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Solitonic Gravastars in a U(1) Gauge Higgs Model

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Boson stars

In GR and cosmology, classical solutions called “Boson stars” are known.

- They exist in a coupled system consists of bosonic fields and gravitational field.
- A kind of nontopological solitons.

  Localized solutions whose stability is guaranteed by conserved Noether charge.

- Dark matter candidate.
- Supermassive solutions could be possible.
  - Seed of supermassive black hole?

Gravastar (Gravitational Vacuum Star)

On the other hand, as a black hole mimicker, gravastar is proposed.

(Mazur and Mottola 2001)

- A kind of black hole mimickers.
- Event horizon does not exist.
- Filled by vacuum energy inside the star, outside is true vacuum.
  - Inside: de Sitter, Outside: Schwarzschild
- Formation process is not understood.
  - Quantum effects?

Summary of today's talk

Using the model

$$S=\int\sqrt{-g}d^4x\left\{\frac{R}{16\pi G}-g^{\mu\nu}(D_\mu\psi)^*(D_\nu\psi)-g^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi)\right.$$
$$\left.-\frac{\lambda}{4}\left(|\phi|^2-\eta^2\right)^2-\mu|\phi|^2|\psi|^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\},$$

we show that boson stars filled with vacuum energy exist.

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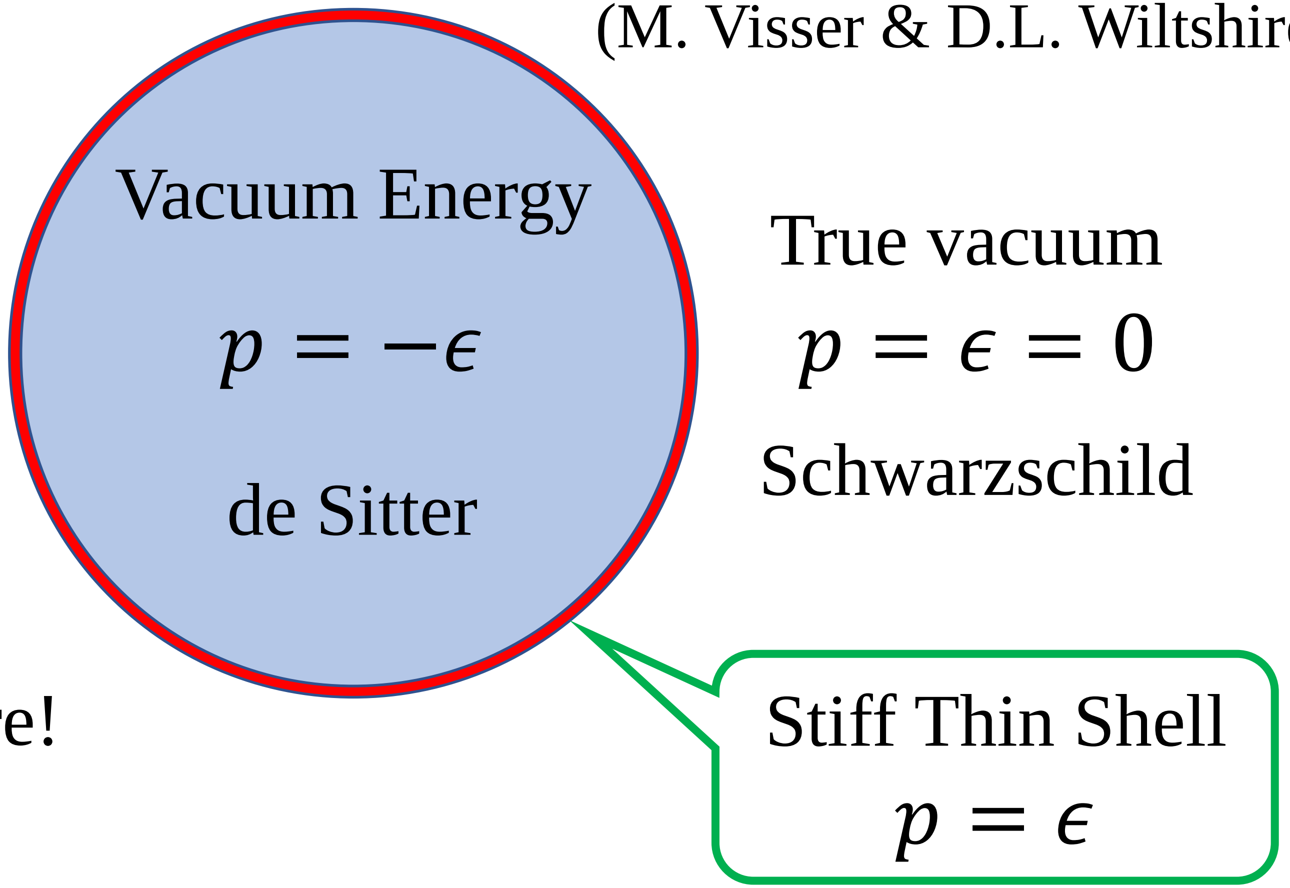
Solitonic gravastars

Gravastar (Gravitational Vacuum Star)

- In general, gravastar solution is artificially constructed by using Israel's junction condition.

(M. Visser & D.L. Wiltshire 2003)



Assuming
This structure!

Gravastar (Gravitational Vacuum Star)

- In general, gravastar solution is artificially constructed by using Israel's junction condition.

In our model, we can build the structure of the Gravastar naturally.

This situation!

Stiff Thin Shell
$p = \epsilon$

Contents

U(1) Gauge Higgs Model

We consider the model

$$S = \int \sqrt{-g} d^4x \left\{ \frac{R}{16\pi G} + \mathcal{L}_M \right\}$$

$$D_\mu \psi \coloneqq \partial_\mu \psi + ieA_\mu \psi$$

$$D_\mu \phi \coloneqq \partial_\mu \phi + ieA_\mu \phi$$

$$\mathcal{L}_M = -g^{\mu\nu}\left(D_\mu \psi\right)^* (D_\nu \psi) - g^{\mu\nu}\left(D_\mu \phi\right)^* (D_\nu \phi) - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 - \mu|\psi|^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- Consist of complex scalar field ψ, U(1) gauge field A_μ, and complex Higgs scalar field ϕ.
  - Spontaneously symmetry breaking $\Rightarrow$ ψ and A_μ acquire mass.

- $U(1)_{local} \times U(1)_{global}$ gauge invariant.

$$\psi \to \psi' = e^{i\xi(x)+\beta}\psi \quad \phi \to \phi' = e^{i\xi(x)+\gamma}\phi \quad A_\mu \to A'_\mu = A_\mu - ie^{-1}\partial_\mu \xi$$

- Complex scalar field ψ and ϕ couple to same gauge field A_μ.
  - Both ψ and ϕ are source of the gauge field A_μ.

- Even if the gravitational field does not exist, spherically symmetric solitons (Q-balls) can be constructed.

Field equations

$$\mathcal{L} = -\left(D_\mu\psi\right)^*(D^\mu\psi) - \left(D_\mu\phi\right)^*(D^\mu\phi) - \frac{\lambda}{4}(|\phi|^2-\eta^2)^2 - \mu|\psi|^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Klein-Gordon equations

$$D_\mu D^\mu\psi - \mu\psi|\phi|^2 = 0$$

$$D_\mu D^\mu\phi - \frac{\lambda}{2}\phi(|\phi|^2-\eta^2) - \mu|\psi|^2\phi = 0$$

Maxwell equations

$$\partial_\mu F^{\mu\nu} = j^\nu_\psi + j^\nu_\phi$$

$$j^\mu_\phi = ie\left\{\phi^*(D^\mu\phi) - (D^\mu\phi)^*\phi\right\},$$
$$j^\mu_\psi = ie\left\{\psi^*(D^\mu\psi) - (D^\mu\psi)^*\psi\right\}$$

Energy-momentum tensor

$$T_{\mu\nu} = 2\left(D_\mu\psi\right)^*(D_\nu\psi) + 2\left(D_\mu\phi\right)^*(D_\nu\phi) - g_{\mu\nu}\left[(D_\alpha\psi)^*(D^\alpha\psi) + (D_\alpha\phi)^*(D^\alpha\phi) + \frac{\lambda}{4}(|\phi|^2-\eta^2)^2 + \mu|\psi|^2|\phi|^2\right] + \left(F_{\mu\alpha}F^\alpha_\nu - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$T^t_t \coloneqq \epsilon$: energy density

Field equations

$$D_\mu D^\mu \psi - \mu\psi|\phi|^2 = 0\,,\, D_\mu D^\mu \phi - \frac{\lambda}{2}\phi(|\phi|^2-\eta^2) - \mu|\psi|^2\phi = 0\,,\, \partial_\mu F^{\mu\nu} = j_\psi^\nu + j_\phi^\nu$$

Energy density

$$\epsilon = \left(e\widetilde{A_t} - \Omega\right)^2\tilde{\psi}^2 + e^2\widetilde{A_t}^2\tilde{\phi}^2 + \left(\tilde{\psi}'\right)^2 + \left(\tilde{\phi}'\right)^2 + \frac{\lambda}{4}\left(\tilde{\phi} - \eta^2\right)^2 + \mu\tilde{\psi}^2\tilde{\phi}^2 + \frac{1}{2}\left(\widetilde{A_t}'\right)^2$$

Stationary and spherically symmetric ansatz

$$\psi(t,r) = e^{i\omega t}\tilde{\psi}(r)\,,\,\phi(t,r) = e^{i\omega' t}\tilde{\phi}(r), A = A_t(r)dt$$

Gauge fixing : $\xi(x) = -\omega' t$ $\Omega \coloneqq \omega - \omega'$

$$\psi(t,r) = e^{i\Omega t}\tilde{\psi}(r)\,,\,\phi(t,r) = \tilde{\phi}(r), \widetilde{A_t}(r) := A_t + e^{-1}\omega'$$

$$\frac{d^2\tilde{\psi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\psi}}{dr} - \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\psi}} \quad \frac{d^2\tilde{\phi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\phi}}{dr} - \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\phi}} \quad \frac{d^2\tilde{A}_t}{dr^2} = -\frac{2}{r}\frac{d\tilde{A}_t}{dr} + \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{A}_t}$$

$$U_{\mathrm{eff}}(\tilde{\psi},\tilde{\phi},\tilde{A}_t) \coloneqq -\frac{\lambda}{4}\left(\tilde{\phi}^2-\eta^2\right)^2 - \mu\tilde{\psi}^2\tilde{\phi}^2 + \left(e\tilde{A}_t - \Omega\right)^2\tilde{\psi}^2 + e^2\tilde{A}_t^2\tilde{\phi}^2$$

