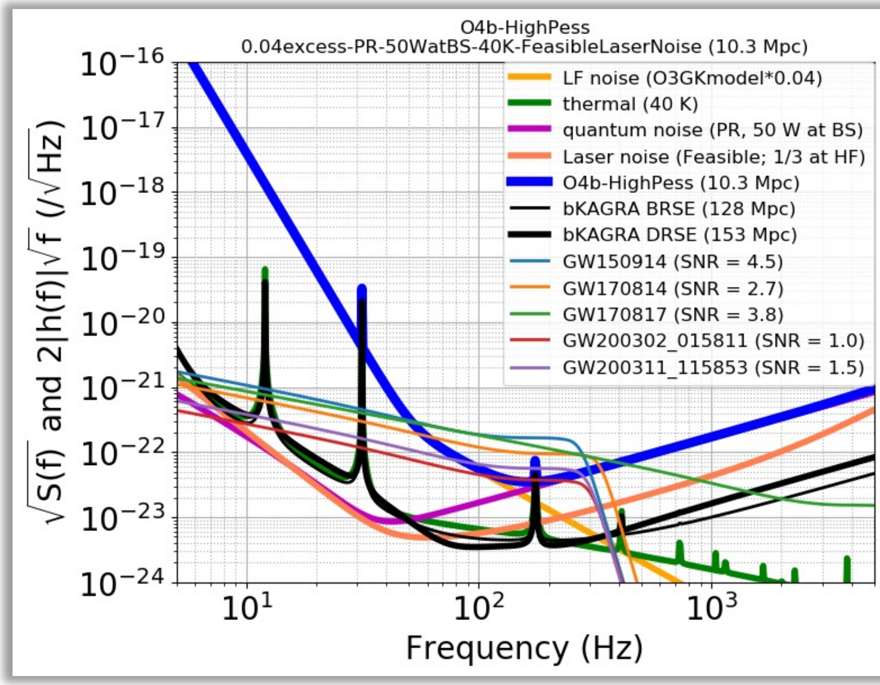
Gravitational Waves: New Eyes to observe the Universe





Gravitational Waves: New Eyes to observe the Universe

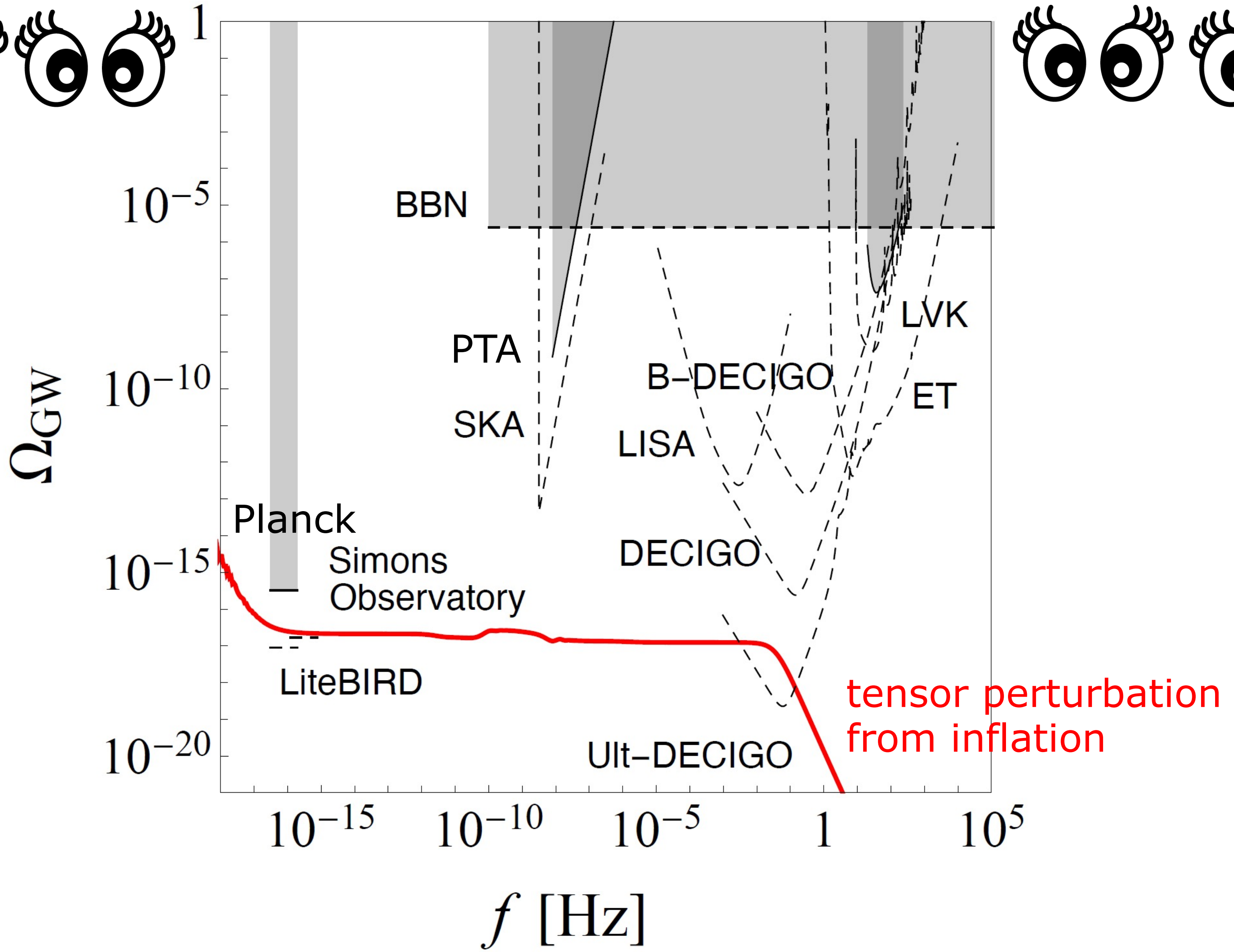


Figure drawn by Sachiko Kuroyanagi

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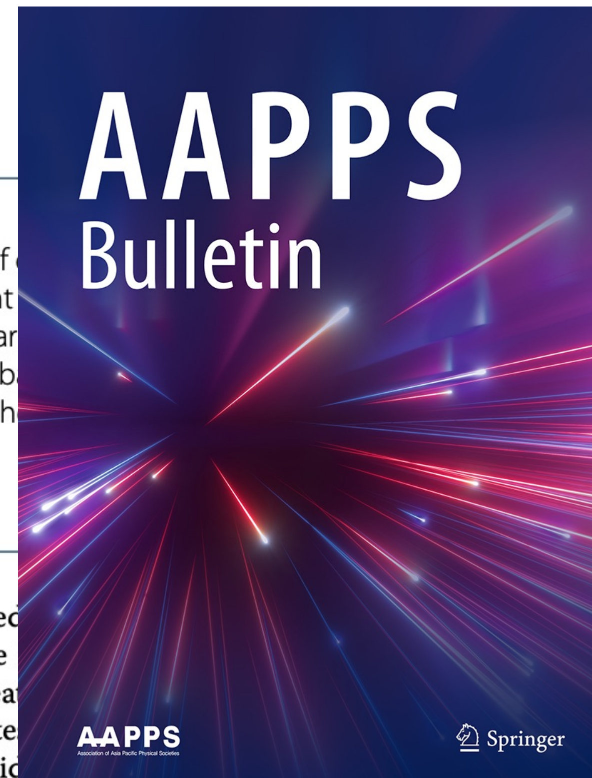
AAPPS Bulletin

REVIEW ARTICLE **Open Access**

Check for updates

Implication of pulsar timing array experiments on cosmological gravitational wave detection

Jun'ichi Yokoyama[1,2,3]



Abstract
Gravitational waves provide a new probe of the Universe which can reveal a number of
astrophysical phenomena that cannot be observed by electromagnetic waves. Different
waves are detected by different means. Among them, precision measurements of pulsar
detector for gravitational waves with light-year scale wavelengths. In this review, first a b
stochastic gravitational wave background using pulsar timing array is introduced, and th
the latest observational result of 12.5-year NANOGrav data are described.

Keywords: Gravitational waves, Pulsar timing array, Stochastic background

AAPPS
Association of Asia Pacific Physical Societies

1 Introduction

Gravitational waves are rip
dicted by Einstein in his th
1916. In Newtonian theory o
both space and time are rigid
affected by any material content existing in the Universe,
and gravity is a nonlocal force. In contrast, Einstein advo

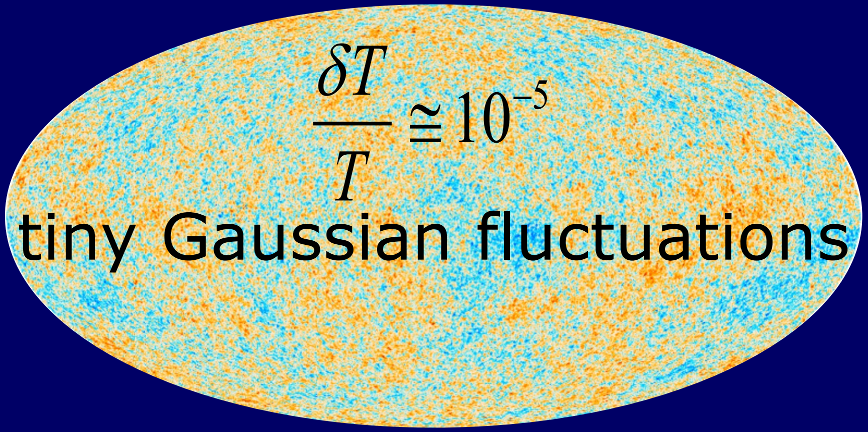
zed
he
ea
ate
cic
$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$, taking the speed of light unity.

Gravitational waves can be expressed by a small per-

Inflation realizes
not only

Homogeneous
Background Radiation
@T=2.73K
CMB

but also



by quantum fluctuations.

This is where Quantum
Physics comes in.

Curvature perturbation

- Regions with more expansion is more curved

$$\varsigma \equiv \frac{\delta a}{a} = \delta N = H\delta t = H\frac{\delta\phi}{\dot{\phi}}$$

$$a(t) \propto e^{N} = e^{Ht}$$

"inflaton" and its fluctuation

- In the standard slow-roll inflation:

In each Hubble time H^{-1}, quantum fluctuations with an amplitude $\delta\varphi \approx \pm\frac{H}{2\pi}$ and the initial wavelength $\lambda \approx H^{-1}$ is generated and stretched by inflation continuously.

The standard calculation of curvature perturbation in inflationary cosmology is based on

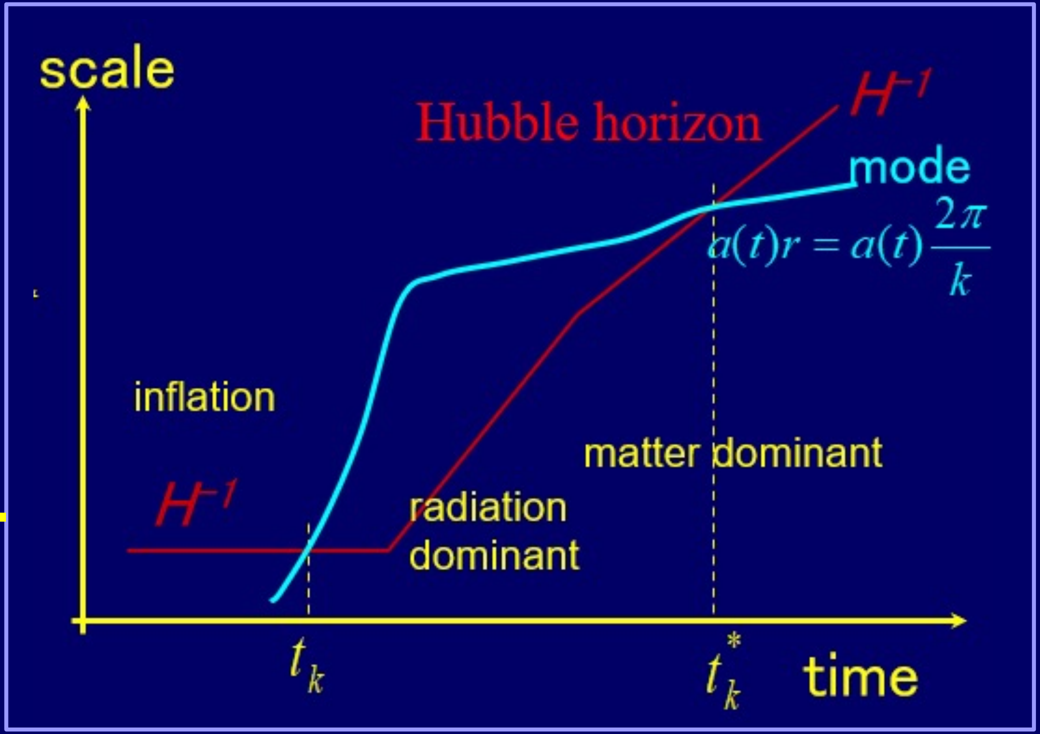
- linear perturbation theory
- tree level quantum field theory
- superhorizon conservation

⬇

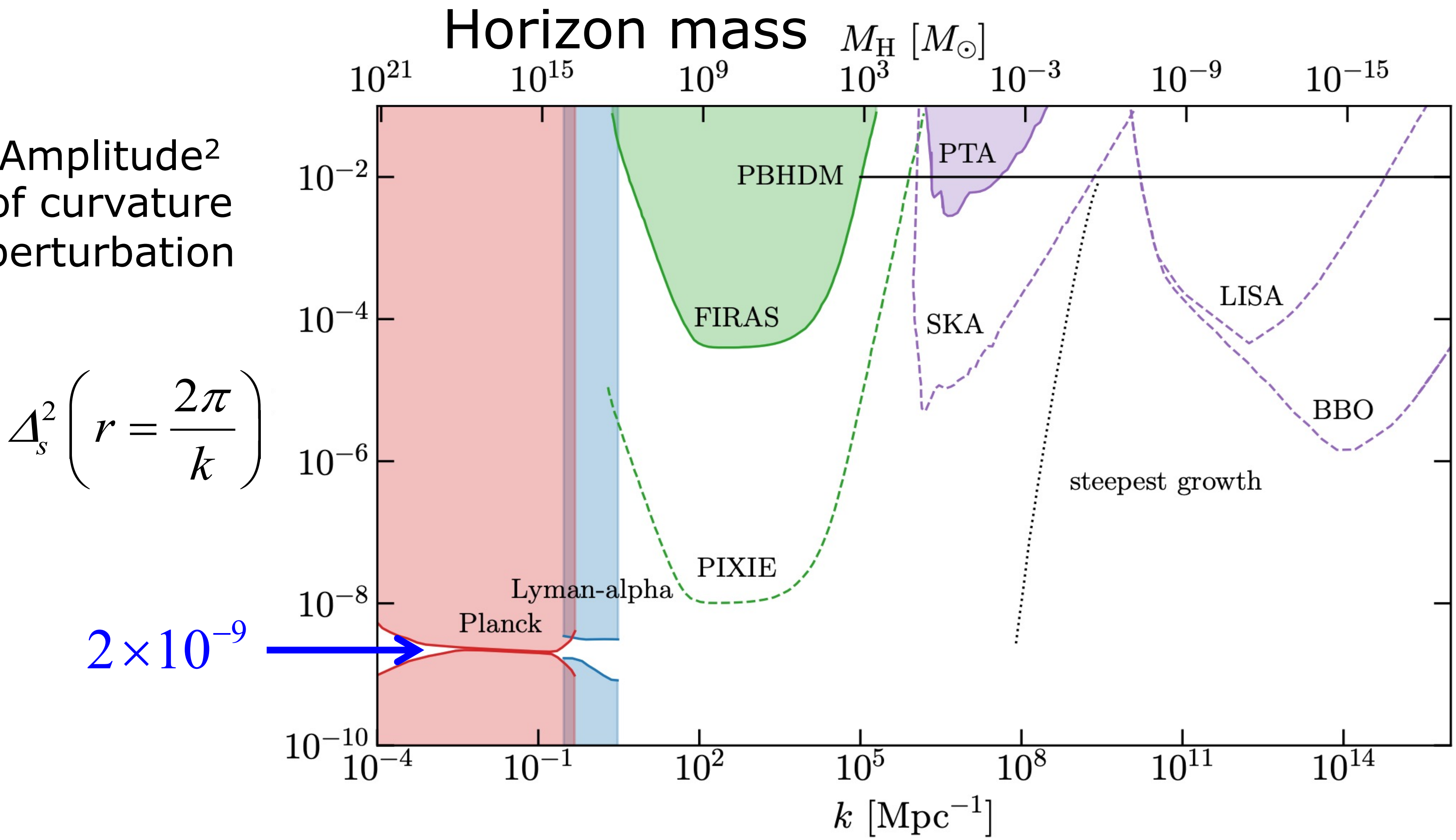
One-to-one correspondence between the scale of fluctuation and its generation time.

