# Gravitational Waves in Gauss Bonnet Cosmologies

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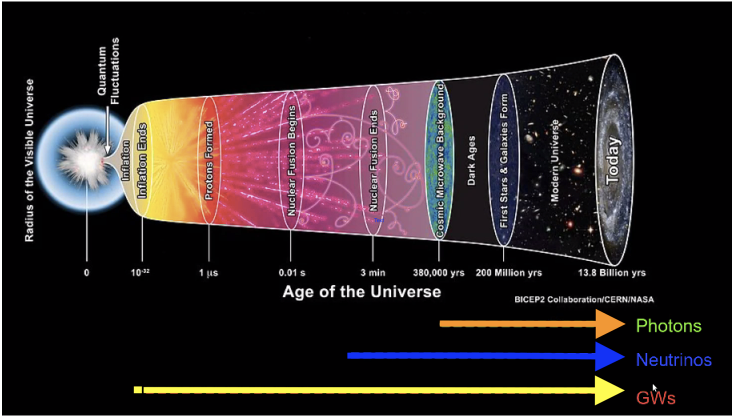
*Based on JCAP 08 (2023) 024 e-Print: 2303.05813 [Biswas, Kar, BH Lee, HC Lee, WW Lee, Scopel, Velasco-Sevilla, Yu] and work to appear soon*

*Osaka, 6 November 2023*

## Outline

# Motivation

- As we all know, GW offer us a unique opportunity to test theory well beyond photons and neutrinos can do.
- In particular, theories of Modified Gravity.



## Specific Gauss-Bonnet Model

$$S = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[ \frac{R}{2\kappa} - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi - V(\phi) + \ f(\phi) R^2_{\mathrm{GB}} + \mathcal{L}_m^{\mathrm{rad}} \right],$$

where $\kappa \equiv 8\pi G = 1/M_{PL}^2$ and $R^2_{\mathrm{GB}} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet (GB) term.
The coupling between the scalar field, $\phi$ and the GB term is driven by a function of the scalar field $f(\phi)$, which is arbitrary.
In this work,

$$f(\phi) = \alpha e^{\gamma\phi},$$

which is the so called dilatonic-Einstein coupling (often appearing in String Theory but can be studied independently).

The modified Friedmann equations can be written as:

$$
\begin{aligned}
H^2 &= \frac{\kappa}{3}\left(\rho_{\{\phi+\mathrm{GB}\}} + \rho_{\mathrm{rad}}\right) \equiv \frac{\kappa}{3}\rho_{\mathrm{tot}}\,, \\
\dot{H} &= -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+\mathrm{GB}\}} + p_{\{\phi+\mathrm{GB}\}}\right) + \left(\rho_{\mathrm{rad}} + p_{\mathrm{rad}}\right)\right] \equiv -\frac{\kappa}{2}\left(\rho_{\mathrm{tot}} + p_{\mathrm{tot}}\right), \\
\ddot{\phi} &+ 3H\dot{\phi} + V' - f' R^2_{\mathrm{GB}} = 0\,,
\end{aligned}
$$

where $\rho_{\mathrm{tot}}$ and $p_{\mathrm{tot}}$ can be interpreted as the total energy density and the pressure of the Universe.

Gauss-Bonnet theories have been used in the past for

- Inflation [Jinsu Kim talk]
- Quintessence
- Recently for WIMPS [2303.05813 ]

- In [2303.05813 ], we have considered a model for which $V(\phi) = 0$ because we wanted to study conditions not coming from inflation but from BBN
- We put constraints on the model parameters using the fact that in a modified cosmological scenario the WIMP annihilation cross section at freeze-out $\langle\sigma v\rangle_f$ required to predict the correct relic abundance is modified compared to the standard value $3\times10^{-26}$ cm$^3 s^{-1}$