Boundary conditions

$r \to 0$: regularity

$$\tilde{\psi}' = 0, \tilde{\phi}' = 0, \widetilde{A_t}' = 0$$

$r \to \infty : \epsilon \to 0$

$$\tilde{\psi} = 0, \tilde{\phi} = \eta, \widetilde{A_t} = 0$$

Analogy with Newtonian mechanics

$$\frac{d^2\tilde{\psi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\psi}}{dr} - \frac{\partial U_{\text{eff}}}{\partial\tilde{\psi}} \quad \frac{d^2\tilde{\phi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\phi}}{dr} - \frac{\partial U_{\text{eff}}}{\partial\tilde{\phi}} \quad \frac{d^2\tilde{A}_t}{dr^2} = -\frac{2}{r}\frac{d\tilde{A}_t}{dr} + \frac{\partial U_{\text{eff}}}{\partial\tilde{A}_t}$$

$$U_{\text{eff}}(\tilde{\psi},\tilde{\phi},\tilde{A}_t) \coloneqq -\frac{\lambda}{4}\left(\tilde{\phi}^2-\eta^2\right)^2 - \mu\tilde{\psi}^2\tilde{\phi}^2 + \left(e\tilde{A}_t-\Omega\right)^2\tilde{\psi}^2 + e^2\tilde{A}_t^2\tilde{\phi}^2$$

Regarding the radius r as "time", amplitude $(\tilde{\psi},\tilde{\phi},\tilde{A}_t)$ as "position of particle", the system can be understood as Newtonian mechanics.

$$S_{\text{eff}} = \int r^2 dr\left(\left(\frac{d\tilde{\psi}}{dr}\right)^2 + \left(\frac{d\tilde{\phi}}{dr}\right)^2 - \frac{1}{2}\left(\frac{d\tilde{A}_t}{dr}\right)^2 - U_{eff}(\tilde{\psi},\tilde{\phi},\tilde{A}_t)\right)$$

Vacuum:
$U_{\text{eff}}(0,\eta,0) = 0$

Analogy with Newtonian mechanics

$$\frac{d^2\tilde{\psi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\psi}}{dr} - \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\psi}} \quad \frac{d^2\tilde{\phi}}{dr^2} = -\frac{2}{r}\frac{d\tilde{\phi}}{dr} - \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\phi}} \quad \frac{d^2\tilde{A}_t}{dr^2} = -\frac{2}{r}\frac{d\tilde{A}_t}{dr} + \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{A}_t}$$

$$U_{\mathrm{eff}}(\tilde{\psi},\tilde{\phi},\tilde{A}_t) := -\frac{\lambda}{4}\left(\tilde{\phi}^2-\eta^2\right)^2 - \mu\tilde{\psi}^2\tilde{\phi}^2 + \left(e\tilde{A}_t-\Omega\right)^2\tilde{\psi}^2 + e^2\tilde{A}_t^2\tilde{\phi}^2$$

If the radius r is enough large, then the friction terms are almost negligible.

Using analogy of the Newton mechanics, soliton solutions can be interpreted as trajectories that connect two stationary points of U_{eff}.

$$\frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\psi}} = 0, \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{\phi}} = 0, \frac{\partial U_{\mathrm{eff}}}{\partial\tilde{A}_t} = 0 \quad \text{and} \quad \left(\tilde{\psi},\tilde{\phi},\tilde{A}_t\right) = (0,\eta,0) : \text{True vacuum}$$

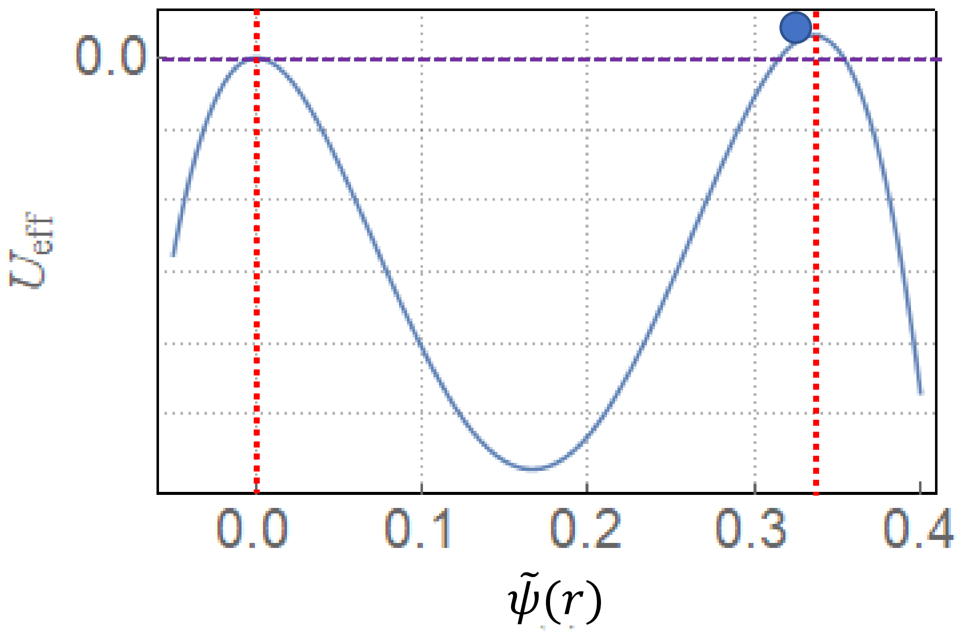
Then, effective energy

$$E_{\mathrm{eff}} := \left(\frac{d\tilde{\psi}}{dr}\right)^2 + \left(\frac{d\tilde{\phi}}{dr}\right)^2 - \frac{1}{2}\left(\frac{d\tilde{A}_t}{dr}\right)^2 + U_{\mathrm{eff}}(\tilde{\psi},\tilde{\phi},\tilde{A}_t)$$

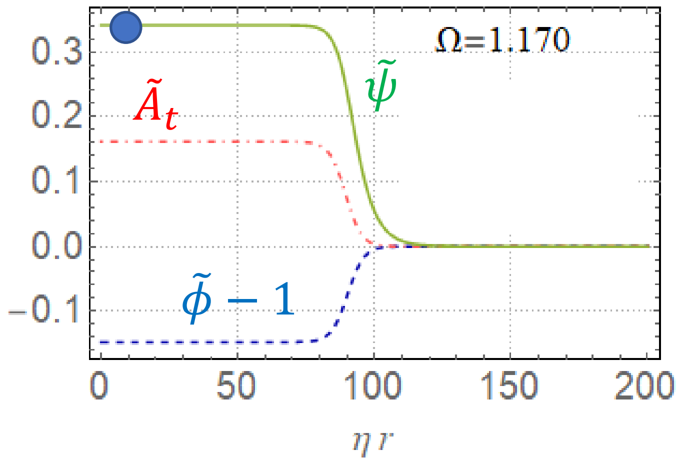
is conserved during the “particle” motion.

Correspondence to the field configurations

Impression of U_{eff}.



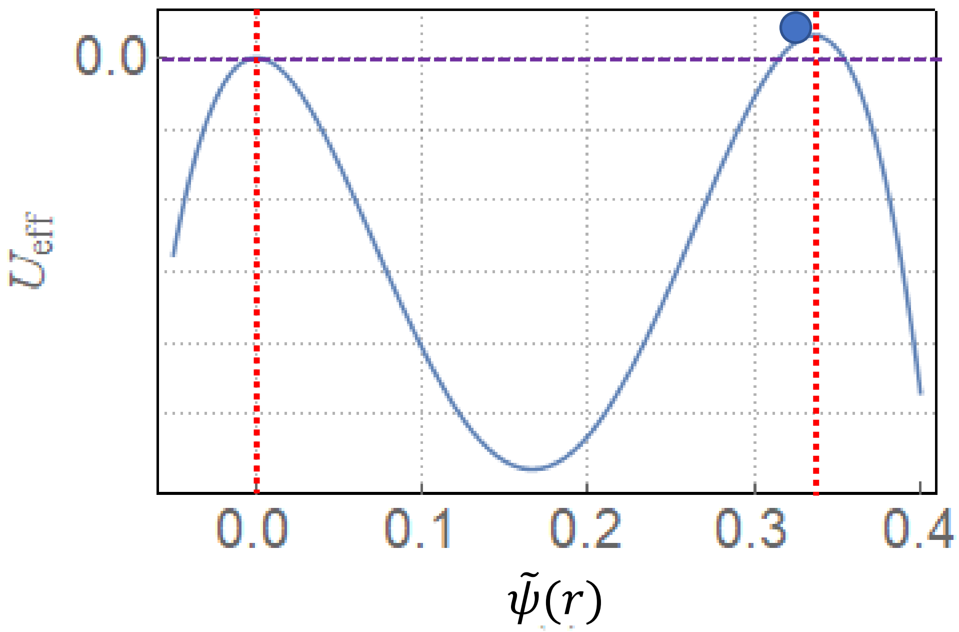
Corresponding field configurations



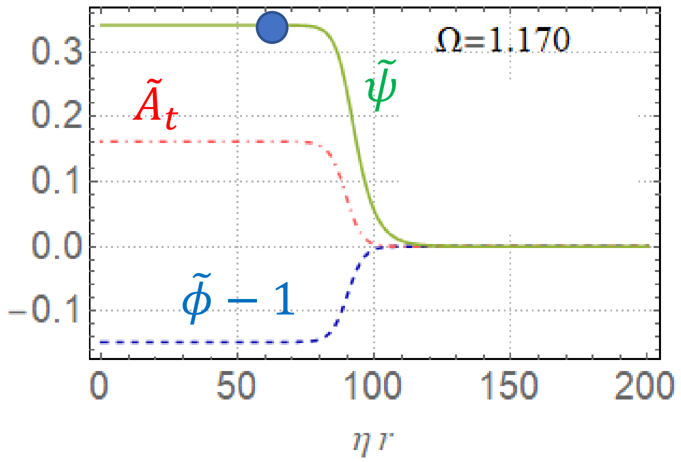
1. Put a “particle” around a stationary point with “zero initial velocity” $\tilde{\psi}' = 0, \tilde{\phi}' = 0, \tilde{A}_t' = 0$

Correspondence to the field configurations

Impression of U_{eff}.