⬇

We can probe models inflation by observing perturbation spectrum.



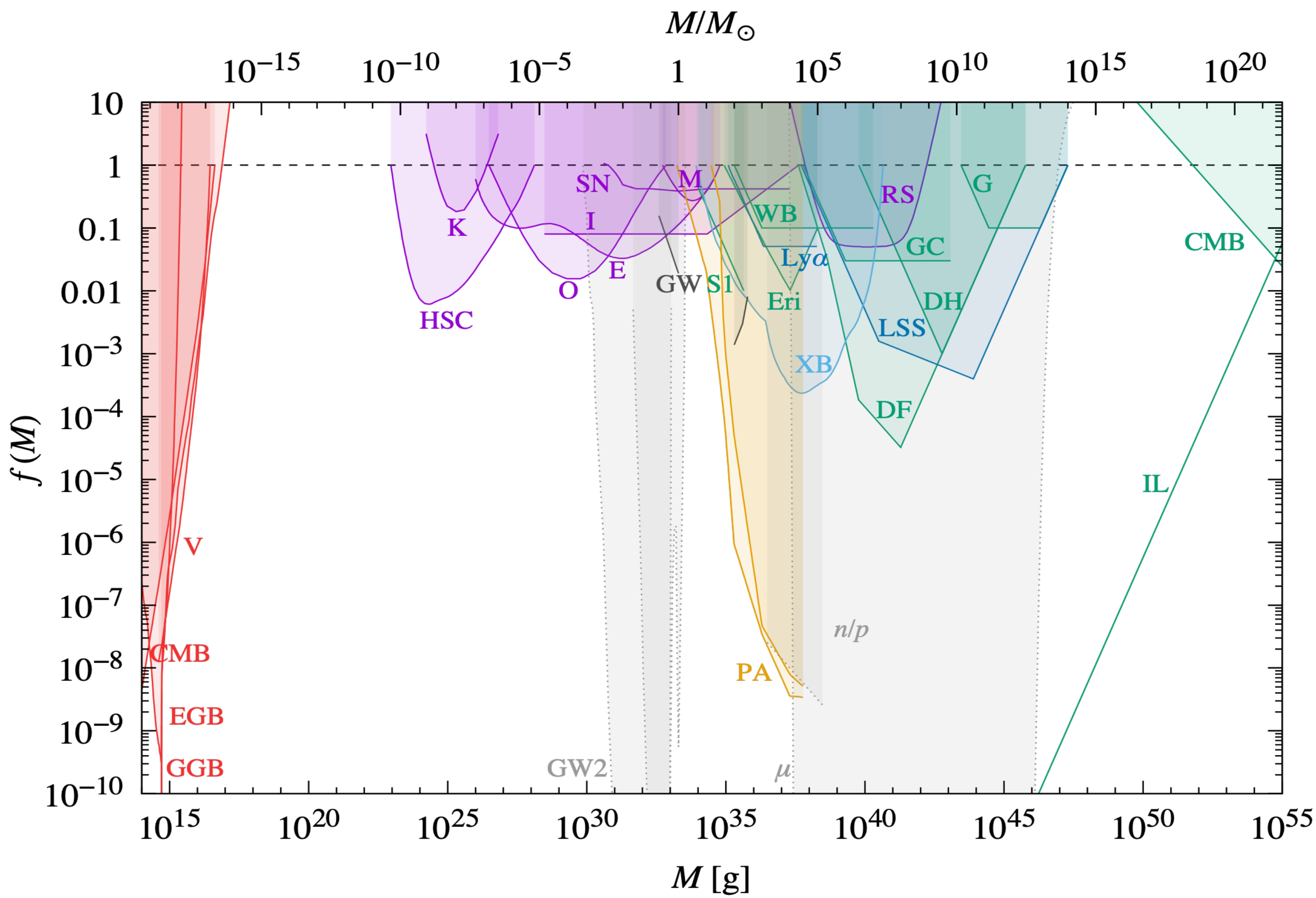
Observationally, we note that the amplitude of curvature perturbations is severely constrained only on large scales probed by CMB.



Green and Kavanagh (2007.10722)

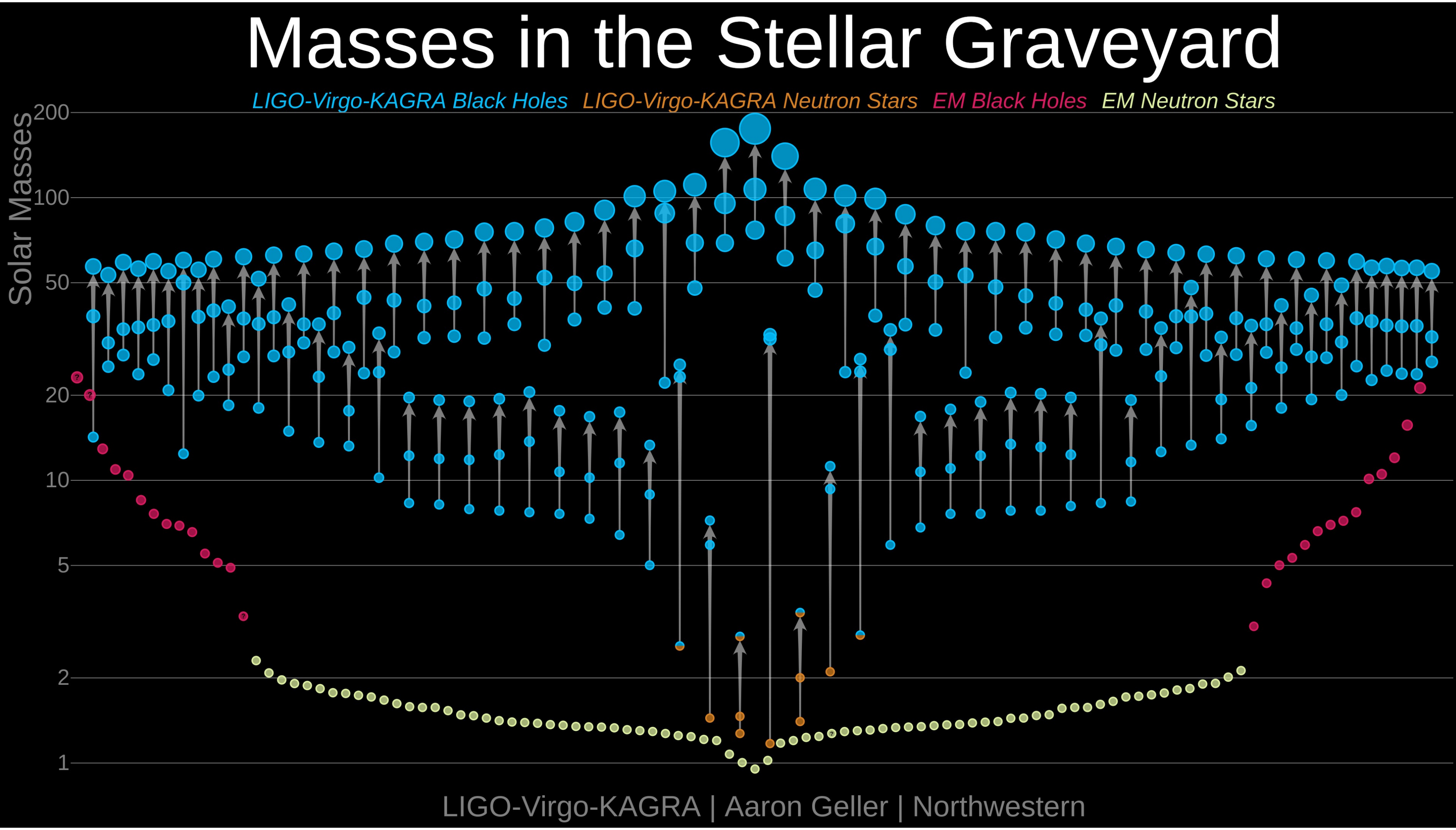
If we realize large-amplitude fluctuations on small scales, Priomrdial Black Holes (PBHs) may have been produced when the region with large fluctuation entered the Hubble horizon.

Constraints on the fraction of PBH dark matter



Carr, Kohri, Sendouda, JY: *Rep. Prog. Phys.* 84(2021) 116902

Black Holes found by Gravitational Wave Observations

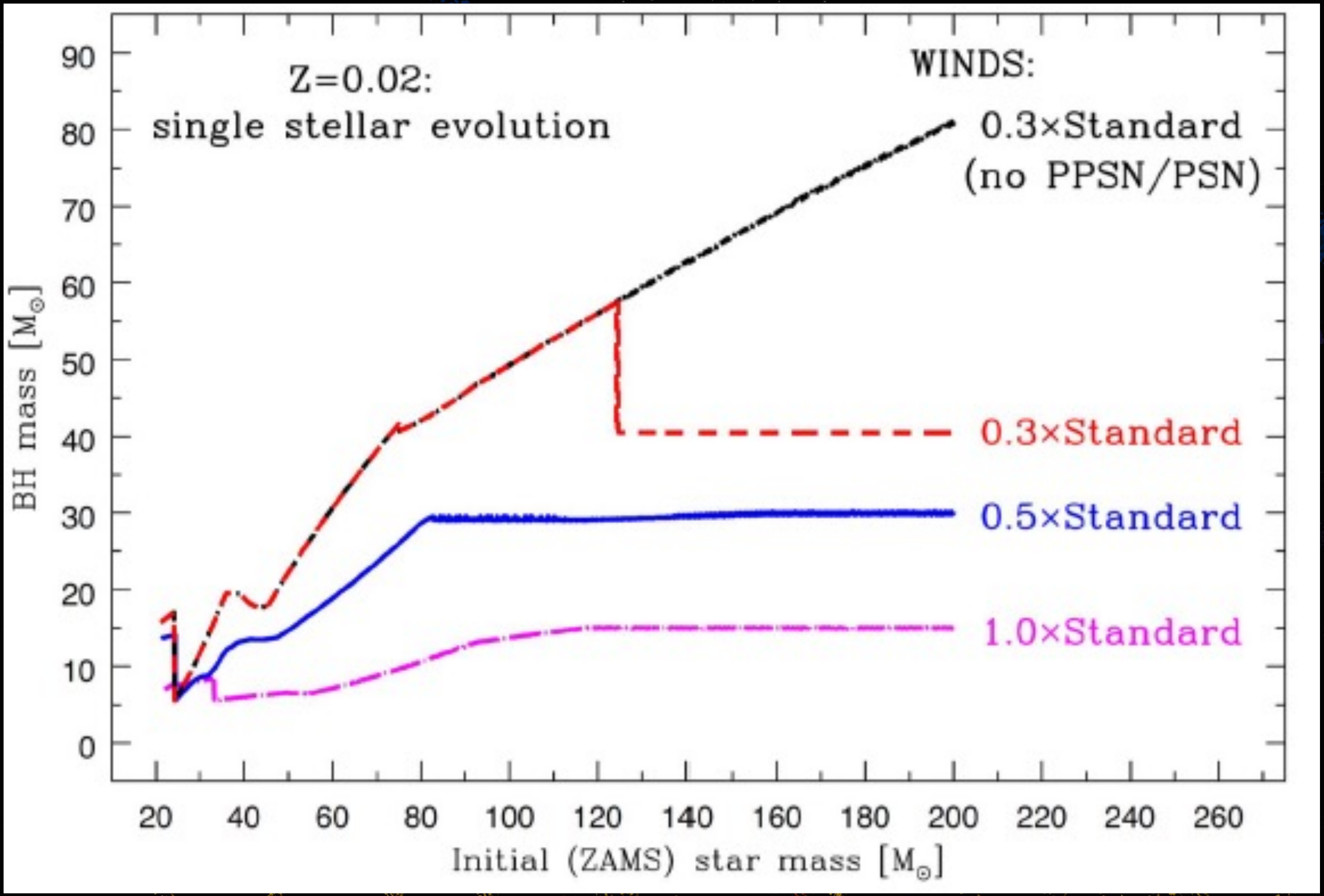


There are many massive black holes in our Universe!

The uninitiated tend to think that there are many massive black holes with $\sim 30 M_\odot$ in the universe, but one must take into account that the heavier the object, the stronger the gravitational wave signal, and therefore the farther it can be seen.

We need to make a volume-limited sample in astronomy removing the aforementioned bias, but the sources are burst events in binary systems so it is highly nontrivial to make such a sample.

What is the origin of these heavy black holes?

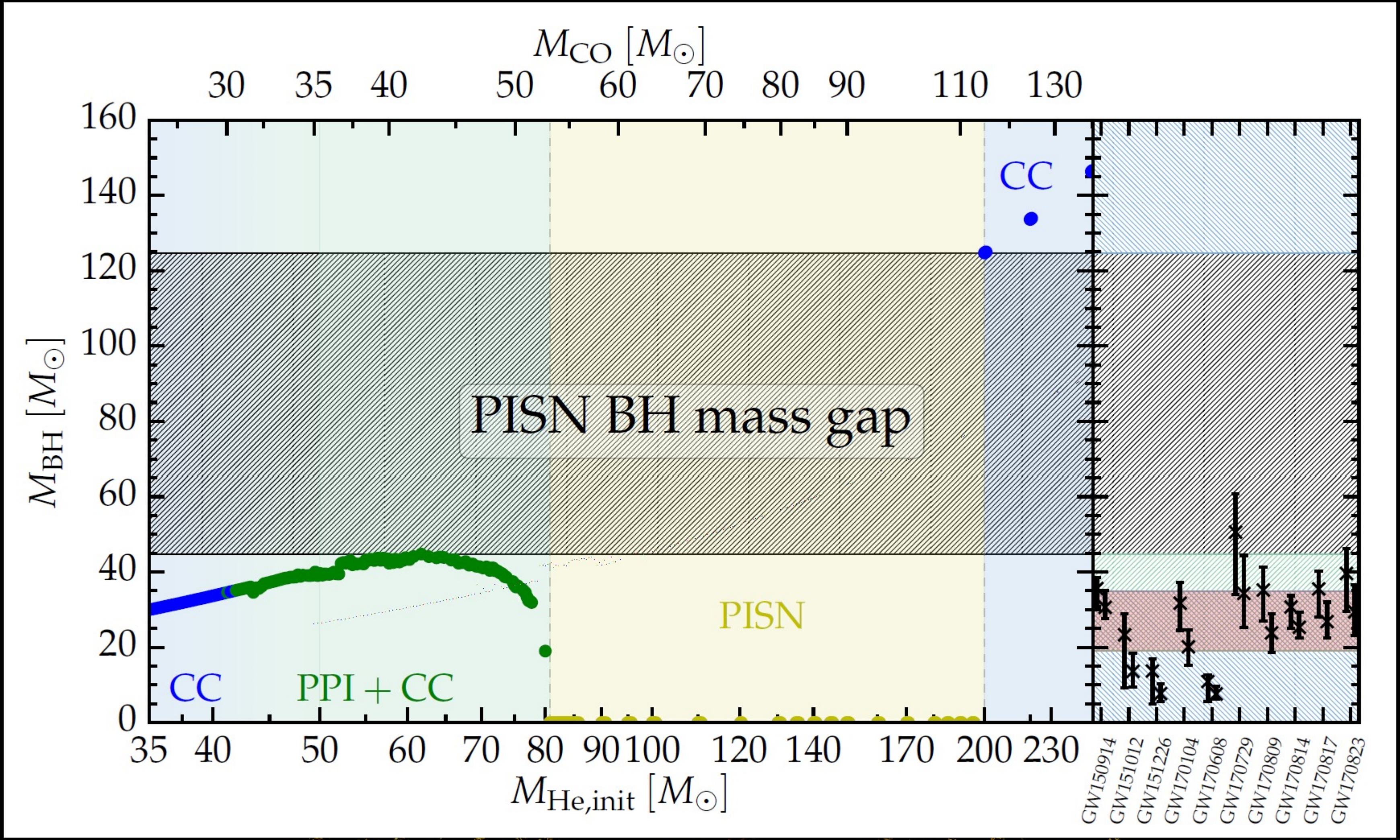


Black holes produced from collapse of normal stars have masses at most $15M_{\odot}$ due to the mass loss by stellar winds

PPSN=Pair-instability Pulsation SuperNova
PSN=PISN=Pair-instability SuperNova

(Belczynski et al 2020)

What is the origin of these heavy black holes?