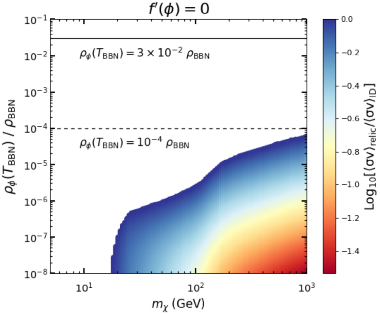
- Equations for $H^2$ and $\dot{H}$ can be re-arranged into a set of three coupled differential equations for the quantities $\phi$, $\dot{\phi}$ and $H$.
- We fix the boundary condition at BBN ($T = T_{\rm BBN} = 1$ MeV) $\phi(T_{\rm BBN}) \equiv \phi_{\rm BBN}$, $\dot{\phi}(T_{\rm BBN}) \equiv \dot{\phi}_{\rm BBN}$ and $H(T_{\rm BBN}) \equiv H_{\rm BBN}$.
- We can in fact parameterize the physical observables in terms of only the following parameters

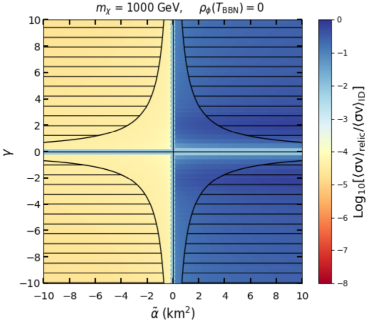
$$\phi'_{\rm BBN} = \phi_{\rm BBN} + \phi_0, \;\; \alpha' = \alpha\, e^{-\gamma\phi_0}, \;\; \gamma.$$

or equivantly

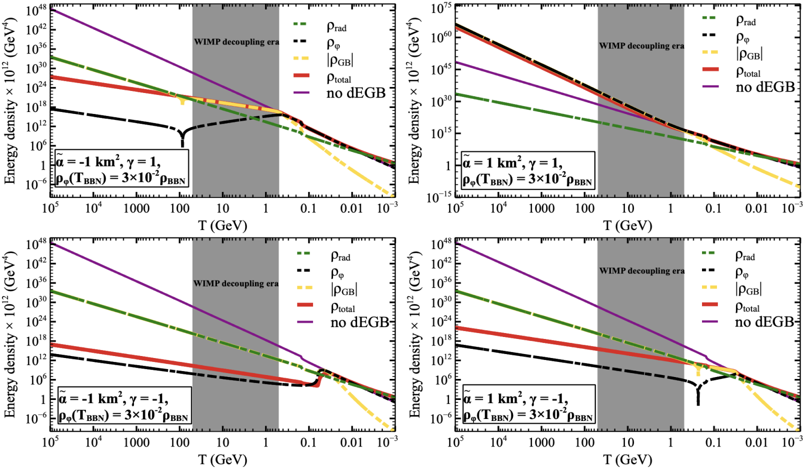
$$\rho_{\rm BBN}, \quad \alpha' = \alpha\, e^{-\gamma\phi_0}, \;\; \gamma.$$

- To give the sense of the motivation we can look at how our GB scenario can open up the parameter space for the WIMP masses and relic abundance in comparison to GR



What we found interesting is that considering different evolutions of the universe, via the GB $f$ function leads to open up the WIMP parameter space.



## GW Production from SM Plasma

- Physical processes ranging from microscopic particle collisions to macroscopic hydrodynamic fluctuations induce gravitational waves in any plasma in thermal equilibrium [1504.02569, J. Ghiglieri and M. Laine]. 2011.04731, A. Ringwald, J. Schütte-Engel and C.Tamarit / F. Muia, F. Quevedo, A. Schachner, G. Villa 2303.01548, JCAP
- For the largest wavelengths the emission rate is proportional to the shear viscosity, $\eta(T, \hat{k})$, of the plasma. In the Standard Model at $T > 160$ GeV, the shear viscosity is dominated by the most weakly interacting particles, right-handed leptons, and is relatively large.
- The evolution of the density of the GW is simply given by

$$(\partial_t + 4H)\rho(t)_{\mathrm{GW}} = 4\frac{T^4}{\overline{M}_{\mathrm{P}}^2}\int \frac{d^3k}{(2\pi)^3}\eta(T,k),$$

- All the information of the plasma is encoded in $\eta(T, k)$.