Corresponding field configurations

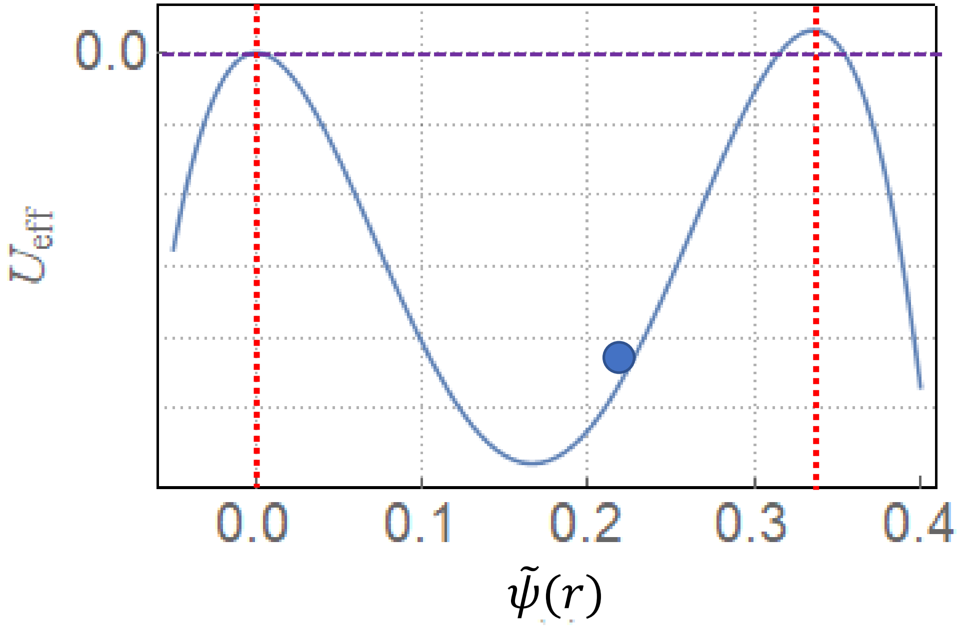


1. Put a "particle" around a stationary point with "zero initial velocity" $\tilde{\psi}' = 0, \tilde{\phi}' = 0, \tilde{A}_t' = 0$

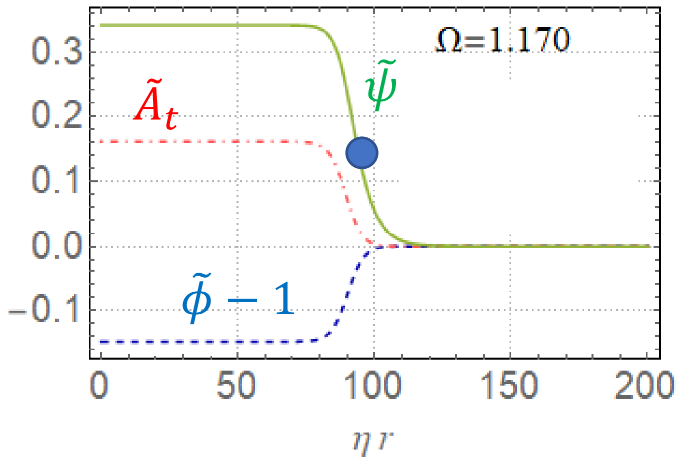
2. The "particle" stays a long "time" around the stationary point.

Correspondence to the field configurations

Impression of U_{eff}.



Corresponding field configurations



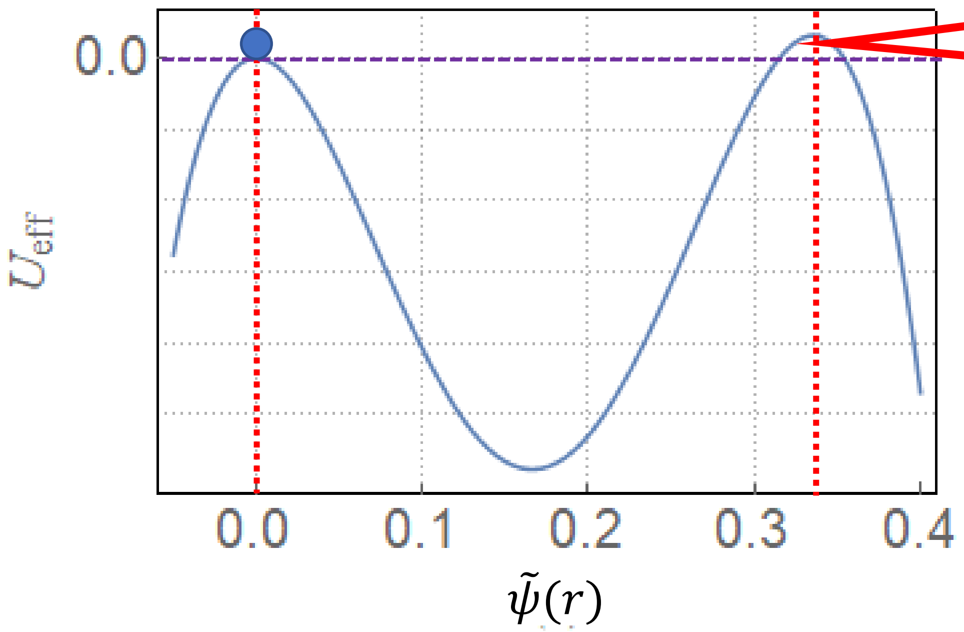
1. Put a “particle” around a stationary point with “zero initial velocity” $\tilde{\psi}' = 0, \tilde{\phi}' = 0, \tilde{A}_t' = 0$

2. The “particle” stays a long “time” around the stationary point.

3. The “particle” roll downs on the U_{eff}.

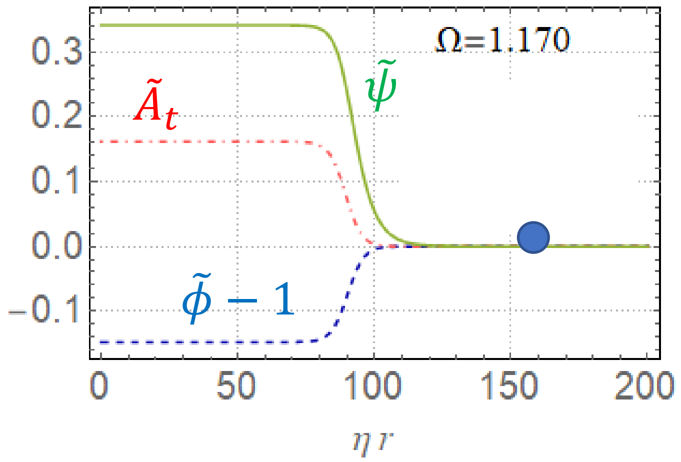
Correspondence to the field configurations

Impression of U_{eff}.



Potential difference is consumed by the friction term.

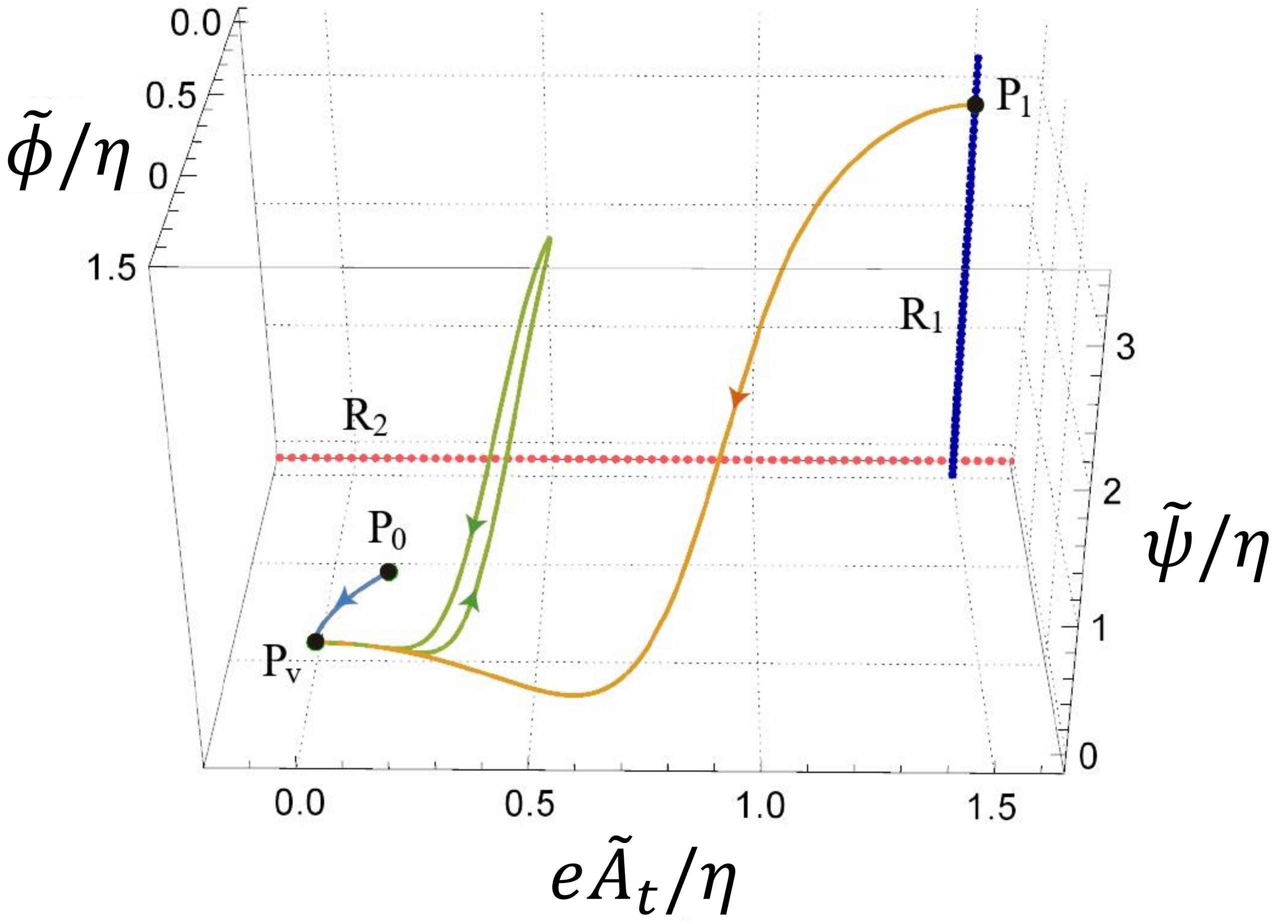
Corresponding field configurations



1. Put a “particle” around a stationary point with “zero initial velocity” $\tilde{\psi}' = 0, \tilde{\phi}' = 0, \tilde{A}_t' = 0$
2. The “particle” stays a long “time” around the stationary point.
3. The “particle” roll downs on the U_{eff}.
4. Finally, the “particle” stays on the vacuum stationary point.