NO black holes more massive than $45M_\odot$?

PISN=Pair-instability SuperNova

(Renzo et al 2020)

What is the origin of these heavy black holes?

First stars, PopIII stars with no Metals

1. Relatively more massive at formation
2. Smaller radius than current stars for the same mass
3. Smaller mass loss due to stellar winds

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **442**, 2963–2992 (2014) doi:10.1093/mnras/stu1022

Possible indirect confirmation of the existence of Pop III massive stars by gravitational wave

Tomoya Kinugawa,★ Kohei Inayoshi, Kenta Hotokezaka, Daisuke Nakauchi and Takashi Nakamura
Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan

2014

Accepted 2014 May 21. Received 2014 May 16; in original form 2014 February 20

ABSTRACT
We perform population synthesis simulations for Population III (Pop III) coalescing compact binary which merges within the age of the Universe. We found that the typical mass of

What is the origin of these heavy black holes?

Primordial Black Hole Scenario

PRL **116,** 201301 (2016) PHYSICAL REVIEW LETTERS week ending 20 MAY 2016

Did LIGO Detect Dark Matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski,
Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess
Department of Physics and Astronomy, Johns Hopkins University

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DOI: 1(

PRL **117,** 061101 (2016) PHYSICAL REVIEW LETTERS week ending 5 AUGUST 2016

Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914

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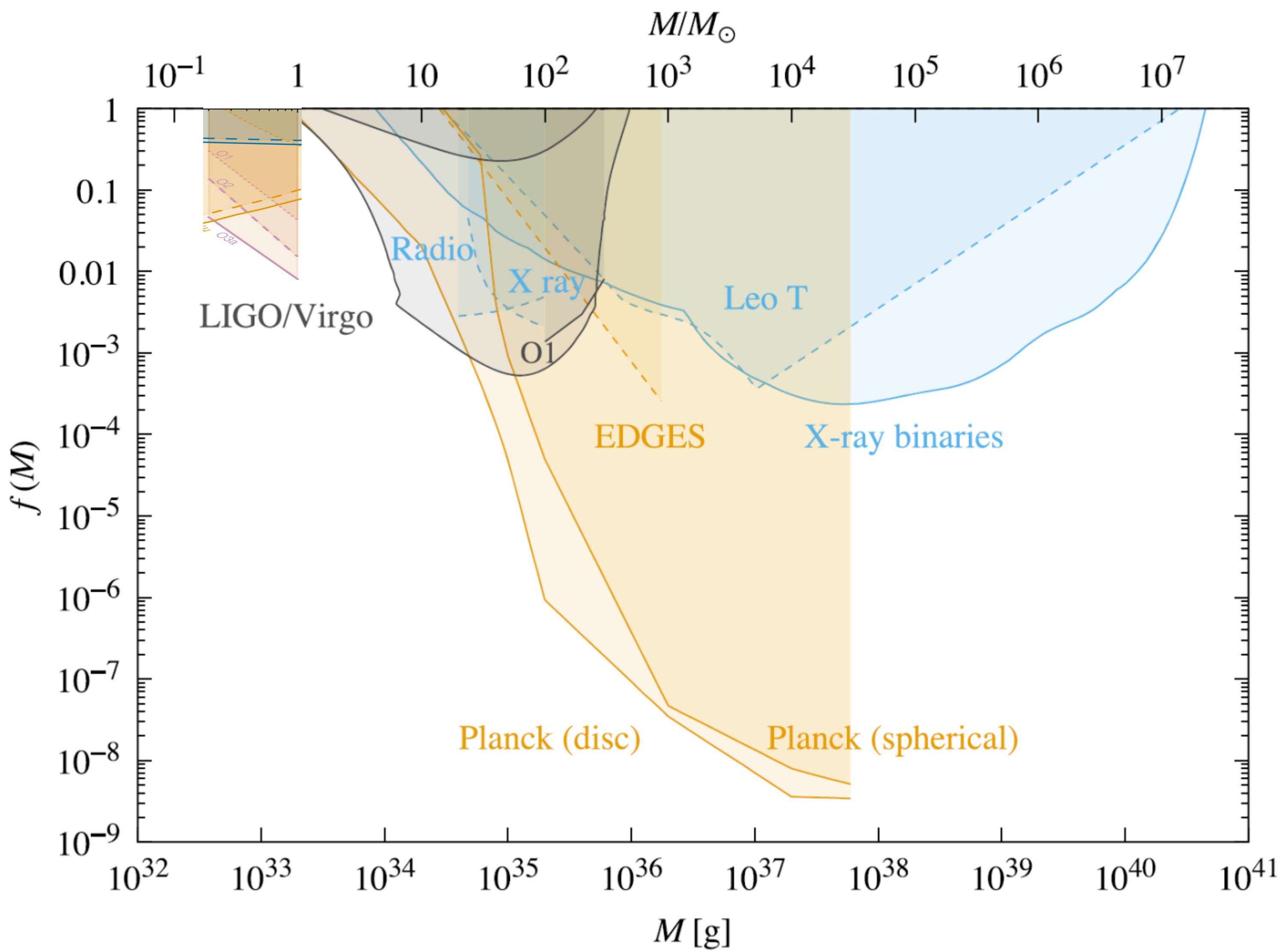
(Received 3 April 2016; published 2 August 2016)

We point out that the gravitational-wave event GW150914 observed by the LIGO detectors can be explained by the coalescence of primordial black holes (PBHs). It is found that the expected PBH merger rate would exceed the rate estimated by the LIGO Scientific Collaboration and the Virgo Collaboration if PBHs were the dominant component of dark matter, while it can be made compatible if PBHs constitute a fraction of dark matter. Intriguingly, the abundance of PBHs required to explain the suggested lower bound on the event rate, > 2 events $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, roughly coincides with the existing upper limit set by the nondetection of the cosmic microwave background spectral distortion. This implies that the proposed PBH scenario may be tested in the not-too-distant future.



$M/M_\odot$
10^{-1}
1
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
1
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$f\,(M)$
Radio
X ray
Leo T
LIGO/Virgo
O1
EDGES
X-ray binaries
Planck (disc)
Planck (spherical)
10^{32}
10^{33}
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10^{35}
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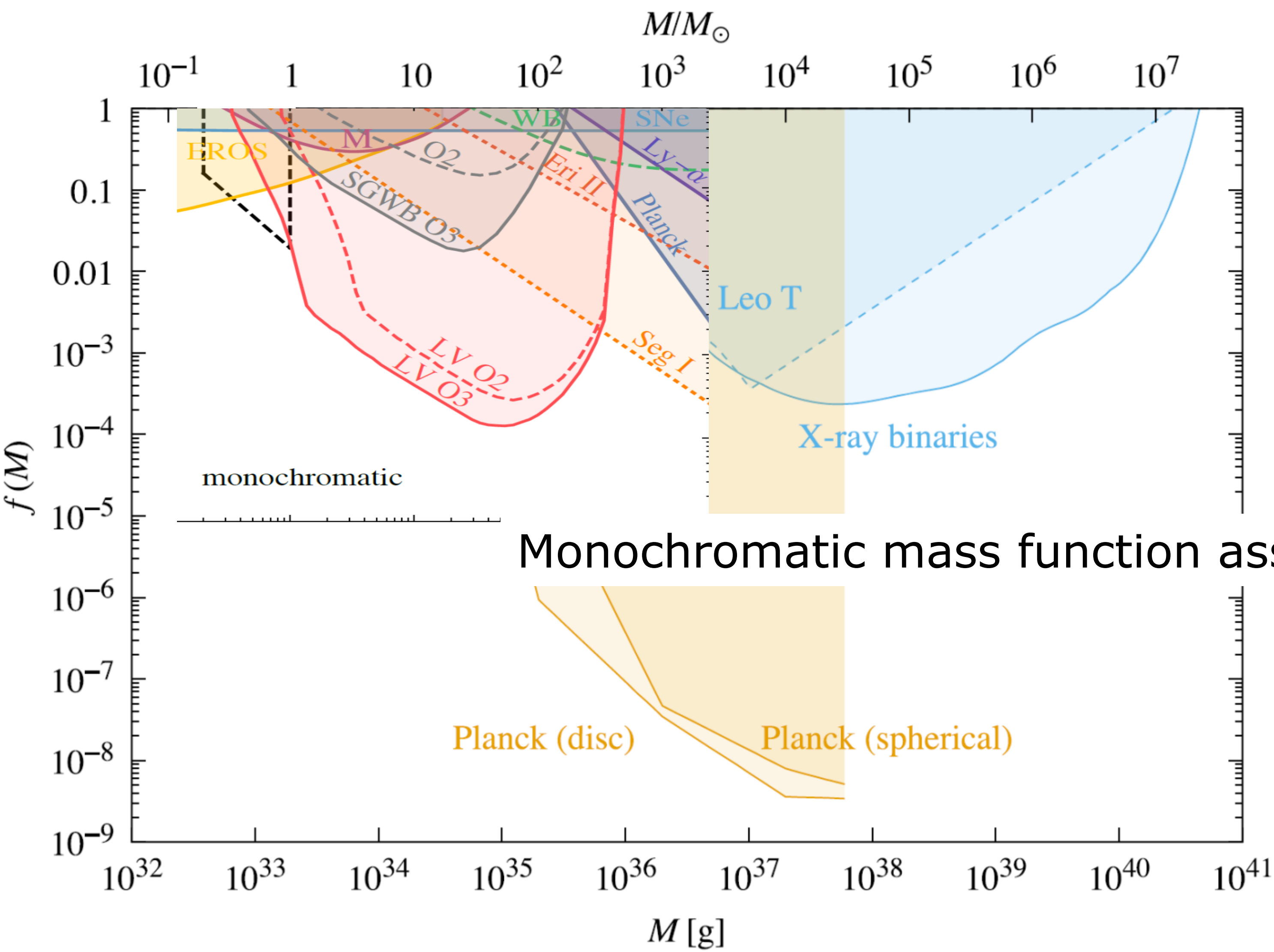
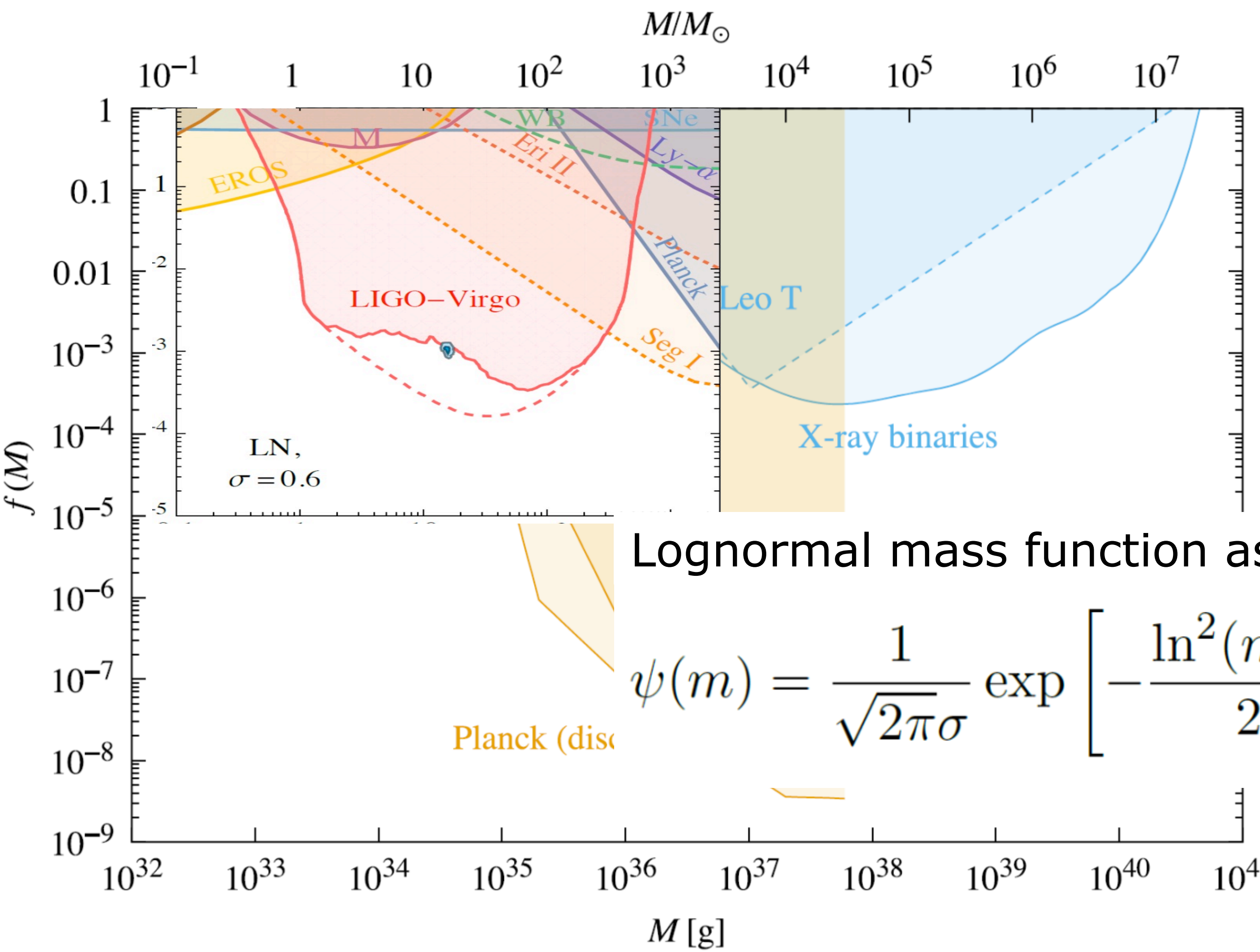
10^{39}
10^{40}
10^{41}
M [g]



Monochromatic mass function assumed

(Hütsi, Raidal, Vaskonen, Veermäe 2021)



