Gravitational waves from phase transitions during inflation

Haipeng An (Tsinghua University)

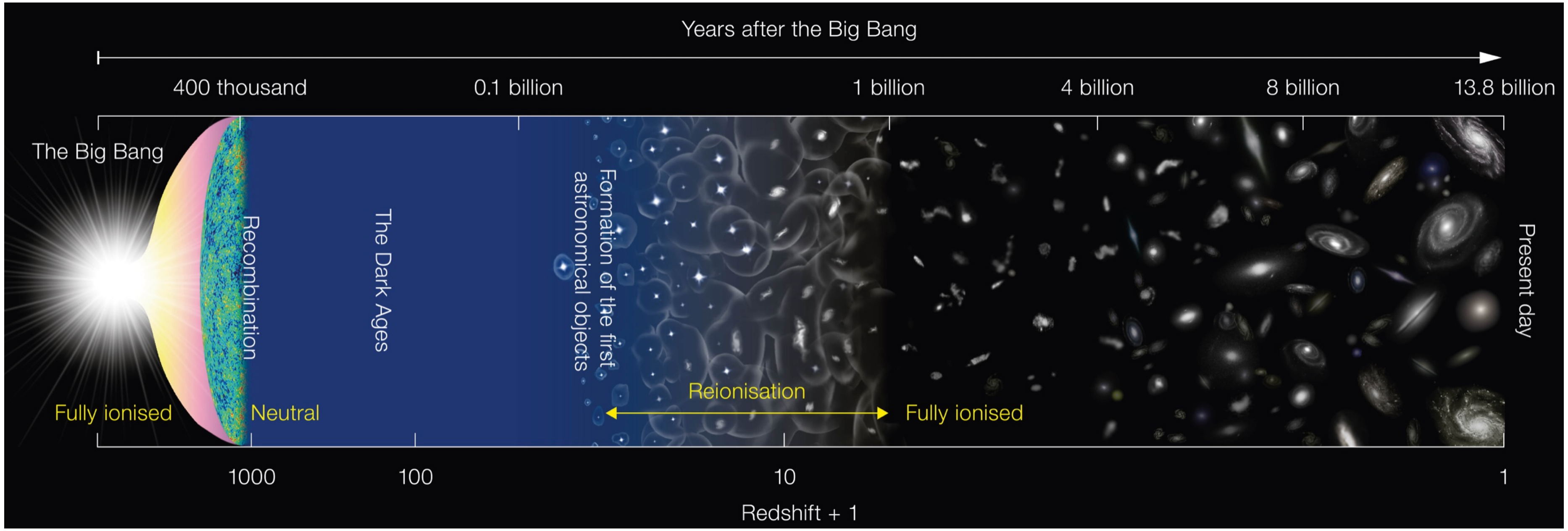
Gravitational Wave Probes of Physics Beyond Standard Model

November 6-9, 2023

NITEP, Osaka Metropolitan University, Osaka, Japan

2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou
2208.14857 w/ Xi Tong and Siyi Zhou
2304.02361 w/Chen Yang
2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

Motivations for inflation

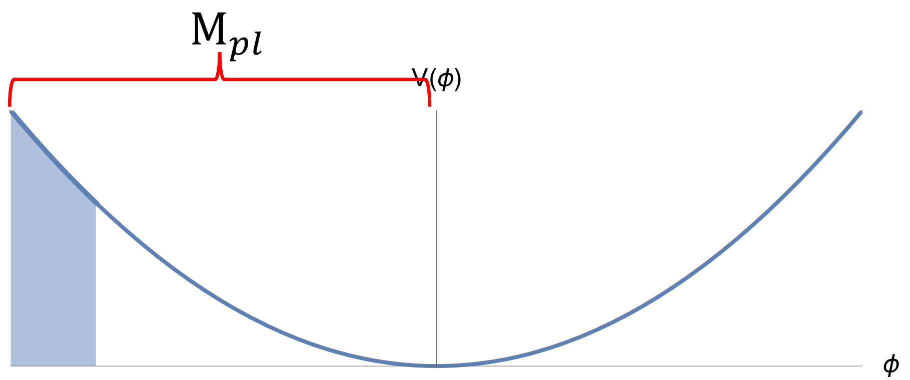


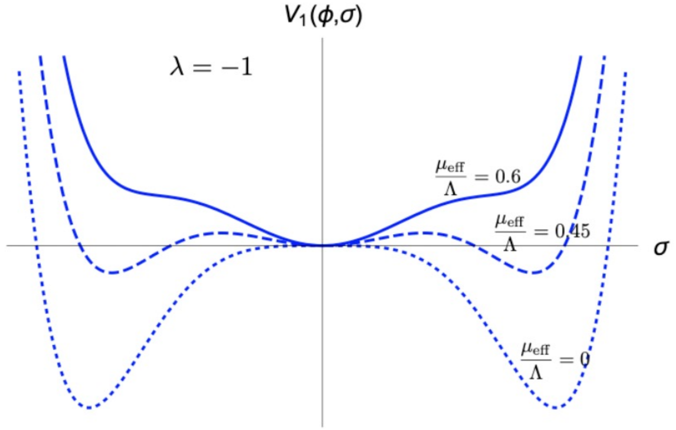
- Causality problem
- Flatness problem
- Magnetic monopole problem