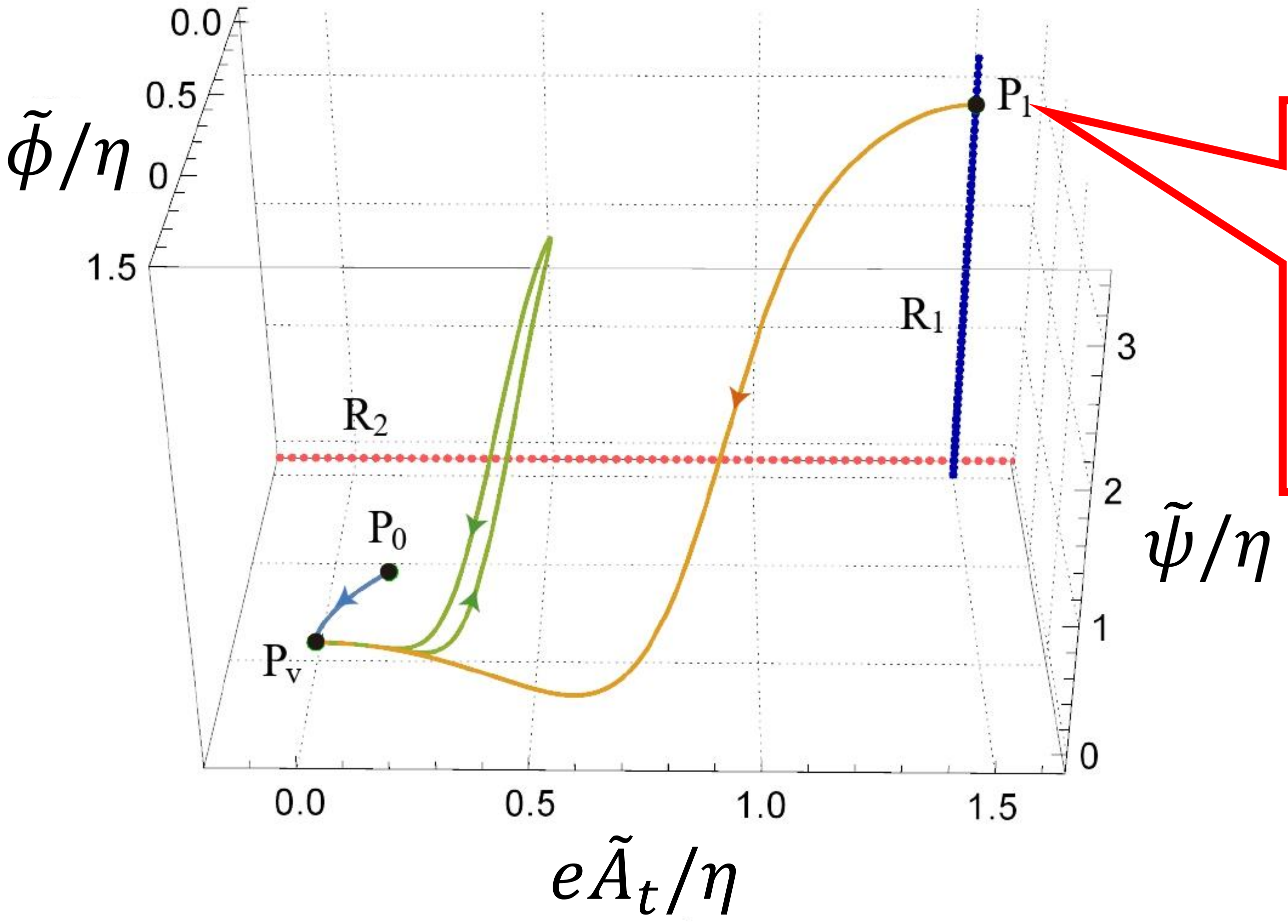
Various stationary points and trajectories

In the U(1) Gauge Higgs model, several stationary points exist.



Various stationary points and trajectories

In the U(1) Gauge Higgs model, several stationary points exist.



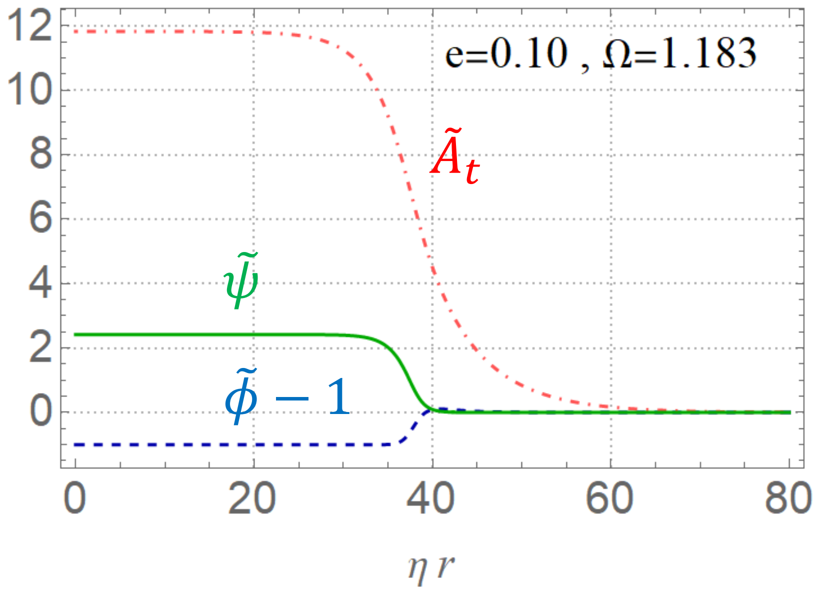
We focus on a solution which start from the stationary point P_1.

$\tilde{\psi} = \tilde{\psi}_0 = \text{const.}\, \tilde{\phi} = 0, e\tilde{A}_t = \Omega$

Solution start from the stationary point P_1

$$\tilde{\psi} = \tilde{\psi}_0 = \text{const.}\, \tilde{\phi} = 0, \tilde{A}_t = \frac{\Omega}{e}$$

$$\epsilon = \frac{\lambda}{4}, p_r = p_\theta = -\frac{\lambda}{4}$$



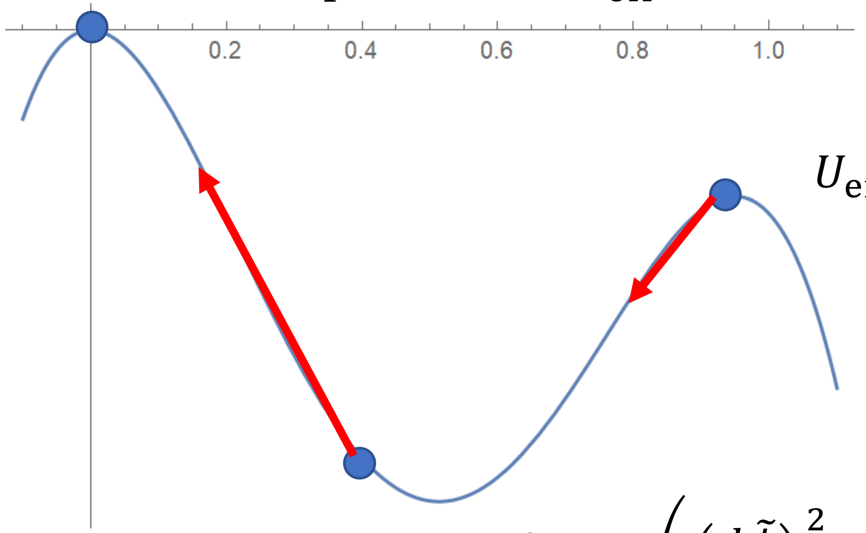
Solution is filled with vacuum energy!

Solution start from the stationary point P_1

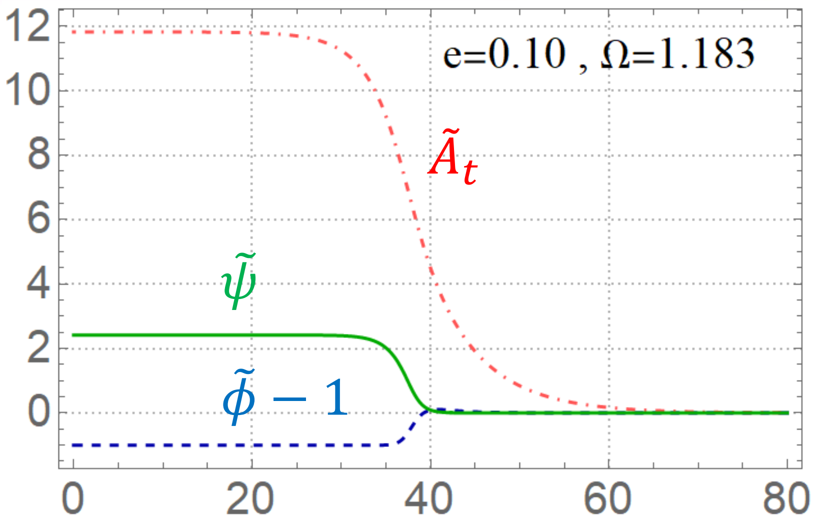
Value of $U_{\mathrm{eff}}(P_1)$ is lower than $U_{\mathrm{eff}}(P_v)$.