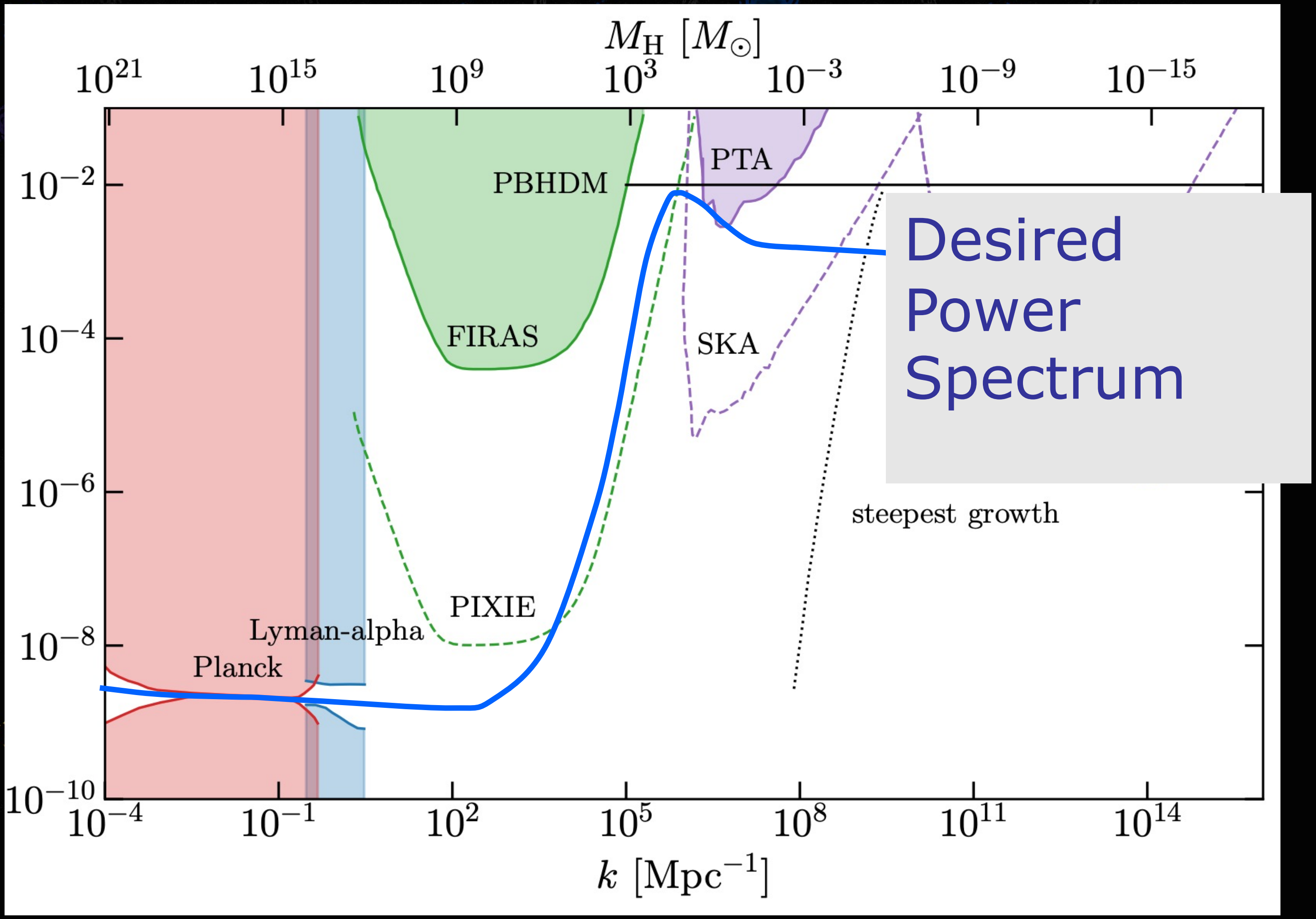
Lognormal mass function assumed

$$\psi(m) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\ln^2(m/m_c)}{2\sigma^2}\right]$$

(Hütsi, Raidal, Vaskonen, Veermäe 2021)

Primordial Black Hole Scenario

We must enhance the amplitude of the power spectrum by 7 digits on the relevant scales.



Primordial Black Hole Scenario

can be realized in a simple single field model

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

2008

Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

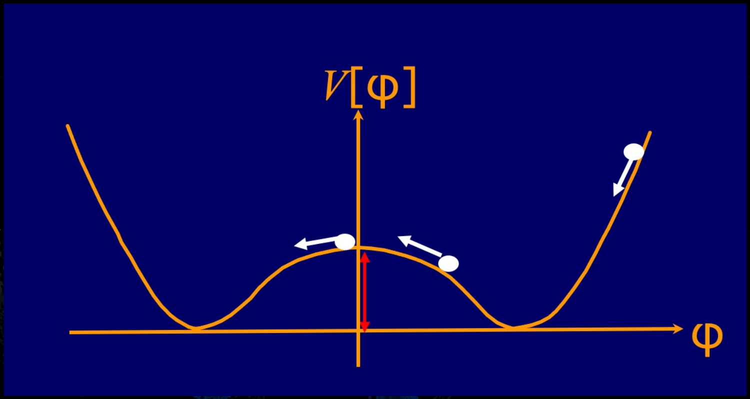
Ryo Saito[1,2], Jun'ichi Yokoyama[2,3] and Ryo Nagata[2]

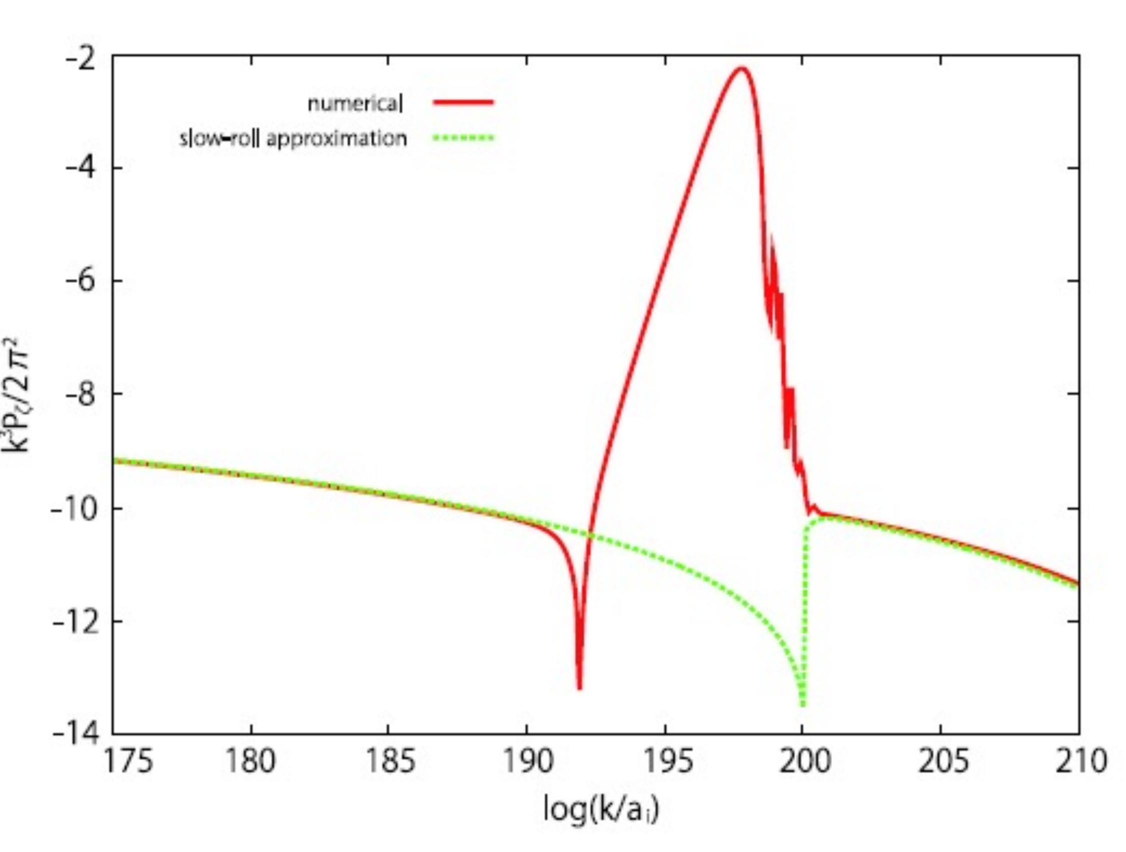
[1] Department of Physics, Graduate School of Science, University of Tokyo

with a Coleman Weinberg potential

$$V(\varphi) = \frac{\lambda}{4}\varphi^4 \left(\ln \left| \frac{\varphi}{v} \right| - \frac{1}{4} \right) + \frac{\lambda}{16} v^4$$

$$(\lambda, v) = (5.4 \times 10^{-14}, 0.355\,139 M_G)$$





In this model, the would-be decaying mode grows at the onset of new inflation after chaotic inflation, which is now known as ultra slow-roll (USR) inflation

Render unto Caesar the things that are Caesar’s

Render unto Gravitational Waves the things that are discovered by Gravitational Waves

1. PBHs are produced when a large-amplitude perturbed region entered the Hubble horizon.
2. Their mass is of order of the horizon mass

$$M_{PBH} \sim \frac{c^3 t}{G} \sim M_{\odot}\left(\frac{t}{10^{-5}\,\mathrm{sec}}\right)$$

3. Tensor perturbations or gravitational waves are produced by second-order density perturbations.

$$f_{\mathrm{GW}} = 4 \times 10^{-10}\ \mathrm{Hz}\left(\frac{M_{\mathrm{PBH}}}{10^{36}\ \mathrm{g}}\right)^{-1/2}\left(\frac{g_{*p}}{10.75}\right)^{-1/12}$$

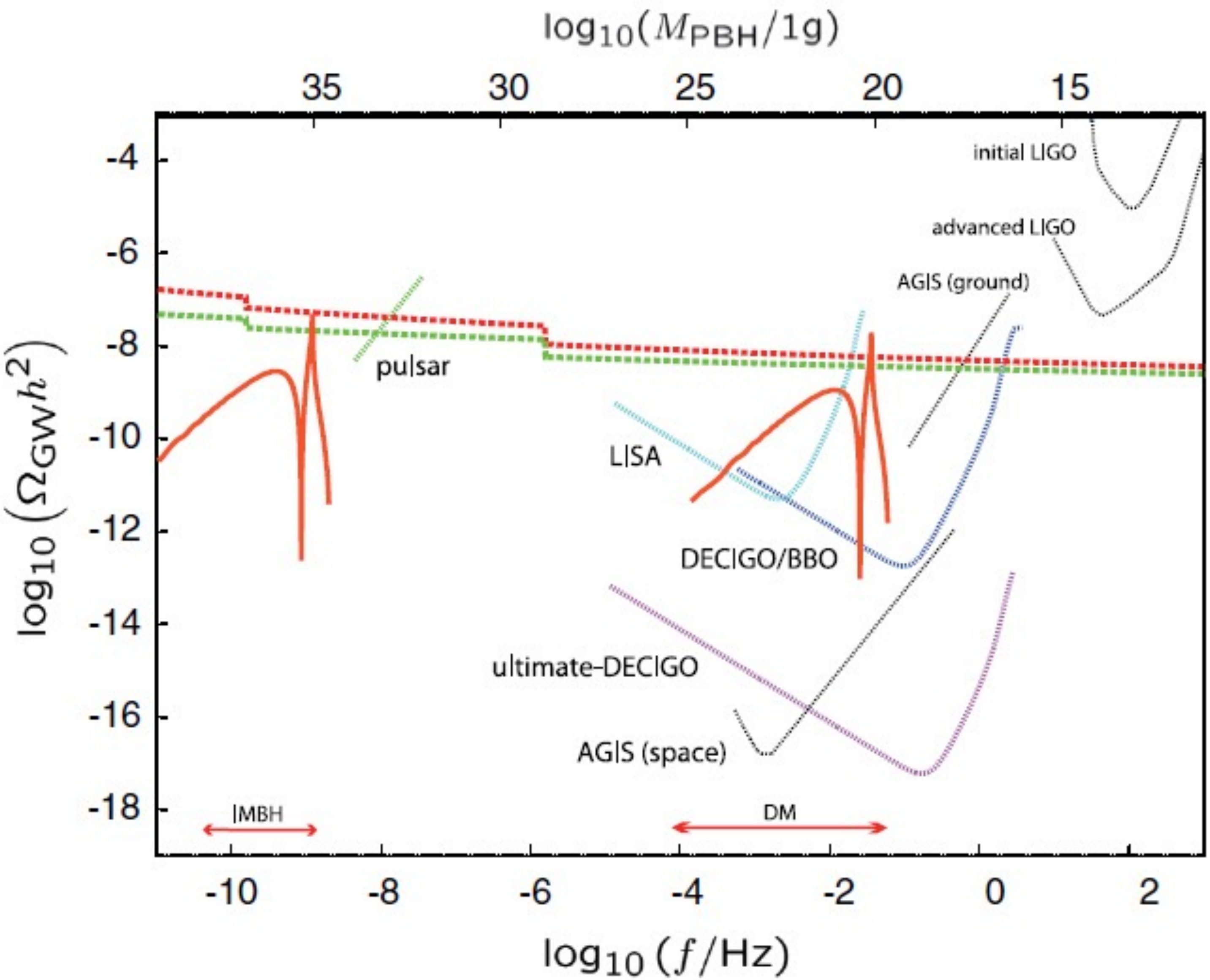
PRL **102,** 161101 (2009) PHYSICAL REVIEW LETTERS week ending 24 APRIL 2009

Gravitational-Wave Background as a Probe of the Primordial Black-Hole Abundance

Ryo Saito[1,2] and Jun'ichi Yokoyama[2,3]

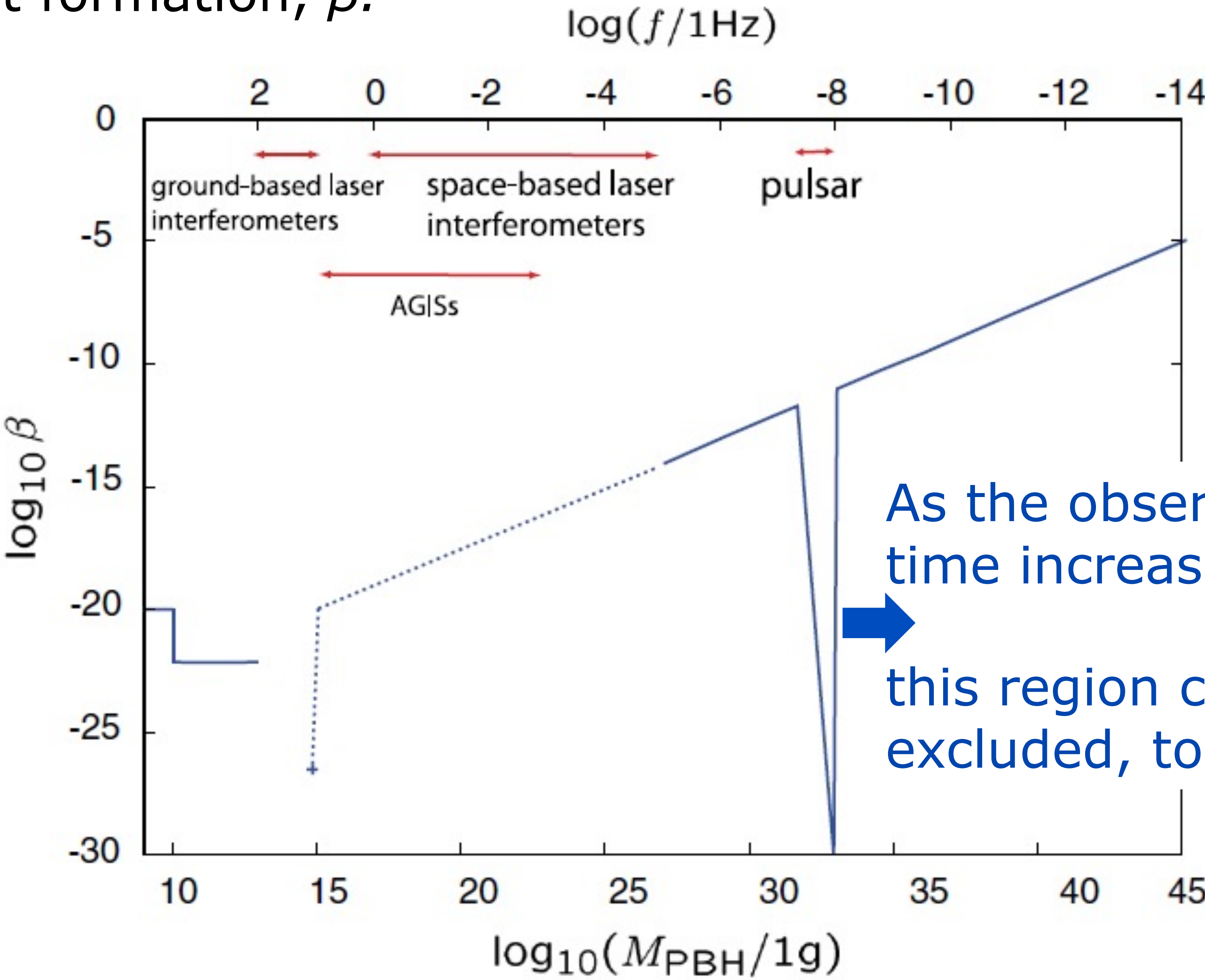
[1]*Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*

[2]*Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*



$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-5}, 10^2 M_\odot)$ (left) and $(10^{-1}, 10^{20}$ g) (right)

Constraint on the fractional energy density of PBHs at formation, *β*.