Near to the peak of the GW signal $\eta(T,\hat{k})$ can be computed using the HTL (Hard Thermal Logarithmic) methods Braaten, Pisarksi, Soft Amplitudes in Hot Gauge Theories: A General Analysis, Nucl. Phys B, 1990

$$\eta(T,\hat{k}) = \frac{1}{16\pi}\hat{k} f_B(\hat{k}) \sum_{i=1}^{3} d_i \hat{m}_{D_i}^2 \ln\left(4\frac{1}{\hat{m}_{D_i}}\hat{k}^2 + 1\right), \quad k \gtrsim 3T,$$

where the Debye masses are

$$m_{D_i}^2 = \begin{cases} d_1 \frac{11}{6} g_1^2 T^2,\ d_1 = 1, \\ d_2 \frac{11}{6} g_2^2 T^2,\ d_2 = 3, \\ d_3 2 g_3^2 T^2,\ d_3 = 8. \end{cases}$$

$\hat{k} := k/T$, $\hat{m}_{D_i} = m_{D_i}/T$.

$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}} \frac{a_{\text{Today}}}{a}, \quad a(T) = a_0 \frac{T_0}{T} \frac{g_{*0}^{1/3}}{g_*(T)^{1/3}},$$

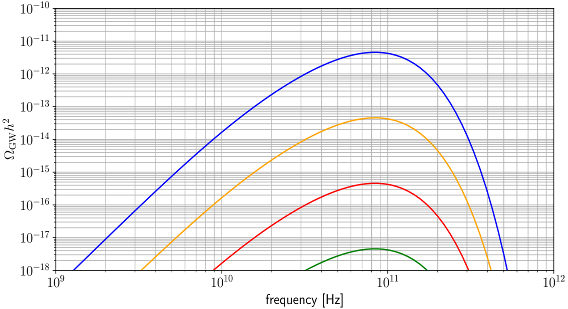
$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}}\, a_0 \frac{T}{a_0 T_0} \left(\frac{g_*(T)}{g_{0*}}\right)^{1/3} = 2\pi \frac{1}{T_0} f_{\text{Today}} \left(\frac{g_*(T)}{g_{0*}}\right)^{1/3}.$$

- In Standard Cosmology, if the GW are emitted in the radiation era, there is a simple relation with regards the temperature [1504.02569, J. Ghiglieri and M. Laine]:

Note that if one now assumes $\eta(T,\hat{k})$, $\hat{k}$ and $g_*(T)$ are independent of the temperature [a good approximation up to the order of magnitude], we have

$$\frac{\Omega_{\rm GW}(f)h^2}{\Omega_{\gamma_0}h^2} \quad = \quad \lambda\left[\frac{a_{\rm in}T_{\rm in}}{a_0T_0}\right]^4 \frac{T_{\rm in}-T_{\rm end}}{\overline{M}_{\rm P}}\frac{T_{\rm in}^2}{\sqrt{\rho}}\,\hat{k}^3\,\eta\left(T,\hat{k}\right),$$

From top to bottom $T = 10^{14}, 10^{12}, 10^{10}, 10^{8}$ GeV



- In other cosmologies, the evolution of the universe may be diverse and hence the way GW propagate

$$\rho(t_{\rm end})_{\rm GW} = \frac{4}{a^4(t_{\rm end})} \int_{a_{\rm in}}^{a_{\rm end}} da\, \frac{a^3}{H} \int \frac{d^3k}{(2\pi)^3} \frac{T^4}{\overline{M}_{\rm P}^2}\, \eta\left(T, \hat{k}\right),$$

  - $H^2 = \frac{1}{3\overline{M}_{\rm P}^{\,2}} \rho_{\rm Tot.}.$
  - $H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+{\rm GB}\}} + \rho_{\rm rad}\right)$, in GB cosmologies and therefore can change drastically the way we can see GW today.
  - For an arbitrary evolution

$$\frac{\Omega_{\rm GW}(f)h^2}{\Omega_{\gamma_0}h^2} = \Omega_\gamma \frac{\lambda}{\overline{M}_{\rm P}} \int_{T_{\rm end}}^{T_{\rm in}} dT \left(\frac{g_{*0}}{g*(T)}\right)^{4/3} T^2\, \hat{k}^3 \frac{\eta(T,\hat{k})}{\sqrt{\rho_{\rm Tot.}}}\, F(T),$$

$$F(T) \approx 1,$$

  assuming there is not a huge change in the degrees of freedom.