$U_{\mathrm{eff}}(P_v) = 0$

Impression of U_{eff}.



$U_{\mathrm{eff}}(P_1) = -\frac{\lambda}{4}$



The "particle" must climb up on the U_{eff} with "zero initial velocity".

$$S_{\mathrm{eff}} = \int r^2 dr \left(\left(\frac{d\tilde{\psi}}{dr}\right)^2 + \left(\frac{d\tilde{\phi}}{dr}\right)^2 - \frac{1}{2}\left(\frac{d\tilde{A}_t}{dr}\right)^2 - U_{eff}(\tilde{\psi}, \tilde{\phi}, \tilde{A}_t) \right)$$

The EOM of $\tilde{A}_t$ has an anti-friction term.

Owing to the gauge field $\tilde{A}_t$, the "particle" climb up on the U_{eff}!

As you seen, using the model

$$\mathcal{L} = -\left(D_\mu\psi\right)^*(D^\mu\psi) - \left(D_\mu\phi\right)^*(D^\mu\phi) - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 - \mu|\psi|^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

spherically symmetric localized solutions (Q-balls) filled with the vacuum energy are possible.

⇓

Coupling gravitational field with the model,
boson stars filled with vacuum energy are also possible!

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"Solitonic gravastars"

Contents

- Introduction
- U(1) Gauge Higgs Model
- Solitonic gravastar solutions
- Shadow of BH vs Gravastar
- Conclusion

Setup

U(1) gauge Higgs model + Einstein gravity

$$S = \int \sqrt{-g} d^4x \left\{ \frac{R}{16\pi G} - g^{\mu\nu} (D_\mu \psi)^* (D_\nu \psi) - g^{\mu\nu} (D_\mu \phi)^* (D_\nu \phi) - \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2 - \mu |\psi|^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$

Static and spherically symmetric spacetime

$$ds^2 = -\sigma(r)^2 \left(1 - \frac{2m(r)}{r}\right) dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

Stationary and spherically symmetric fields (same as Q-balls)

$$\psi(t,r) = e^{i\Omega t} \tilde{\psi}(r) \,, \phi(t,r) = \tilde{\phi}(r), \widetilde{A_t}(r) := A_t + e^{-1}\omega'$$

Boundary conditions

$\cdot\, r \to 0$: Regularity

$$\frac{d\tilde{\psi}}{dr} = 0, \frac{d\tilde{\phi}}{dr} = 0, \frac{d\tilde{A}}{dr} = 0, m = 0, \frac{d\sigma}{dr} = 0$$

$\cdot\, r \to \infty : T_t^t \to 0$

$$\tilde{\psi} = 0, \tilde{\phi} = \eta, \tilde{A}_t = 0, m = m_\infty = \text{const.}\, \sigma = 1$$

Fields equations

All dimension variables are normalized by η.

Klein-Gordon equations for ψ and ϕ

$$\tilde{\psi}'' + \left\{\frac{2}{r}\left(1+\frac{m-rm'}{r-2m}\right)+\frac{\sigma'}{\sigma}\right\}\tilde{\psi}' + \left(1-\frac{2m}{r}\right)\left[\frac{(e\tilde{A}_t-\Omega)^2\tilde{\psi}}{\sigma^2(1-2m/r)} - \mu\tilde{\phi}^2\tilde{\psi}\right] = 0,$$

$$\tilde{\phi}'' + \left\{\frac{2}{r}\left(1+\frac{m-rm'}{r-2m}\right)+\frac{\sigma'}{\sigma}\right\}\tilde{\phi}' + \left(1-\frac{2m}{r}\right)\left[\frac{e^2\tilde{\phi}\tilde{A}_t^2}{\sigma^2(1-2m/r)} - \frac{\lambda}{2}\tilde{\phi}(\tilde{\phi}^2-1) - \mu\tilde{\phi}\tilde{\psi}^2\right] = 0,$$

Maxwell equation

$$\tilde{A}_t'' + \left(\frac{2}{r}-\frac{\sigma'}{\sigma}\right)\tilde{A}_t' + \left(1-\frac{2m}{r}\right)\left[-2e^2\tilde{\phi}^2\tilde{A}_t - 2e^2\tilde{\psi}^2\tilde{A}_t + 2e\Omega\tilde{\psi}^2\right] = 0,$$

$$\left(\begin{aligned}\tilde{A}_t(r) &\coloneqq A_t(r) + \frac{\omega'}{e}\\ \Omega &\coloneqq \omega' - \omega\end{aligned}\right)$$

Einstein equations ($G_t^t = 8\pi G T_t^t$, $G_r^r - G_t^t = 8\pi G(T_r^r - T_t^t)$)

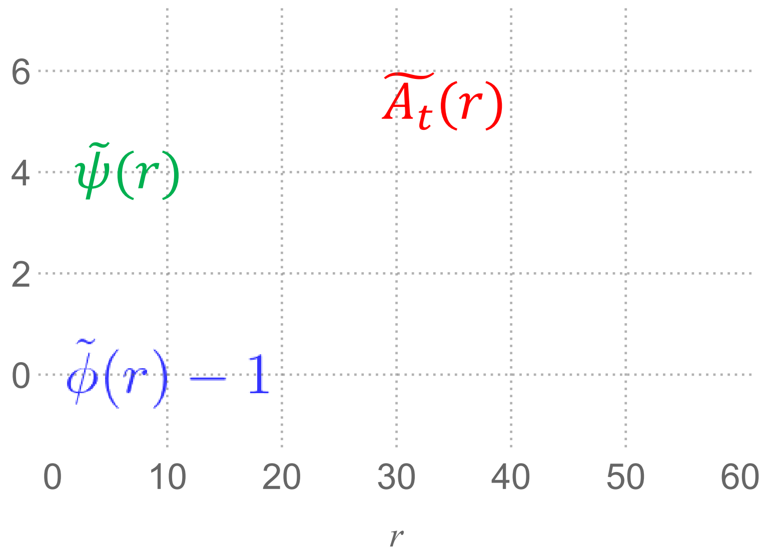
$$\frac{2m'}{r^2} - 8\pi G\eta^2\left[\frac{e^2\tilde{\phi}^2\tilde{A}_t^2}{\sigma^2(1-2m/r)} + \frac{\left(e\tilde{A}_t-\Omega\right)^2\tilde{\psi}^2}{\sigma^2(1-2m/r)} + \left(1-\frac{2m}{r}\right)\{(\tilde{\phi}')^2 + (\tilde{\psi}')^2\} + \frac{\lambda}{4}(\tilde{\phi}^2-1)^2 + \mu\tilde{\phi}^2\tilde{\psi}^2 + \frac{(\tilde{A}_t')^2}{2\sigma^2}\right]$$

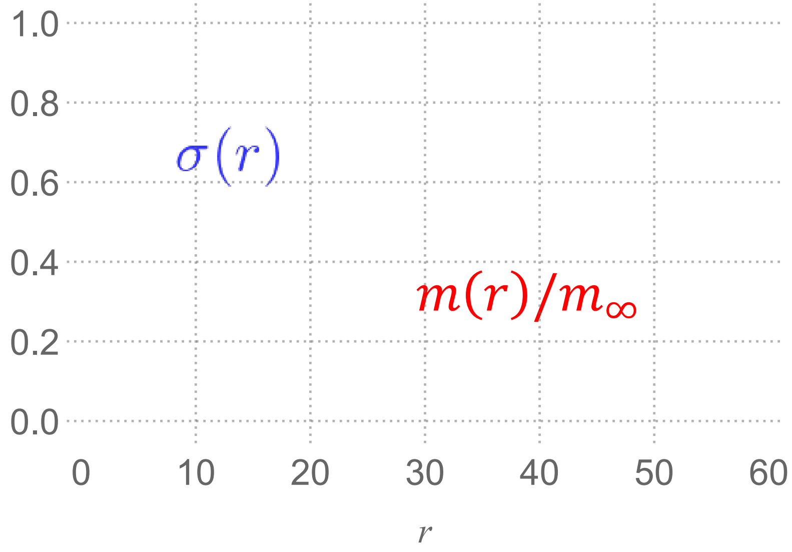
$$\frac{(r-2m)\sigma'}{r^2\sigma} - 8\pi G\eta^2\left[\frac{e^2\tilde{\phi}^2\tilde{A}_t^2}{\sigma^2(1-2m/r)} + \frac{(e\tilde{A}_t-\Omega)^2\tilde{\psi}^2}{\sigma^2(1-2m/r)} + \left(1-\frac{2m}{r}\right)\left\{(\tilde{\phi}')^2+(\tilde{\psi}')^2\right\}\right] = 0$$

$G\eta^2 (= \eta^2/M_{Pl}^2) \to 0$: Return to the model of Q-ball

Boson star solution

$$\Omega/\eta = 0.685$$
$$e = 0.1$$
$$\lambda = 1.0$$
$$\mu = 1.4$$
$$\sqrt{G}\eta = 10^{-2}$$





- All fields are localized.
  - Boson star solution

- Inside the star, $\tilde{\psi} = \tilde{\psi}_0 = \text{const.}, \tilde{\phi} = 0, \widetilde{A_t} = \frac{\Omega}{e}$, and $\sigma = \sigma_0 = \text{const.}$.