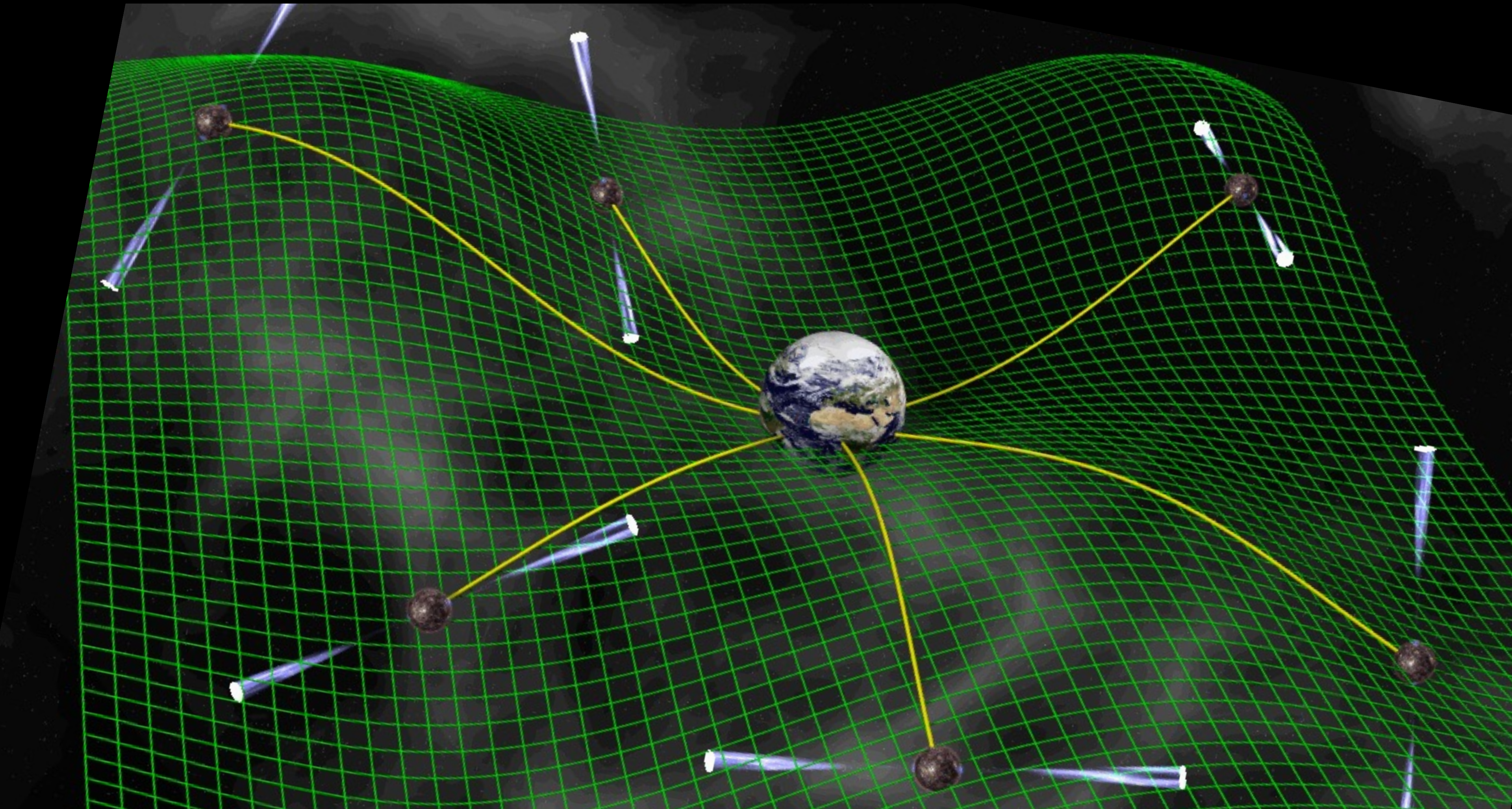
2009

As the observation time increases,

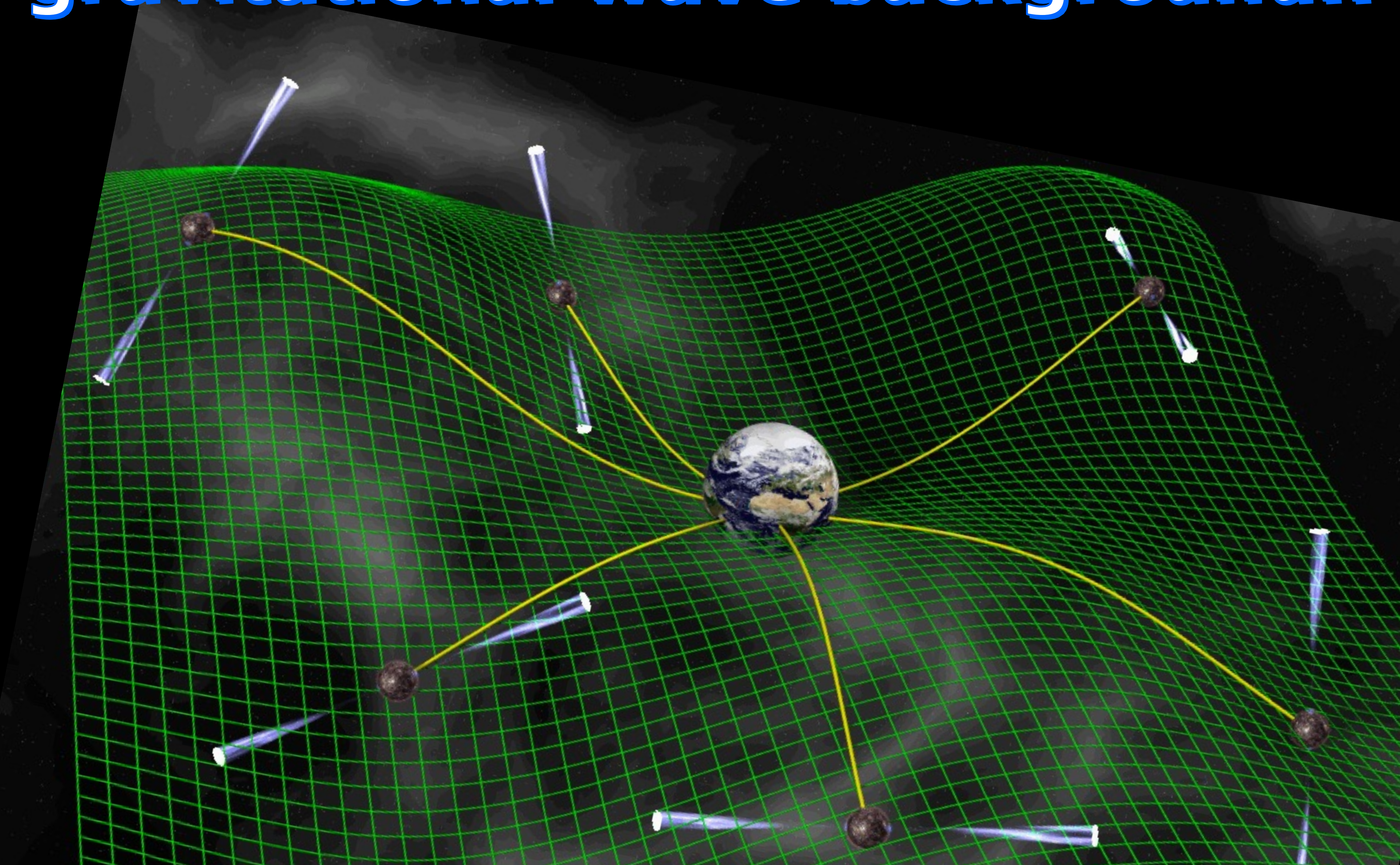
this region can be excluded, too.

(Non-) observation of pulsar timing disturbance would reject PBH hypothesis of LVK black holes.

PTAs would detect gravitational wave signals if LVK black holes are of primordial origin.



2023 PTAs detected stochastic gravitational wave background!!



Correct Proposition (Saito & JY 2009)

(Non-) observation of pulsar timing disturbance would reject PBH hypothesis of LVK black holes.

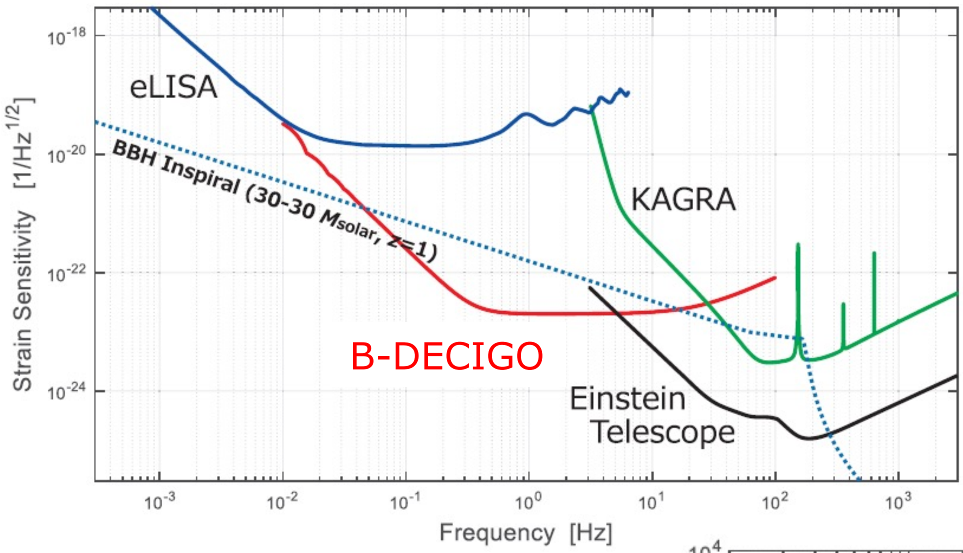
Contraposition is also correct. 対偶命題
PTAs would detect gravitational wave signals if LVK black holes are of primordial origin.

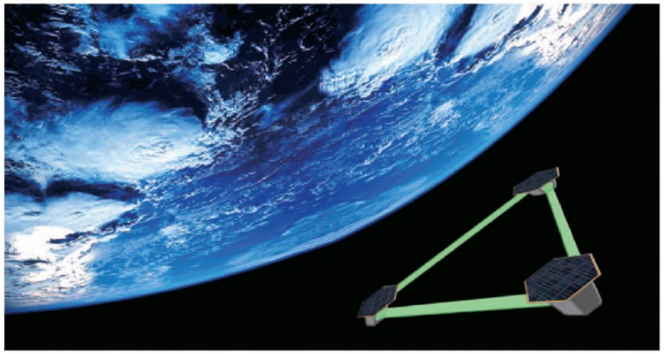
Counter-Proposition
Positive detection of GWs by PTAs would prove the PBH hypothesis of LVK black holes.

is not necessarily correct. We can neither prove nor disprove PBH hypothesis now.

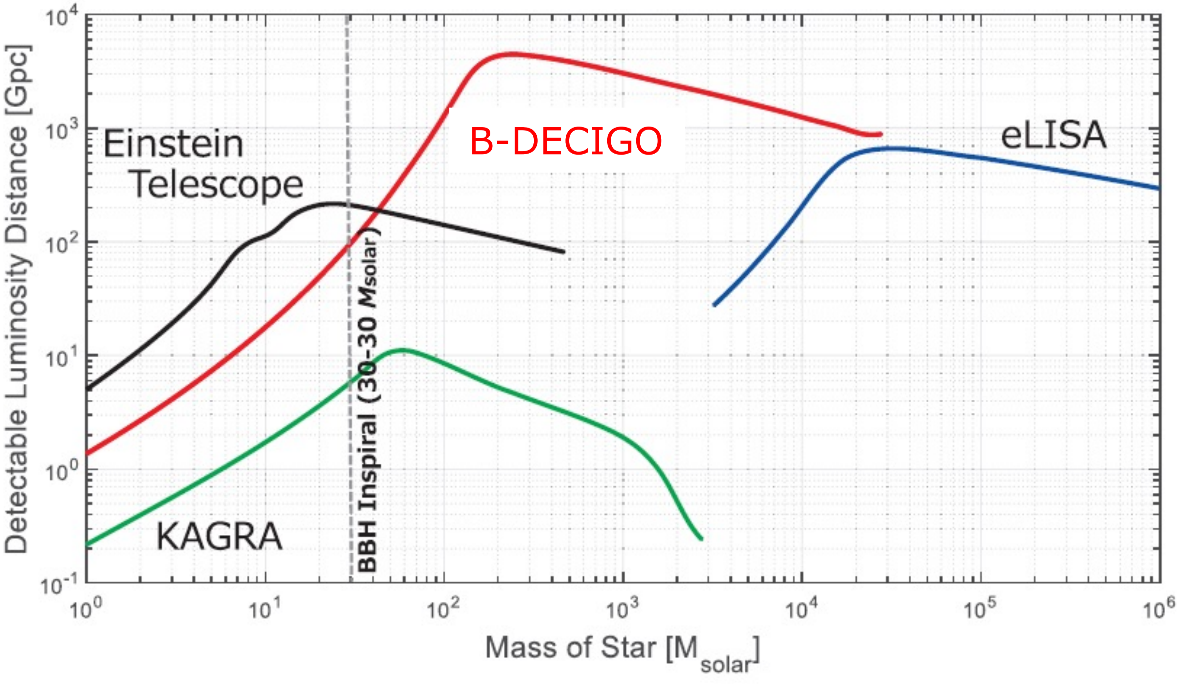
We need more direct way!

A space-based laser interferometer B-DECIGO can do it!

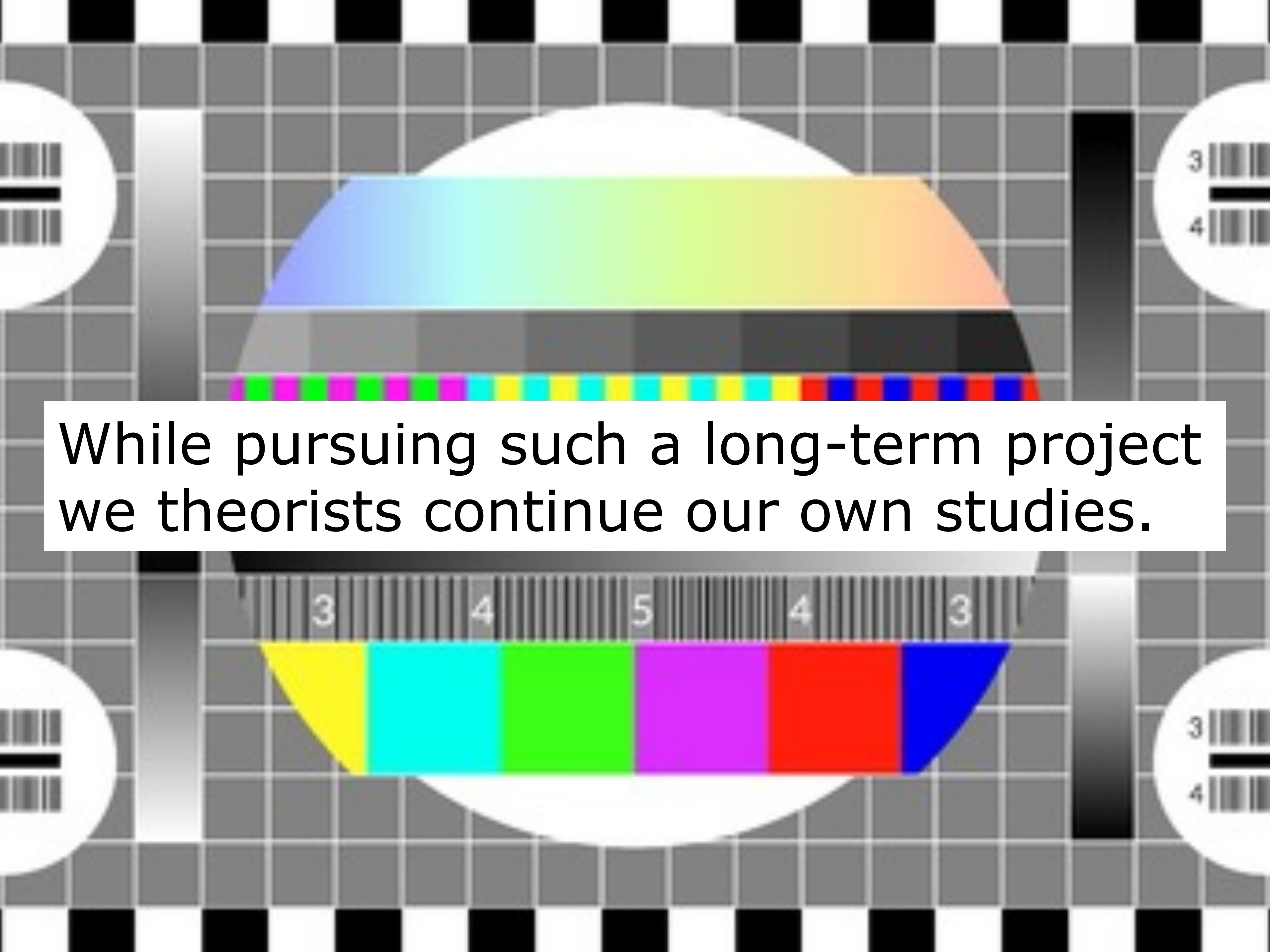




It can observe 30-30 $M_{\odot}$ binary BH coalescence events up to z∼30, where there is no BHs from first stars.