- The basic behaviour is controlled by how big $\rho_{\text{Tot.}}$ increases or decreases with respect to the radiation density, as it can be seen by looking at the temperature dependence on

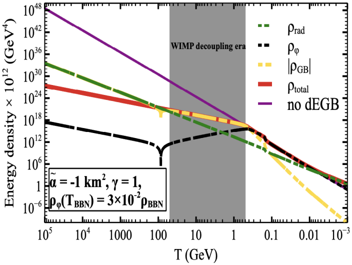
$$\frac{\Omega_{\text{GW}}(f)h^2}{\Omega_{\gamma_0}h^2} \approx \Omega_\gamma \frac{\lambda}{\overline{M}_{\text{P}}} \int_{T_{\text{end}}}^{T_{\text{in}}} dT \left(\frac{g_{*0}}{g*(T)}\right)^{4/3} T^2\,\hat{k}^3 \frac{\eta(T,\hat{k})}{\sqrt{\rho_{\text{Tot.}}}}\,,$$

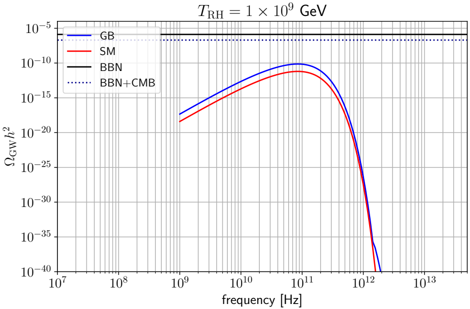
  and remembering $\rho_{\text{rad.}} \propto T^4$.

- The peak frequency has only a minor dependence on the temperature and therefore it does not change much

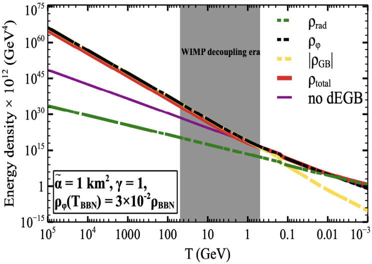
$$\hat{k} = \frac{1}{T} 2\pi f_{\text{Today}} \frac{a_{\text{Today}}}{a}, \quad a(T) = a_0 \frac{T_0}{T} \frac{g_{*0}^{1/3}}{g_*(T)^{1/3}}.$$

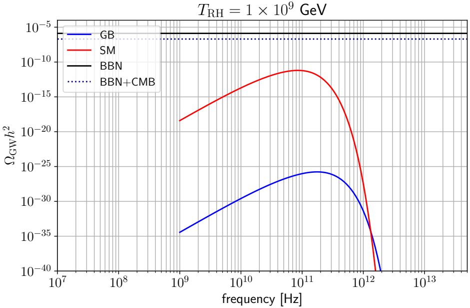
- For $\tilde{\alpha} = -1$, $\gamma = 1$:



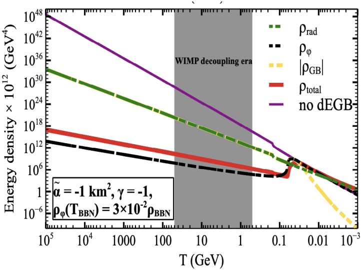



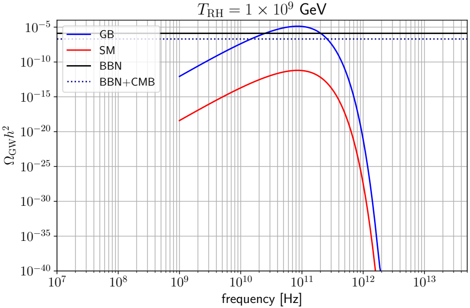

- For $\tilde{\alpha} = 1$, $\gamma = 1$:



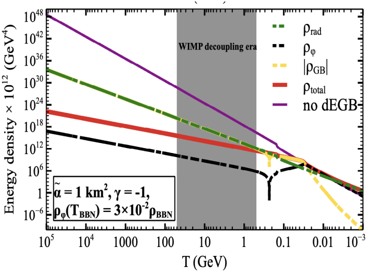



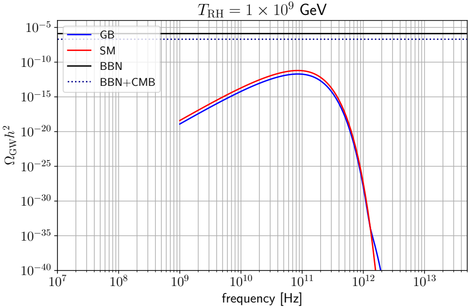
- For $\tilde{\alpha} = -1$, $\gamma = -1$:

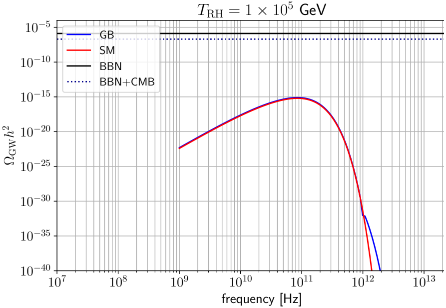


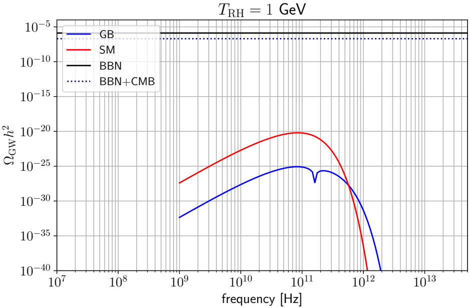



- For $\tilde{\alpha} = 1$, $\gamma = -1$:



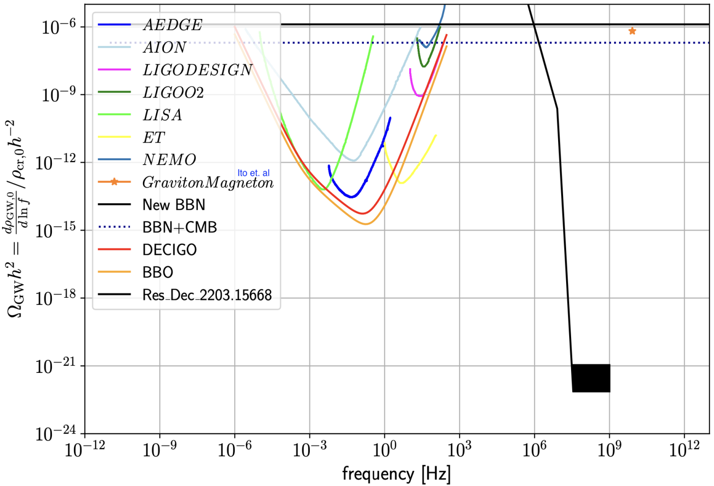



- Hence for some cases, we will be able to set a limit on the reheating temperature
- In these cases, the limit on reheating temperature can be drastically reduced in comparison to the SM
- In other cases, it could be increased and therefore change the panorama of some particle physics processes we know

- We all know however that current and experimental efforts are far below the peak frequency for these kind of signals



- The Resonant Detector proposed in 2203.15668 by Herman, Lehoucq and Füzfa has the potential to start probing the required frequency region.

### Electromagnetic Antennas for the Resonant Detection of the Stochastic Gravitational Wave Background

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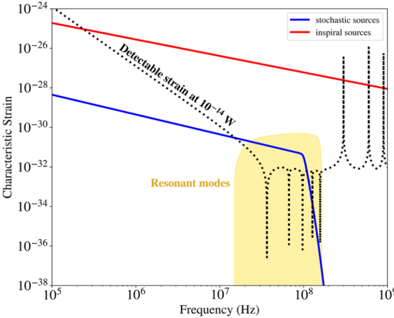
(Dated: December 7, 2022)

Some stochastic gravitational wave background models from the early Universe has a cut-off frequency close to 100 MHz, due to the horizon of the inflationary phase. To detect gravitational waves at such frequencies, resonant electromagnetic cavities are very suitable. In this work, we study the frequency sensitivity of such detectors, and show how we could use them to probe this cut-off frequency and also the energy density per frequency of this stochastic background. This paper paves the way for further experimental studies to probe the most ancient relic of the Universe.




**Please let me know if you know more about proposals in the MHz and GHz region.**

## Conclusions

- GW will probe the evolution of our universe in a way photons or neutrinos cannot
- Modified gravities can be tested without the need of ad-hoc mechanisms, using the GW coming from the Standard Model plasma
- GUT theories can also produce similar GW and modified theories of gravity can be tested in the same way: by confronting its predictions to Standard Cosmology
- No experiments are currently planned to probe the required region in frequency and sensitivity but this kind of studies constitute a solid science case that can provide a motivation to develop experiments probing that region

ありがとう!