Boson star solution

$$\Omega/\eta = 0.685$$
$$e = 0.1$$
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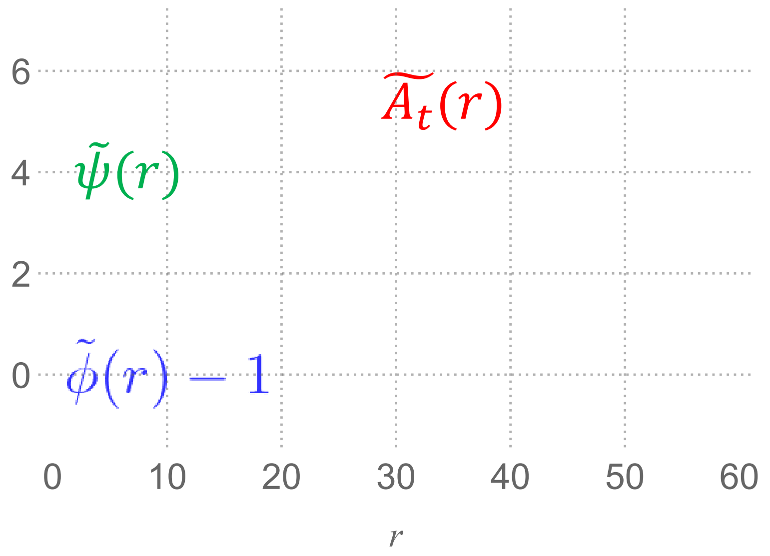
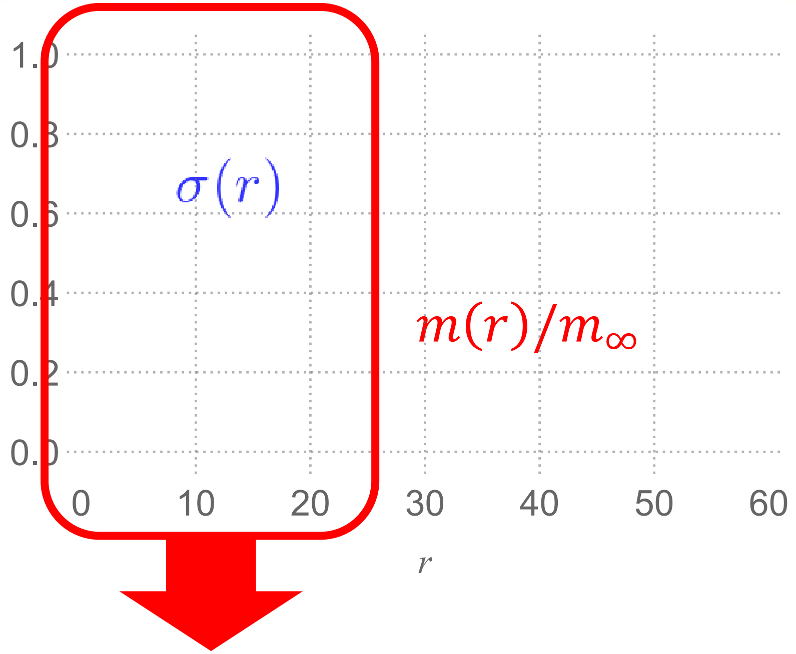
$$\mu = 1.4$$
$$\sqrt{G}\eta = 10^{-2}$$



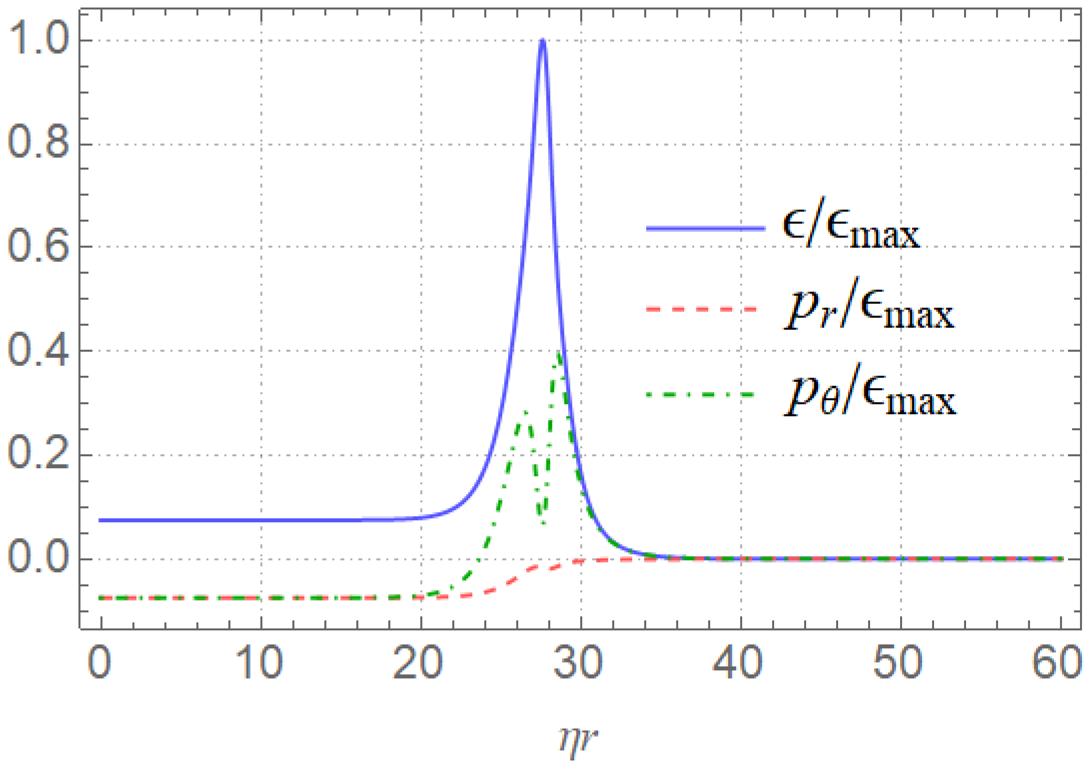


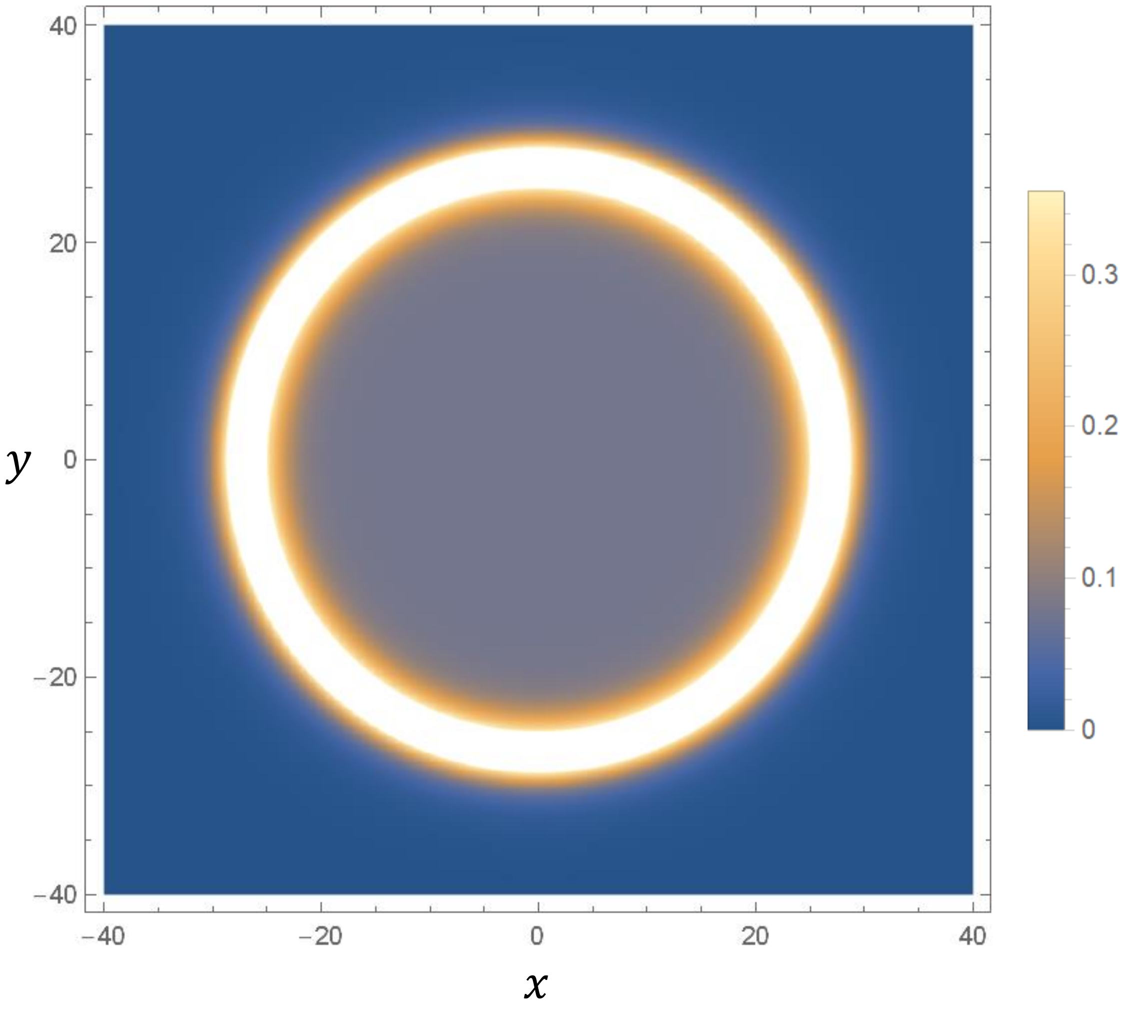
- Inside the boson star is described by de Sitter metric.

$$ds^2 = -\left(1 - \frac{r^2}{l^2}\right) d\tilde{t}^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad \tilde{t} := \sigma_0 t$$