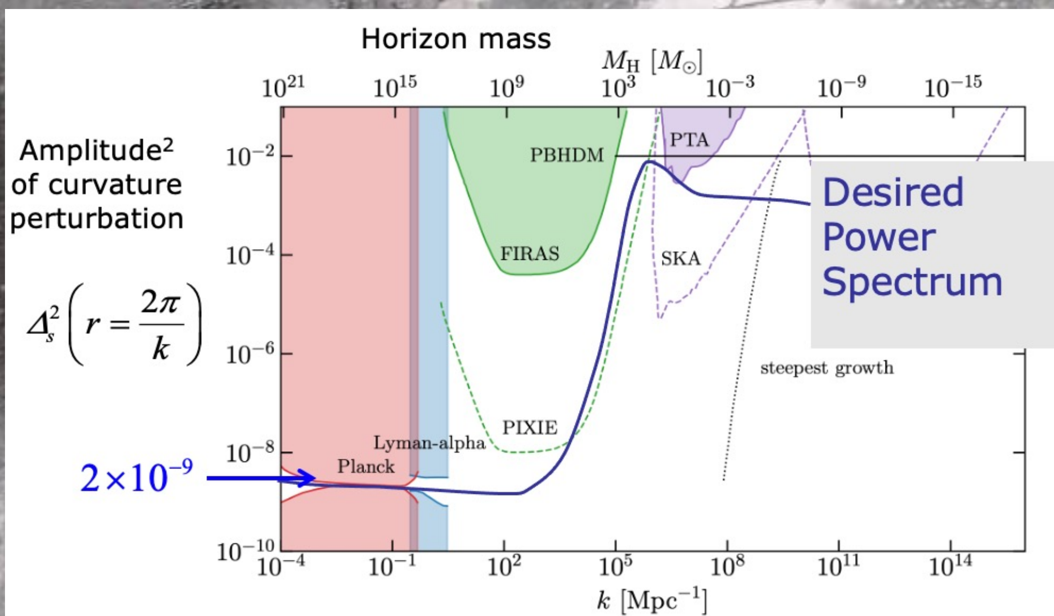


Talk by Seiji Kawamura tomorrow



While pursuing such a long-term project we theorists continue our own studies.

What we wish to argue is that single-field models realizing a desired spectrum suffer from large one-loop correction to the power spectrum and hence not viable.



work with Jason Kristiano

Cosmological perturbation theory

Starting from the action $S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{pl}}^2 R - (\partial_\mu\phi)^2 - 2V(\phi)\right]$,

and assuming quasi de Sitter background $a(t) \propto e^{Ht}$,

we calculate the action for the curvature perturbation ζ to 2nd order.

$$S^{(2)} = M_{\mathrm{pl}}^2 \int \mathrm{d}t\, \mathrm{d}^3x\, a^3\epsilon \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right] \qquad \varepsilon := -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2M_{pl}^2 H^2}$$

ζ behaves like a free massless scalar field with a noncanonical normalization.

Introducing Mukhanov-Sasaki (MS) variable $v = z\zeta M_{\mathrm{pl}}$ with $z = a\sqrt{2\epsilon}$, the second-order action becomes

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right]. \qquad a(\tau) \cong -\frac{1}{H\tau}$$

τ: conformal time

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right].$$

It has a canonical kinetic term, so can easily be quantized.

Mukhanov Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0. \quad \Leftarrow \quad v(x,\tau) = \int \frac{d^3k}{(2\pi)^3}\left[v_k(\tau)\hat{a}_k e^{ik\tau} + v_k^*(\tau)\hat{a}_k^\dagger e^{-ik\tau}\right]$$

$$\frac{z''}{z} = 2a^2H^2\left(1 + \varepsilon + \frac{3}{2}\delta + \frac{1}{2}\delta^2 + \frac{1}{2}\varepsilon\delta + \ldots\right) \qquad a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\varepsilon} \qquad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$$

For slow-roll inflation $\varepsilon \ll 1,\ \delta \ll 1$ we find $\frac{z''}{z} = \frac{2}{\tau^2}$

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right)v_k = 0.$$

$$v_k(\tau) = \frac{\mathcal{A}_k}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} + \frac{\mathcal{B}_k}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)e^{ik\tau}.$$

$\mathcal{A}_k = 1$ and $\mathcal{B}_k = 0$ is the solution corresponding to the Minkowski mode function (vacuum) at high frequency or in the beginning.

Quantization

$$\hat{v}(\boldsymbol{k},\tau) \equiv v_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + v^*_{\boldsymbol{k}}(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}$$

Since v has a canonical kinetic term, conjugate momentum is simply $\hat{\pi}(\boldsymbol{k},\tau) = \hat{v}'(\boldsymbol{k},\tau)$ and the standard quantization

$$\left[\hat{v}(\boldsymbol{k},\tau), \hat{\pi}(\boldsymbol{k}',\tau)\right] = i\hbar\delta(\boldsymbol{k}-\boldsymbol{k}') \quad \Rightarrow \quad \left[\hat{a}_{\mathbf{p}}, \hat{a}^\dagger_{-\mathbf{q}}\right] = (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})$$

can be done with the normalization $v'^*_k v_k - v'_k v^*_k = i$.

Curvature perturbation:

From $\varsigma = \dfrac{v}{M_{pl}z} = \dfrac{v}{M_{pl}a\sqrt{2\varepsilon}}$, we find the mode function

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\varepsilon}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau),$$

where $\star$ denotes horizon crossing condition $\tau = -1/k$.

Quantization

$$\hat{\zeta}(\boldsymbol{k},\tau) \equiv \frac{\hat{v}(\boldsymbol{k},\tau)}{M_{pl} a \sqrt{2\varepsilon}} = \varsigma_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + \varsigma^*_{\boldsymbol{k}}(\tau)\hat{a}^\dagger_{-\boldsymbol{k}} \qquad \left[\hat{a}_{\mathbf{p}}, \hat{a}^\dagger_{-\mathbf{q}}\right] = (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})$$

Vacuum expectation value yields power spectrum $\varsigma(\boldsymbol{p}) \equiv \hat{\zeta}(\boldsymbol{p},\tau)$

$$\langle\zeta(\mathbf{p})\zeta(\mathbf{q})\rangle_{(0)} =: (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})\,\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(0)} \qquad \langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(0)} = |\zeta_p(\tau)|^2$$

In the superhorizon regime $-k\tau \ll 1$, vacuum fluctuation is constant and given by

$$\Delta_s^2\left(r = \frac{2\pi}{k}\right) := \left\langle\left\langle\varsigma(\boldsymbol{k})\varsigma(-\boldsymbol{k})\right\rangle\right\rangle \frac{4\pi k^3}{(2\pi)^3} = \left|\varsigma_{\boldsymbol{k}}\right|^2 \frac{4\pi k^3}{(2\pi)^3} = \frac{H^2}{8\pi^2 M_{pl}^2 \varepsilon}$$

"Classicalilzation" of curvature perturbation

$$\varsigma_{\boldsymbol{k}}(\tau) = \frac{iH}{2M_{pl}\sqrt{\varepsilon k^3}}(1+ik\tau)e^{-ik\tau} \to \frac{iH}{2M_{pl}\sqrt{\varepsilon k^3}}\left[1+O\left(\left(\frac{k}{aH}\right)^2\right)\right]$$ for $k \ll a(t)H$ $\left(-k\tau = \frac{k}{aH}\right)$

$\varsigma_{\boldsymbol{k}}^*(\tau) = -\varsigma_{\boldsymbol{k}}(\tau)$ in the superhorizon limit

So we find $\hat{\zeta}(\boldsymbol{k},\tau) = \varsigma_{\boldsymbol{k}}(\tau)\left(\hat{a}_{\boldsymbol{k}} - \hat{a}^\dagger_{-\boldsymbol{k}}\right)$
and its conjugate momentum

$$\hat{\pi}_\varsigma(\boldsymbol{k},\tau) = (M_{pl}z)^2\hat{\zeta}'(\boldsymbol{k},\tau) = (M_{pl}z)^2\varsigma'_{\boldsymbol{k}}(\tau)\left(\hat{a}_{\boldsymbol{k}} - \hat{a}^\dagger_{-\boldsymbol{k}}\right)$$

The same operator dependence!

When the decaying mode is negligible, $\hat{\zeta}(\boldsymbol{k},\tau)$ and $\hat{\pi}_\varsigma(\boldsymbol{k},\tau)$ have the same operator dependence and apparently commute with each other.

Long-wave quantum fluctuations behave as if classical statistical fluctuations.

→ Origin of large scale structures and CMB anisotropy

More precise statements

$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\pi}_{\varsigma}(\boldsymbol{k}',\tau)\right]=\left[\hat{v}(\boldsymbol{k},\tau),\hat{\pi}(\boldsymbol{k}',\tau)\right]=i\hbar\delta(\boldsymbol{k}-\boldsymbol{k}')$$ always holds.

What we find is

$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\zeta}'(\boldsymbol{k}',\tau)\right]=\frac{1}{2M_{pl}^2a^2\varepsilon}i\hbar\delta(\boldsymbol{k}-\boldsymbol{k}')\searrow$$ decreases exponentially.

in standard slow-roll inflation with $\varepsilon\approx const\ll 1$.

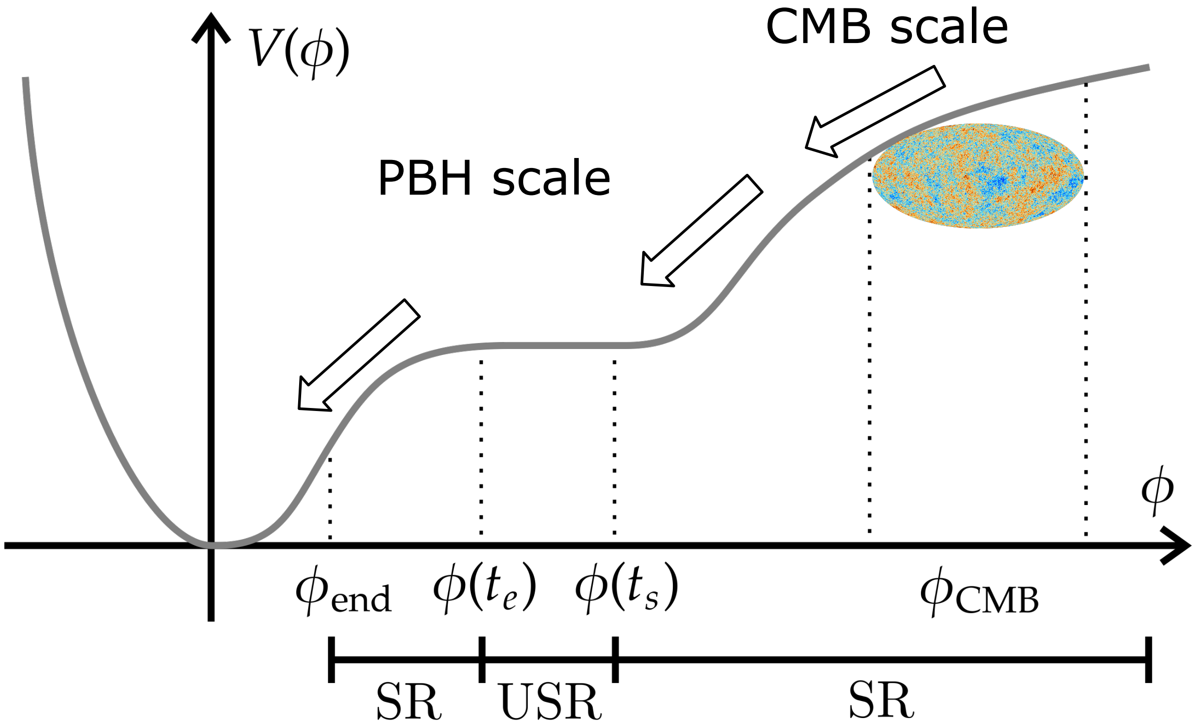
In terms of the mode function

$$\zeta_k'^*\zeta_k-\zeta_k'\zeta_k^*=\frac{i}{2a^2\epsilon M_{\rm pl}^2}\searrow 0$$ in standard slow-roll inflation.

In order to realize temporal enhancement of curvature perturbation, one is tempted to adopt a model in which ε decreases temporarily.

$$\Delta_s^2\left(r=\frac{2\pi}{k}\right):=|\varsigma_{\boldsymbol{k}}|^2\frac{4\pi k^3}{(2\pi)^3}=\frac{H^2}{8\pi^2M_{pl}^2\varepsilon}\ \nearrow\quad\Leftarrow\varepsilon\searrow$$

Ivanov, Naselsky, & Novikov (1994)



SR: slow roll periods

$$H^2+\frac{K}{a^2}=\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2+V[\phi]\right)$$

$$\ddot{\phi}+3H\dot{\phi}+V'[\phi]=0$$

$$\varepsilon\approx const\equiv\varepsilon_{SR}$$

USR: ultra-slow roll period (flat potential)

$$V'[\phi]\cong 0\qquad\ddot{\phi}+3H\dot{\phi}=0\qquad\dot{\phi}\propto a^{-3}(t)\searrow\qquad\epsilon=\frac{\dot{\phi}^2}{2M_{\rm pl}^2H^2}\propto a^{-6}\qquad\Delta_s^2\nearrow$$

Ultra slow-roll (USR) inflation

Kinney (1998,2005), JY & Inoue (2002)
Martin, Motohashi, & Suyama (2013)

$$\epsilon = \frac{\dot{\phi}^2}{2M_{\rm pl}^2 H^2} \propto a^{-6}$$

➡ Second slow-roll parameter: $\eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon} = -6$

In such a regime, contrary to the standard slow-roll inflation, curvature perturbation grows even on superhorizon scale, as it satisfies

$$\frac{d^2\zeta_{\boldsymbol{k}}}{dN^2} + (3-\epsilon+\eta)\frac{d\zeta_{\boldsymbol{k}}}{dN} + \left(\frac{k}{aH}\right)^2 \zeta_{\boldsymbol{k}} = 0, \qquad N = \int H dt$$

In the standard inflation with $\varepsilon, |\eta| \ll 1$, on superhorizon,

$\varsigma_{\boldsymbol{k}} = const$ constant mode

$\varsigma_{\boldsymbol{k}} \propto e^{-3N} = a^{-3}$ decaying mode

➡ "classical" perturbation

In ultra slow-roll inflation with $\varepsilon \ll 1, \eta = -6$, on superhorizon

$\varsigma_{\boldsymbol{k}} = const$ constant mode

$\varsigma_{\boldsymbol{k}} \propto e^{3N} = a^3$ growing mode ➡ quantum nature?

Indeed we find

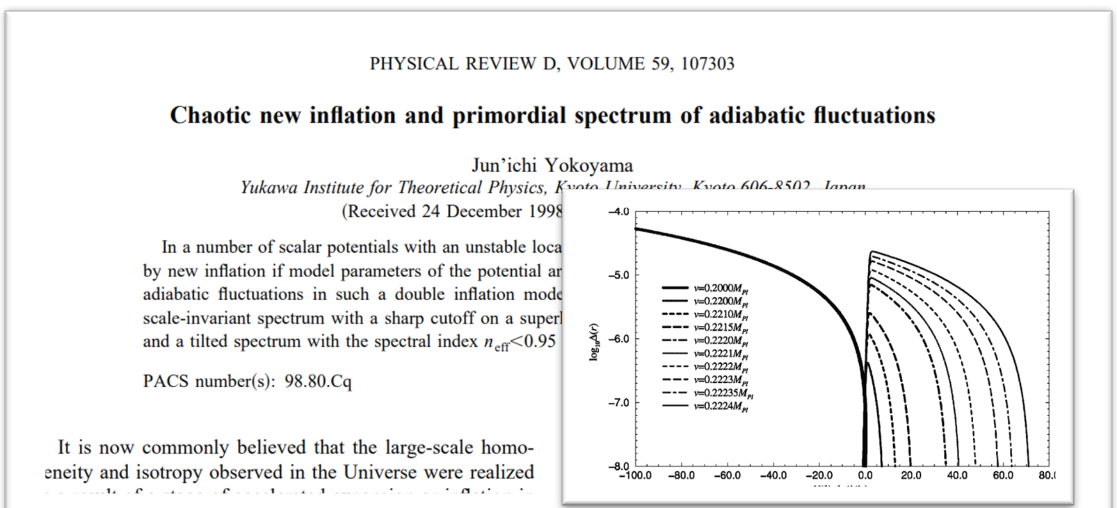
$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\zeta}'(\boldsymbol{k}',\tau)\right]=\frac{1}{2M_{pl}^2a^2\varepsilon}i\hbar\delta(\boldsymbol{k}-\boldsymbol{k}')\propto a^4\nearrow$$

$$\zeta_k'^{*}\zeta_k-\zeta_k'\zeta_k^{*}=\frac{i}{2a^2\epsilon M_{\rm pl}^2}\propto a^4\nearrow$$

which induces significant correction as we will see shortly.

In USR, the standard wisdom does not apply!

NB Such superhorizon growth of perturbation was also found in the chaotic new inflation model (JY 1999) and its analytic interpretation was given in (Saito, JY, Nagata 2008).


PHYSICAL REVIEW D, VOLUME 59, 107303

Chaotic new inflation and primordial spectrum of adiabatic fluctuations

Jun'ichi Yokoyama
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(Received 24 December 1998

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PACS number(s): 98.80.Cq

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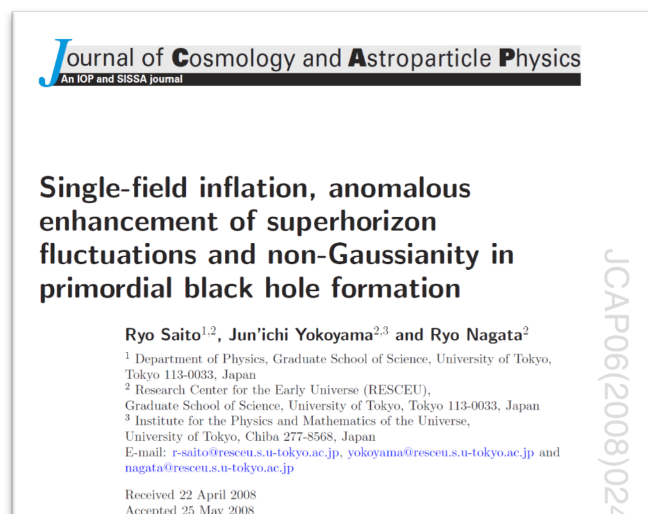
Journal of Cosmology and Astroparticle Physics
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Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

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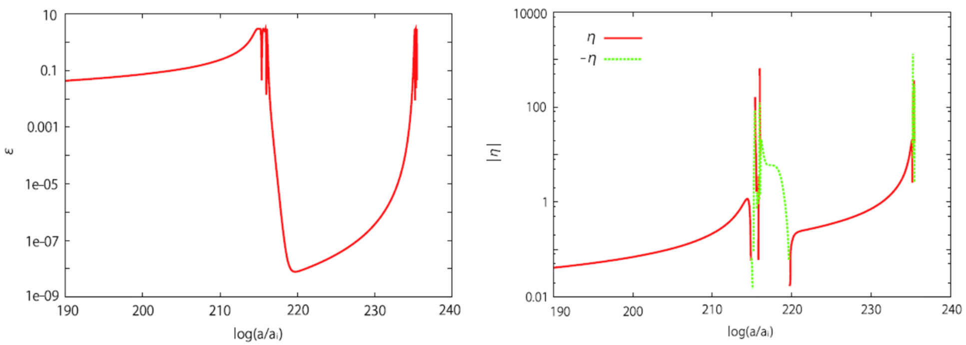

Figure 2. The evolution of slow-roll parameters ϵ (left) and η (right) with the values of the parameters $(\lambda, v) = (5.4 \times 10^{-14}, 0.355\,139 M_G)$. In the right figure, the dashed portions indicate where $\eta < 0$ while the solid portions indicate where $\eta > 0$.

3.1. Evolution of curvature perturbation

Curvature perturbation in the comoving gauge ζ, in terms of which the amplitude of perturbation in the intrinsic spatial curvature of the comoving slicing $\mathcal{R}_c$ is written as

$$\mathcal{R}_c = \frac{4}{a^2}\nabla^2\zeta, \tag{6}$$

evolves according to an equation [20]:

$$\frac{d^2\zeta_k}{dN^2} + (3 - \epsilon + \eta)\frac{d\zeta_k}{dN} + \left(\frac{k}{aH}\right)^2 \zeta_k = 0, \tag{7}$$

where N is the number of e-folds and ζ_k is the Fourier transform of ζ:

Primordial black hole formation

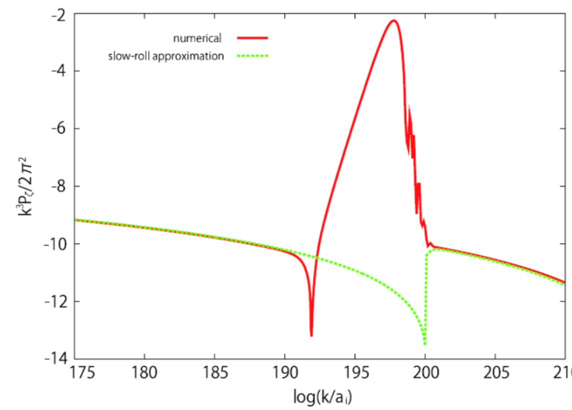

Figure 3. Power spectrum of curvature perturbation (solid line). This spectrum is calculated under the parameters $(\lambda, v) = (5.4 \times 10^{-14}, 0.355\,139 M_G)$. We show also a power spectrum estimated by using the formula (10), which is used for a slow-roll inflation model (dashed line).

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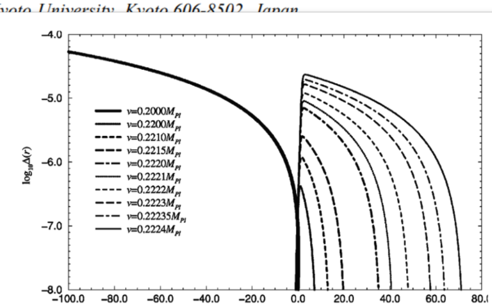

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Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

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Mode function in ultra slow-roll inflation

Mukhnanov Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0.$$

$$\frac{z''}{z} = 2a^2H^2\left(1+\varepsilon+\frac{3}{2}\delta+\frac{1}{2}\delta^2+\frac{1}{2}\varepsilon\delta+\ldots\right)$$

cancel

$$a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\varepsilon} \qquad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$$

For slow-roll inflation $\varepsilon \ll 1,\ \delta \ll 1$ we find $\frac{z''}{z} = \frac{2}{\tau^2}$

For ultra slow-roll inflation we find $\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} = -3$

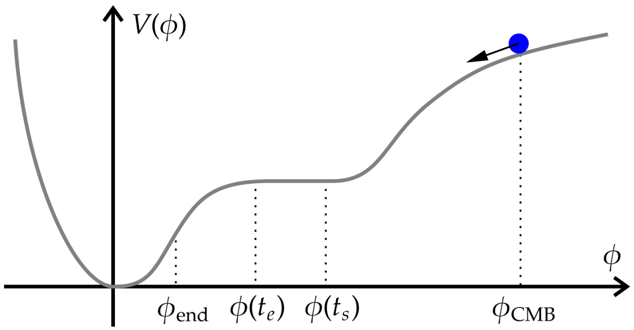
again $\frac{z''}{z} = \frac{2}{\tau^2}$

$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0$ both in SR and USR regimes!

Mode function in ultra slow-roll inflation

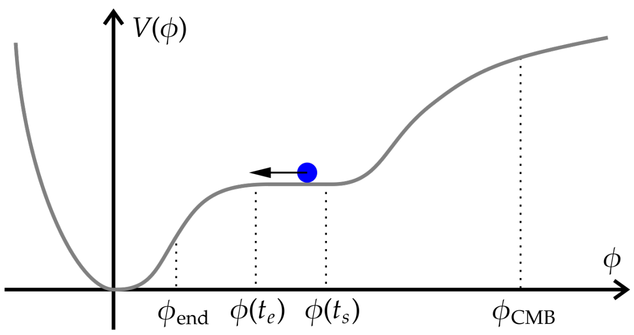
Initial slow-roll regime (CMB scale)

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$



Ultra slow-roll regime (PBH scale)

The mode function in this regime is found by matching ζ_k and ζ_k' at the transition time τ_s.



$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \left(\frac{\tau_s}{\tau}\right)^3 \frac{1}{k^{3/2}} \times \left[\mathcal{A}_k e^{-ik\tau}(1+ik\tau) - \mathcal{B}_k e^{ik\tau}(1-ik\tau)\right]$$

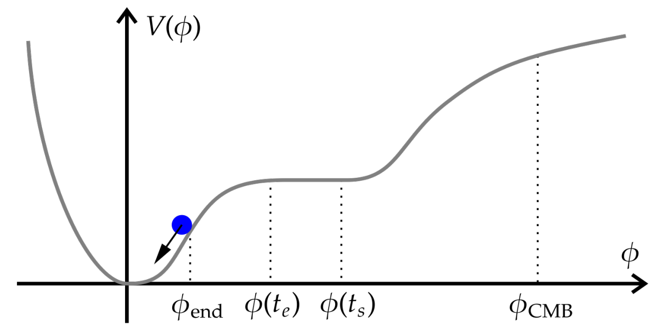
grow $\propto a^3$

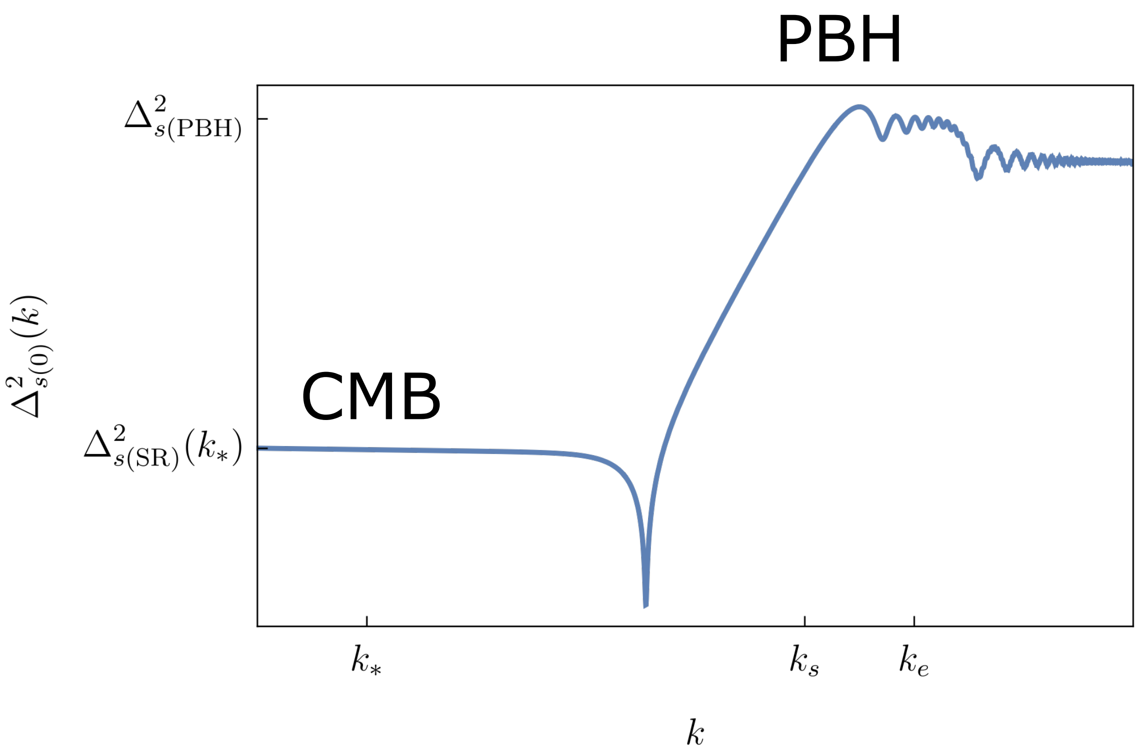
$$\mathcal{A}_k = 1 - \frac{3(1+k^2\tau_s^2)}{2ik^3\tau_s^3}, \quad \mathcal{B}_k = -\frac{3(1+ik\tau_s)^2}{2ik^3\tau_s^3}e^{-2ik\tau_s}$$

After some period of USR inflation, the system returns to SR regime again and inflation is terminated at τ_0 .

At the second transition we perform similar matching again to obtain the full solution of $\zeta_k(\tau)$.

Byrnes et. al. (1811.11158), Liu et. al. (2003.02075), Karam et. al. (2205.13540)





$$\Delta^2_{s(\rm PBH)} \approx \Delta^2_{s(\rm SR)}(k_s)\left(\frac{k_e}{k_s}\right)^6$$

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \left(\frac{\tau_s}{\tau}\right)^3 [\ldots]$$

grow $\propto a^3$

So far we have considered only second-order action of ς from the full action, which led to linear perturbation.