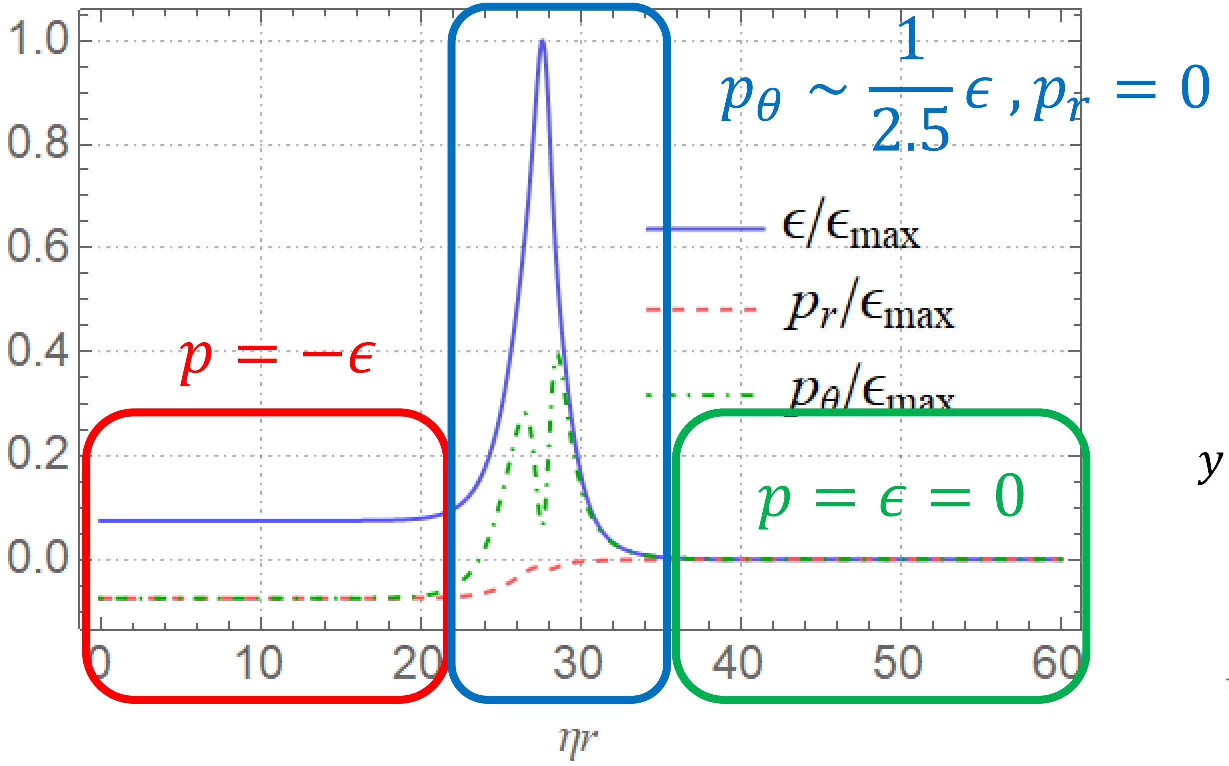
$$l \sim \sqrt{\frac{3}{\Lambda}}, \Lambda = \frac{8\pi G \eta^2 \lambda}{4} \Rightarrow l \sim \left(\frac{3}{2\pi G \eta^2 \lambda}\right)^{\frac{1}{2}} = 10^2 \left(\frac{3}{2\pi\lambda}\right)^{\frac{1}{2}} \sim 70$$

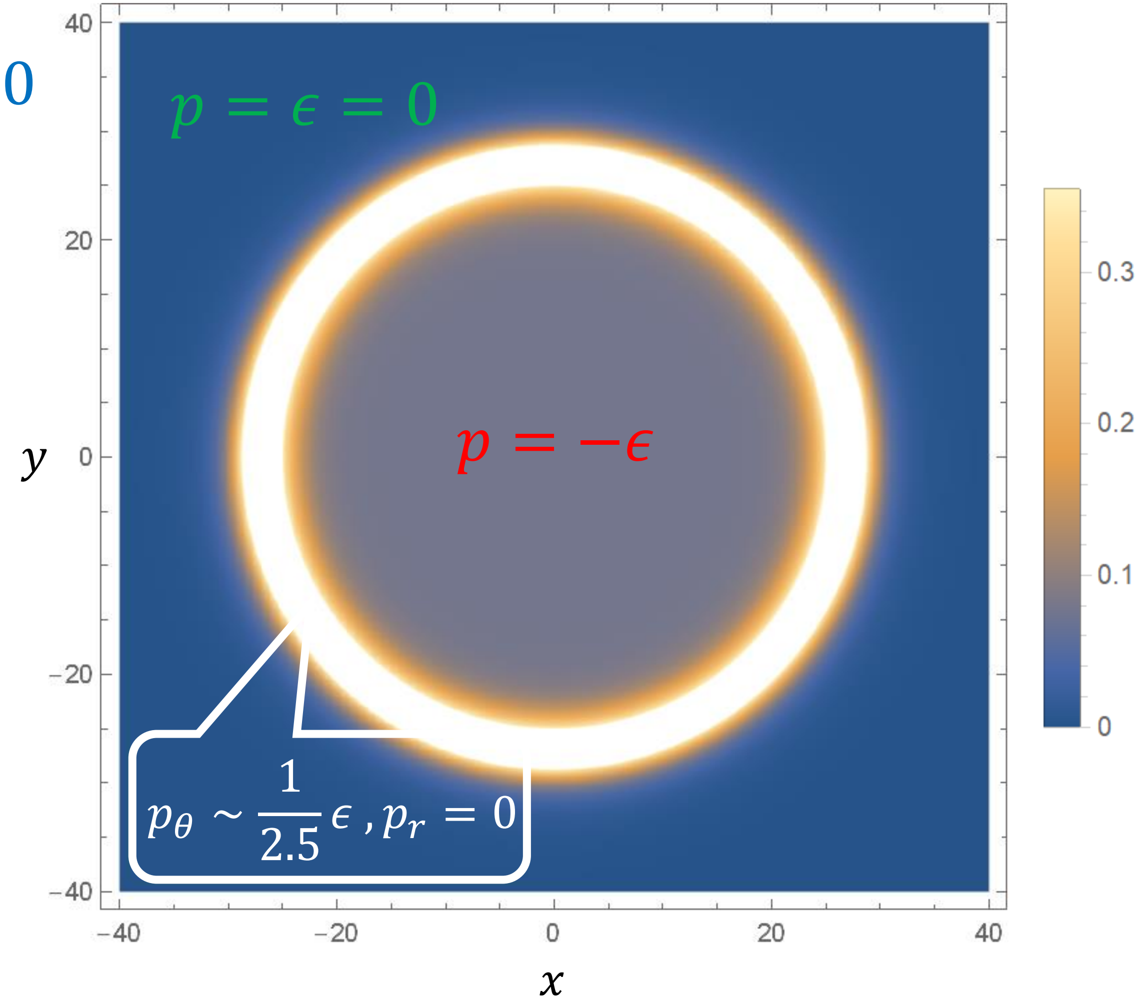
Energy and pressure





Energy and pressure





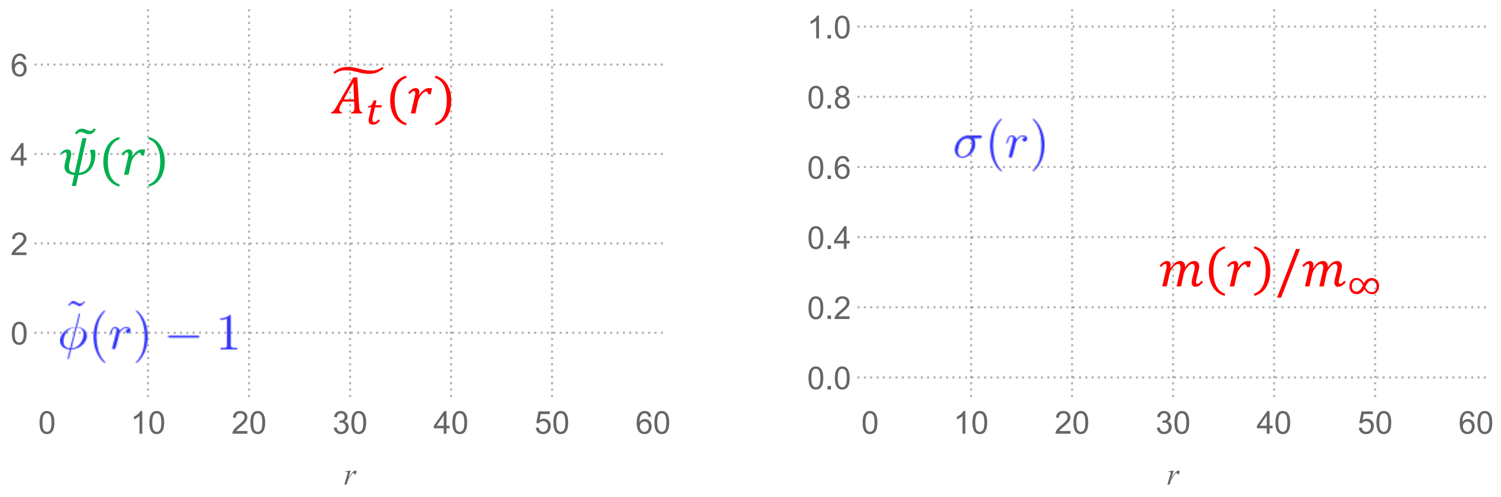
Interior filled with vacuum energy

and

exterior with true vacuum

are connected by the shell with finite width.

Solitonic gravastar



- Interior is described by de Sitter metric by vacuum energy.
- Exterior is described by Schwarzschild metric .
- These two regions are connected by the shell with finite width.

Solitonic gravastars.

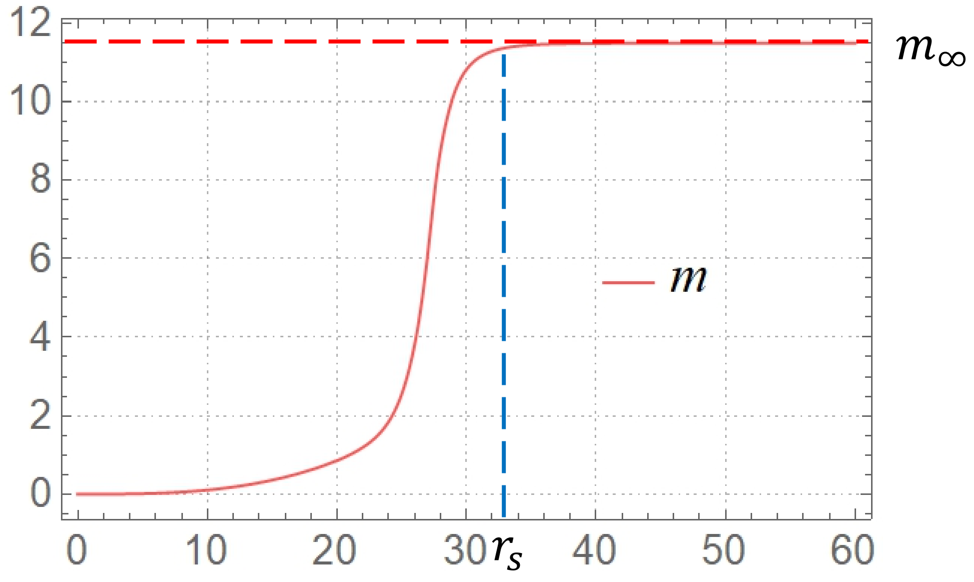
Various quantities

- Mass M_G $M_G \sim 1150 M_{Pl}$

- Radius r_s $m(r_s) \coloneqq 0.99 m_\infty$

$$R \coloneqq r_s/(\sqrt{G}\eta) \sim 3300 R_{Pl}$$

99% of M_G
is included within R.



- Stability

$$\frac{E_{BS}}{E_{free}} = 0.4141 < 1$$

The solitonic gravastar is energetically stable!

- Compactness C

$$C \coloneqq \frac{2m_\infty}{r_s} = \frac{2M_G/M_{pl}}{R/R_{Pl}}$$

$C \geq \frac{2}{3} \Rightarrow$ Photon Sphere

$$C = \frac{2300}{3300} \sim 0.7 > \frac{2}{3}$$

Photon sphere exist!

Contents

- Introduction
- U(1) Gauge Higgs Model
- Solitonic gravastar solutions
- Shadow of BH vs Gravastar
- Conclusion

Geodesic equation

Static and spherically symmetric spacetime

$$ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \ \Rightarrow \text{Killing vectors } \xi_t, \xi_\varphi$$

$$E := (\xi_t)_\mu \frac{dx^\mu}{d\zeta}, \qquad L := (\xi_\varphi)_\mu \frac{dx^\mu}{d\zeta} \quad \text{are conserved.}$$

$$\epsilon := -g_{\mu\nu} \frac{dx^\mu}{d\zeta} \frac{dx^\nu}{d\zeta} \text{ is conserved along geodesics.}$$

ξ: affine parameter

Geodesic equation can be reduced to

$$\tilde{\epsilon} := \frac{\epsilon}{E^2}, \tilde{L} := \frac{L}{E}$$

$$\left(\frac{dr}{d\tilde{\zeta}}\right)^2 + \underbrace{\frac{1}{g_{rr}}\left(\tilde{\epsilon} + \frac{1}{g_{tt}} + \frac{\tilde{L}^2}{g_{\varphi\varphi}}\right)}_{:= V_{\text{eff}}(r)} = 0$$

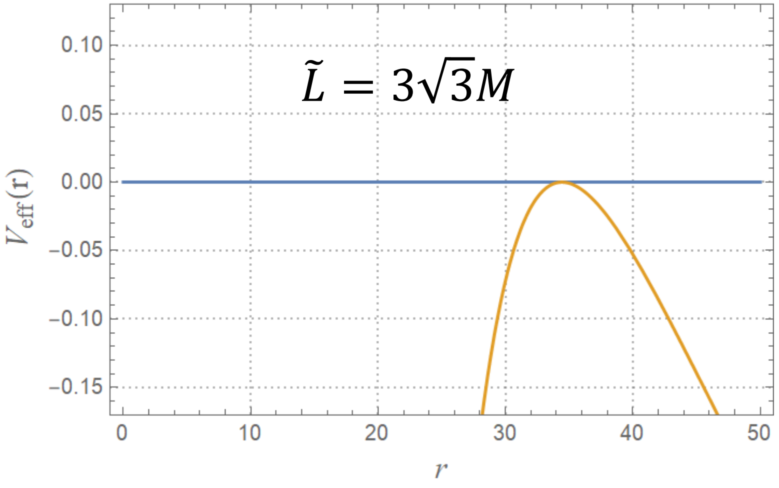
$\tilde{\epsilon} = 0$ for null geodesic.

Effective potential for solitonic gravastar

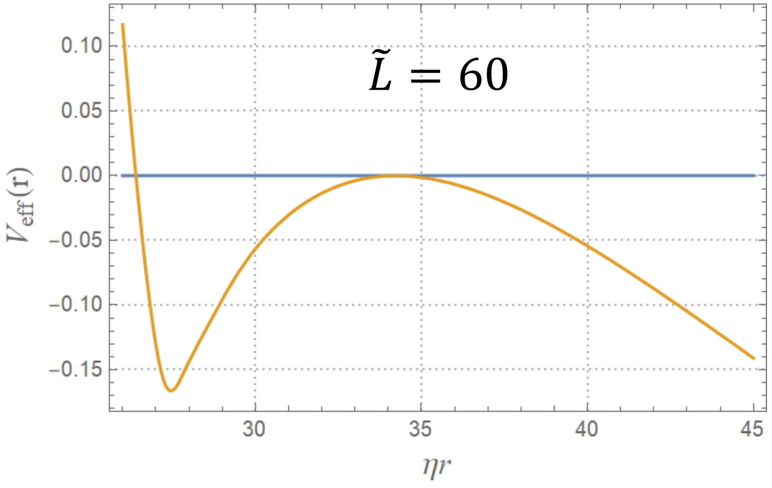
Null geodesic equation

$$\left(\frac{dr}{d\tilde{\zeta}}\right)^2 + \underbrace{\frac{1}{g_{rr}}\left(\frac{1}{g_{tt}} + \frac{\tilde{L}^2}{g_{\varphi\varphi}}\right)}_{:= V_{\text{eff}}(r)} = 0$$

- Black hole



- Solitonic Gravastar



Both BH and Solitonic Gravastar, unstable orbit exist.

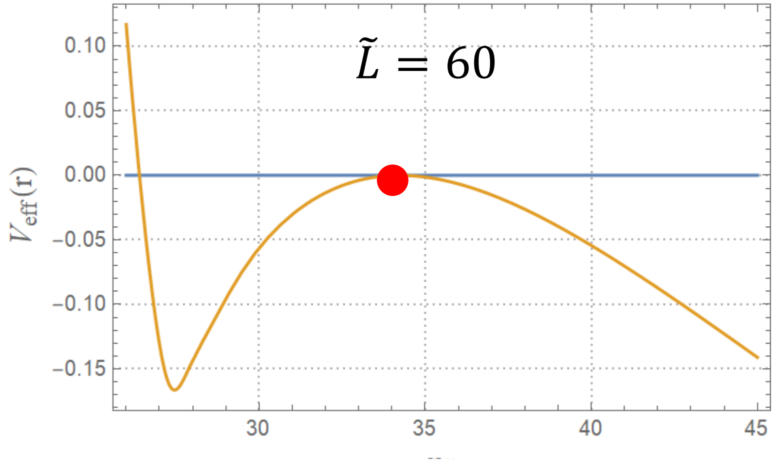
Effective potential for solitonic gravastar

Null geodesic equation

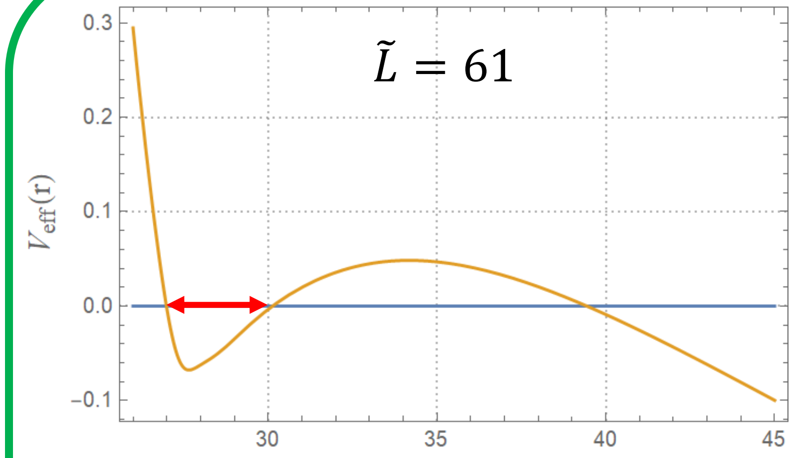
$$\left(\frac{dr}{d\tilde{\zeta}}\right)^2 + \underline{\frac{1}{g_{rr}}\left(\frac{1}{g_{tt}} + \frac{\tilde{L}^2}{g_{\varphi\varphi}}\right)} = 0$$

$$:= V_{\rm eff}(r)$$

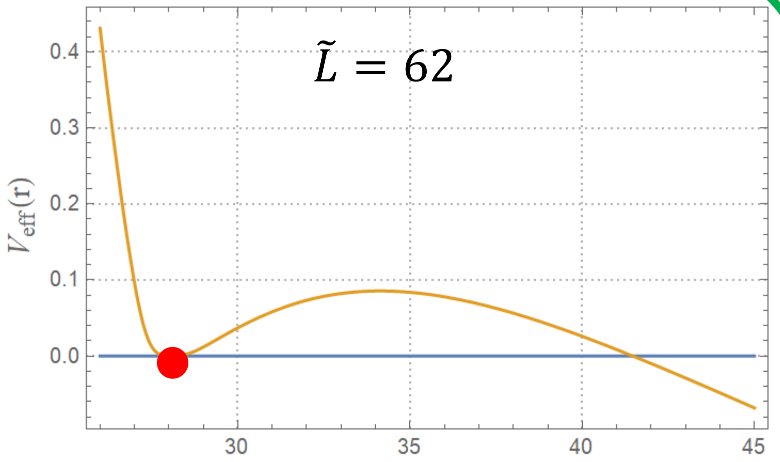
➢ Solitonic Gravastar



Unstable orbit



Bound orbit



Stable orbit

These orbits do not exist for the BH.

Contents

Conclusions

- Gravastars are proposed as black hole mimickers.
- We constructed “solitonic gravastars”.
  - de Sitter region (inside) and Schwarzschild region (outside) are connected by a shell with finite width.
- The solitonic gravastras are enough compact to have photon sphere.

Future works

- Stability and formation process by numerical simulation.
- Analysis of various phenomena.
  - Gravitational wave by collision of the solitonic gravastars.
  - Hawking radiation.

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