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\rm pl}^2 R - (\partial_\mu\phi)^2 - 2V(\phi)\right].$$

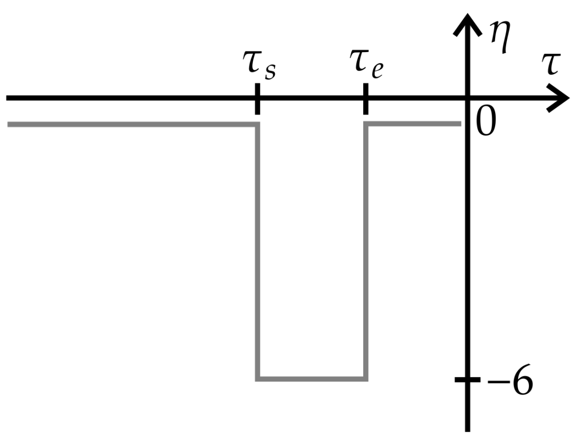
$$\longrightarrow \quad S^{(2)} = M_{\rm pl}^2 \int \mathrm{d}t\ \mathrm{d}^3x\ a^3\epsilon\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]$$

Third order terms generate non-Gaussianity and one-loop correction to the power spectrum

$$\chi = \epsilon\partial^{-2}\dot{\zeta}$$

$$\longrightarrow \quad S^{(3)}[\zeta] = M_{\rm pl}^2 \int \mathrm{d}t\ \mathrm{d}^3x\ a^3\left[\epsilon^2\dot{\zeta}^2\zeta + \frac{1}{a^2}\epsilon^2(\partial_i\zeta)^2\zeta - 2\epsilon\dot{\zeta}\partial_i\zeta\partial_i\chi - \frac{1}{2}\epsilon^3\dot{\zeta}^2\zeta + \frac{1}{2}\epsilon\zeta(\partial_i\partial_j\chi)^2 + \frac{1}{2}\epsilon\dot{\eta}\dot{\zeta}\zeta^2\right]$$

The most relevant is the last term as η changes abruptly at transitions.



$$H_{\rm int}(\tau) = -\frac{1}{2}M_{\rm pl}^2\int \mathrm{d}^3x\epsilon\eta' a^2\zeta'\zeta^2$$

Unlike in particle physics, whose focus is transition amplitude, we wish to evaluate an expectation value or a correlation function.

$$H_{\rm int}(\tau) = -\frac{1}{2} M_{\rm pl}^2 \int {\rm d}^3 x \epsilon \eta' a^2 \zeta' \zeta^2$$

In-in formalism

$$\langle \mathcal{O}(\tau) \rangle = \left\langle \left[\bar{\rm T} \exp\left(i \int_{-\infty}^{\tau} {\rm d}\tau' H_{\rm int}(\tau') \right) \right] \hat{\mathcal{O}}(\tau) \left[{\rm T} \exp\left(-i \int_{-\infty}^{\tau} {\rm d}\tau' H_{\rm int}(\tau') \right) \right] \right\rangle$$

$\hat{\mathcal{O}}(\tau) = \zeta(\mathbf{p}_1)\zeta(\mathbf{p}_2)$: evaluated toward the end of inflation $\tau = \tau_0 \ (\longrightarrow 0)$.

Perturbative expansion

$$\langle \mathcal{O}(\tau) \rangle = \langle \mathcal{O}(\tau) \rangle_{(0,2)}^{\dagger} + \langle \mathcal{O}(\tau) \rangle_{(1,1)} + \langle \mathcal{O}(\tau) \rangle_{(0,2)}$$

$$\langle \mathcal{O}(\tau) \rangle_{(0,2)} = -\int_{-\infty}^{\tau} {\rm d}\tau_1 \int_{-\infty}^{\tau_1} {\rm d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau) H_{\rm int}(\tau_1) H_{\rm int}(\tau_2) \right\rangle$$

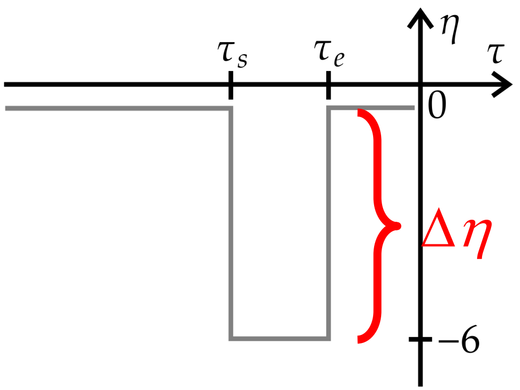
$$\langle \mathcal{O}(\tau) \rangle_{(1,1)} = \int_{-\infty}^{\tau} {\rm d}\tau_1 \int_{-\infty}^{\tau} {\rm d}\tau_2 \left\langle H_{\rm int}(\tau_1) \hat{\mathcal{O}}(\tau) H_{\rm int}(\tau_2) \right\rangle$$

After substituting $H_{\rm int}(\tau)$ to the perturbative expansion, we find

$$\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(1,1)} = \frac{1}{4}M_{\rm pl}^4 \int_{-\infty}^{0} \mathrm{d}\tau_1\ a^2(\tau_1)\epsilon(\tau_1)\eta'(\tau_1) \int_{-\infty}^{0} \mathrm{d}\tau_2\ a^2(\tau_2)\epsilon(\tau_2)\eta'(\tau_2) \int \prod_{a=1}^{6}\left[\frac{\mathrm{d}^3 k_a}{(2\pi)^3}\right]\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$\times\,\delta^3(\mathbf{k}_4+\mathbf{k}_5+\mathbf{k}_6)\,\langle\zeta'(\mathbf{k}_1,\tau_1)\zeta(\mathbf{k}_2,\tau_1)\zeta(\mathbf{k}_3,\tau_1)\zeta(\mathbf{p})\zeta(-\mathbf{p})\zeta'(\mathbf{k}_4,\tau_2)\zeta(\mathbf{k}_5,\tau_2)\zeta(\mathbf{k}_6,\tau_2)\rangle\,,$$

$$\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(0,2)} = -\frac{1}{4}M_{\rm pl}^4 \int_{-\infty}^{0} \mathrm{d}\tau_1\ a^2(\tau_1)\epsilon(\tau_1)\eta'(\tau_1) \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2\ a^2(\tau_2)\epsilon(\tau_2)\eta'(\tau_2) \int \prod_{a=1}^{6}\left[\frac{\mathrm{d}^3 k_a}{(2\pi)^3}\right]\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$\times\,\delta^3(\mathbf{k}_4+\mathbf{k}_5+\mathbf{k}_6)\,\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\zeta'(\mathbf{k}_1,\tau_1)\zeta(\mathbf{k}_2,\tau_1)\zeta(\mathbf{k}_3,\tau_1)\zeta'(\mathbf{k}_4,\tau_2)\zeta(\mathbf{k}_5,\tau_2)\zeta(\mathbf{k}_6,\tau_2)\rangle\,.$$

Time integral is nonvanishing only at τ_s and τ_e and the latter makes a dominant contribution.



As a result, we find

$$\langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(1)} = \langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(1,1)} + 2\mathrm{Re}\,\langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(0,2)}$$
$$= \frac{1}{4}M_{\rm pl}^4\epsilon^2(\tau_e)a^4(\tau_e)(\Delta\eta(\tau_e))^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\Big[4\zeta_p\zeta_p^*\zeta_p'\zeta_p'^*\zeta_k\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p^*\zeta_p'^*\zeta_p\zeta_k'\zeta_k^*\zeta_q\zeta_q^*$$
$$+ 8\zeta_p\zeta_p^*\zeta_p'\zeta_p^*\zeta_k'^*\zeta_k\zeta_q\zeta_q^*$$
$$- \mathrm{Re}\left(4\zeta_p\zeta_p\zeta_p'^*\zeta_p'^*\zeta_k\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p\zeta_p'^*\zeta_p^*\zeta_k'\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p\zeta_p'^*\zeta_p^*\zeta_k'^*\zeta_k\zeta_q\zeta_q^*\right)\Big]_{\tau=\tau_e}$$

$$\boldsymbol{q} \equiv \boldsymbol{k} - \boldsymbol{p}$$

The leading term is given by

$$\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(1)} = \frac{1}{4} M_{\rm pl}^4 \epsilon^2(\tau_e) a^4(\tau_e)(\Delta\eta(\tau_e))^2 \times 16 \int \frac{d^3k}{(2\pi)^3} \left[|\zeta_p|^2 |\zeta_q|^2 \ \mathrm{Im}(\zeta_p' \zeta_p^*) \ \mathrm{Im}(\zeta_k' \zeta_k^*) \right]_{\tau=\tau_e} .$$

$$\mathrm{Im}(\zeta_k' \zeta_k^*) = \frac{i}{2}(\zeta_k'^* \zeta_k - \zeta_k' \zeta_k^*) = \frac{-1}{4a^2 \epsilon(\tau_e) M_{\rm pl}^2}$$

takes a big value at the end of USR regime as we argued.

$$\mathrm{Im}(\varsigma_k' \varsigma_k^*) = \frac{-1}{4a^2(\tau_e)\varepsilon_{SR}(a_s/a_e)^6 M_{pl}^2} = \frac{-1}{4\varepsilon_{SR} M_{pl}^2} (H\tau_e)^2 \left(\frac{\tau_s}{\tau_e}\right)^6 = -\frac{k_e^4}{k_s^6} \left(\frac{H^2}{4M_{pl}^2 \varepsilon_{SR}}\right)$$

at $\tau = \tau_e$, where we have used $k_e = a(\tau_e)H = -\frac{H}{H\tau_e} = -\frac{1}{\tau_e}$.

The leading term is given by

$$\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(1)} = \frac{1}{4}M_{\rm pl}^4\epsilon^2(\tau_e)a^4(\tau_e)(\Delta\eta(\tau_e))^2 \times 16\int\frac{d^3k}{(2\pi)^3}\left[|\zeta_p|^2|\zeta_q|^2\ \mathrm{Im}(\zeta_p'\zeta_p^*)\ \mathrm{Im}(\zeta_k'\zeta_k^*)\right]_{\tau=\tau_e}.$$

$$\mathrm{Im}(\zeta_k'\zeta_k^*) = \frac{i}{2}(\zeta_k'^*\zeta_k - \zeta_k'\zeta_k^*) = \frac{-1}{4a^2\epsilon(\tau_e)M_{\rm pl}^2}$$

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at $\tau = \tau_e$, where we have used $k_e = a(\tau_e)H = -\frac{H}{H\tau_e} = -\frac{1}{\tau_e}$.

This is in contrast to the standard SR inflation in which $\mathrm{Im}(\zeta_k'\zeta_k^*)$ becomes exponentially small.

The leading term is given by

$$\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(1)} = \frac{1}{4}M_{\rm pl}^4\epsilon^2(\tau_e)a^4(\tau_e)(\Delta\eta(\tau_e))^2 \times 16\int\frac{{\rm d}^3k}{(2\pi)^3}\left[|\zeta_p|^2|\zeta_q|^2\ {\rm Im}(\zeta_p'\zeta_p^*)\ {\rm Im}(\zeta_k'\zeta_k^*)\right]_{\tau=\tau_e}.$$

$${\rm Im}(\zeta_k'\zeta_k^*) = \frac{i}{2}(\zeta_k'^*\zeta_k - \zeta_k'\zeta_k^*) = \frac{-1}{4a^2\epsilon(\tau_e)M_{\rm pl}^2}$$

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at $\tau=\tau_e$, where we have used $k_e = a(\tau_e)H = -\frac{H}{H\tau_e} = -\frac{1}{\tau_e}$.

One-loop correction

$$\Delta^2_{s(1)}(p) = \frac{1}{4}(\Delta\eta(\tau_e))^2\Delta^2_{s({\rm SR})}(p)\int_{k_s}^{k_e}\frac{{\rm d}k}{k}\Delta^2_{s(0)}(k)$$

$$\Delta^2_{s(1)}(p) = \frac{1}{4}(\Delta\eta(\tau_e))^2\left[\Delta^2_{s({\rm SR})}(p)\right]^2\left(\frac{k_e}{k_s}\right)^6\left(1.1+\log\frac{k_e}{k_s}\right)$$

For perturbation theory to be valid, we require one loop correction << tree level (linear theory) result

$$\Delta^2_{s(1)} \ll \Delta^2_{s(\mathrm{SR})}: \quad \underbrace{\frac{1}{4}(\Delta\eta(\tau_e))^2}_{6^2=36}\underbrace{\Delta^2_{s(\mathrm{SR})}(p)}_{2.1\times10^{-9}}\left(\frac{k_e}{k_s}\right)^6\left(1.1+\log\frac{k_e}{k_s}\right) \ll 1.$$

we obtain $\frac{k_e}{k_s} < 15$, or $\Delta^2_{s(\mathrm{PBH})} \ll 0.03\left(\frac{k_s}{k_*}\right)^{-0.03}$.

($n_s = 0.97$ at $k_* = 0.05\mathrm{Mpc}^{-1}$)

- Consider two examples that are of recent interest:
  - PBHs as dark matter with mass $\mathcal{O}(10^{-15})M_{\odot}$ corresponding to scale $\mathcal{O}(10^{14})\mathrm{Mpc}^{-1}$ has a bound $\Delta^2_{s(\mathrm{PBH})} \ll 0.01$.
  - PBHs as LIGO-Virgo BHs with mass $\mathcal{O}(10)M_{\odot}$ corresponding to scale $\mathcal{O}(10^{6})\mathrm{Mpc}^{-1}$ has a bound $\Delta^2_{s(\mathrm{PBH})} \ll 0.02$.

- In both cases, the upper bound contradicts with typical requirement to form a significant abundance of PBHs, which is $\Delta^2_{s(\mathrm{PBH})} \sim \mathcal{O}(0.01)$.

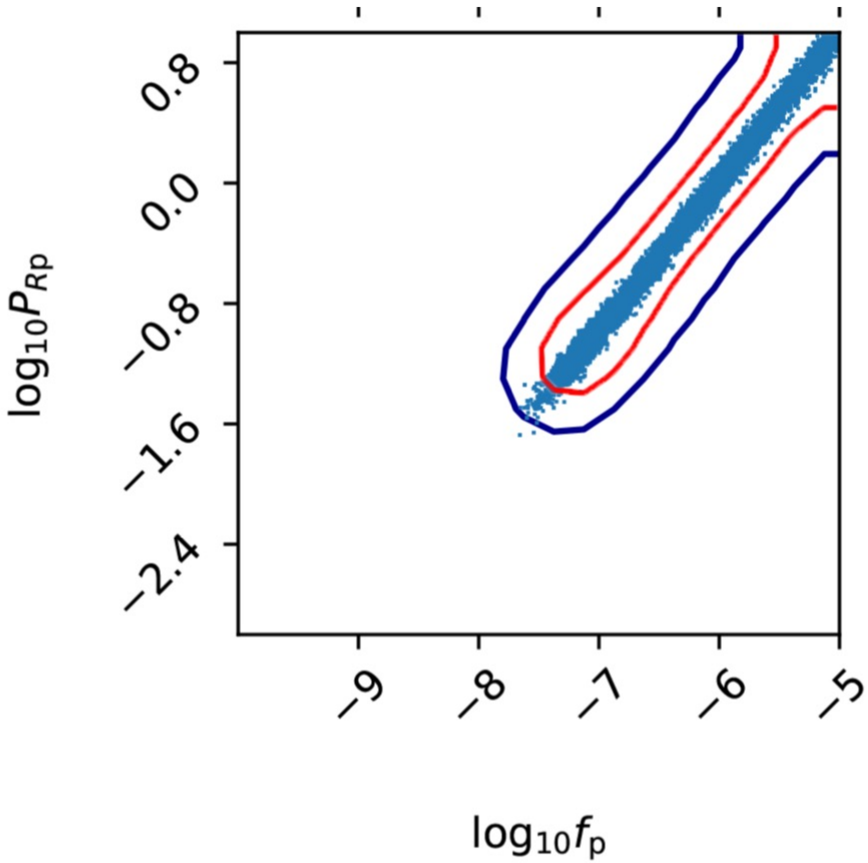
Kristiano and JY 2211.03395

A number of single-field inflation models accommodating PBH formation have the same feature, namely, sharp transition of η.

- Bump or dip: Mishra and Sahni (1911.00057)
- Upward or downward step: Cai et. al. (2112.13836), Inomata et. al. (2104.03972)
- Polynomial shape: Hertzberg and Yamada (1712.09750), Ballesteros et. al. (2001.08220)
- Chaotic new inflation with a Coleman-Weinberg potential: Saito, JY, & Nagata (0804.3470)

All these single-field inflation models producing sizable amount of PBHs are in trouble.

Can PTAs observations be explained by tensor perturbations produced by second-order scalar perturbations from ultra slow-roll inflation ?



2310.20564
Mu, Liu, Cheng, Kuo

$\Delta_s^2 > 0.025$ is required, so inconsistent with our constraint.

See also 2308.08546
Ellis et al.

Conclusion

- Higher order quantum effects are important in single-field inflation models with peaky spectrum due to USR behavior.

- Primordial black hole formation and second order gravitational wave background from inflation may be important clues to quantum effects during inflation.

- PBHs and SGWB from inflation may require multi-field setting.

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鑑