Induced phase transition in spectator sectors

- ϕ: inflaton field σ: spectator field

Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

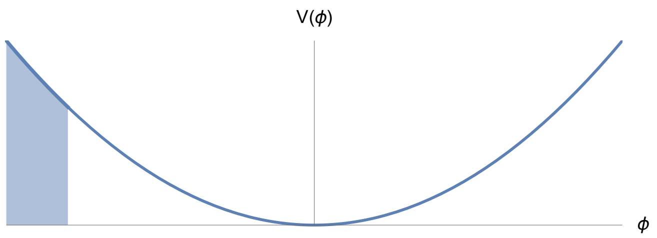


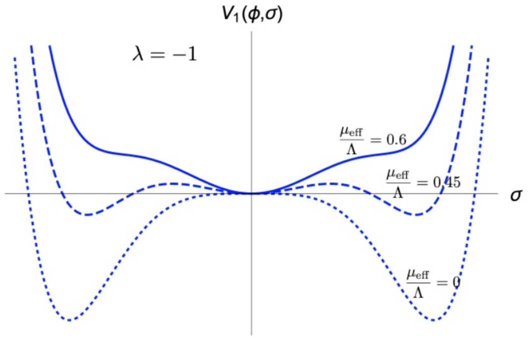


Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

Induced phase transition in spectator sectors

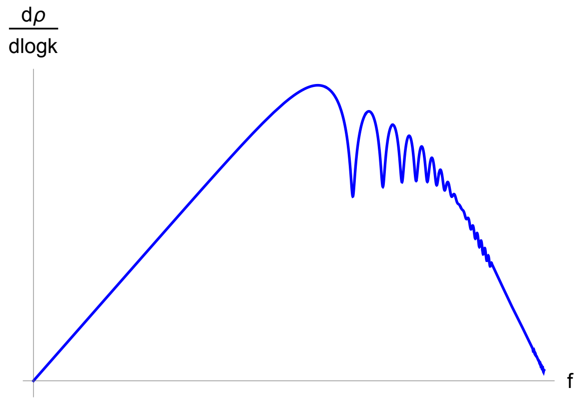
Evolution of inflaton → Phase transitions in spectator sector (First order or second order) → Gravitational waves (Primary GW) / Curvature perturbation → Gravitational waves (Secondary GW)

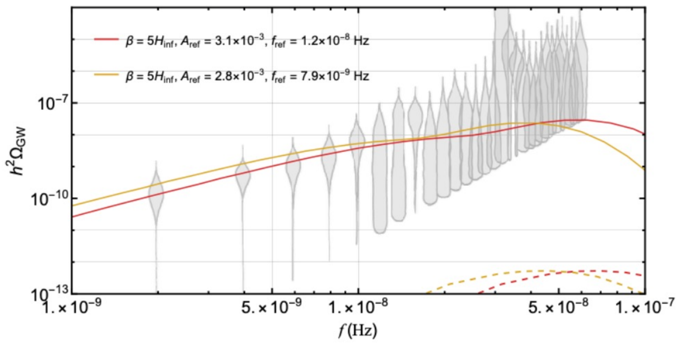


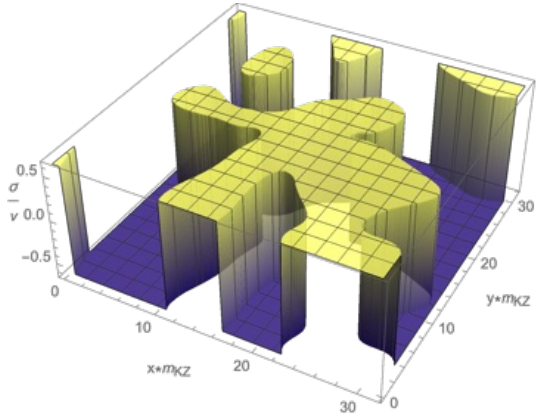


Outline

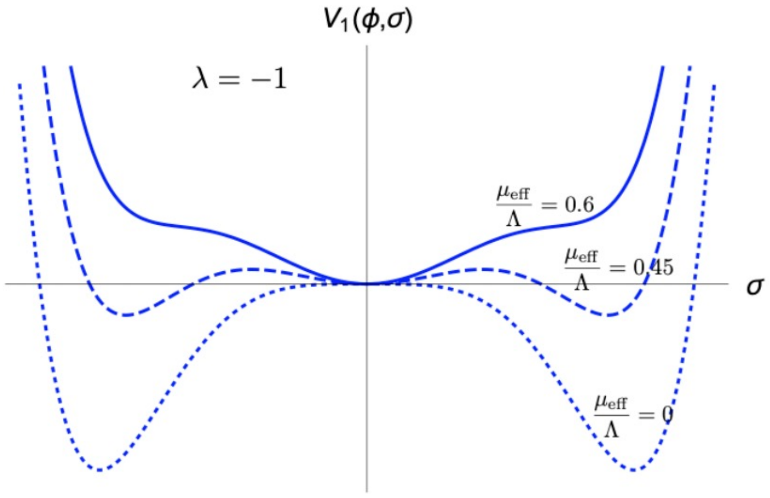
- GWs from first-order phase transitions during inflation.
  - Primary GWs
  - Curvature perturbation and secondary GWs
- GWs from second-order phase transitions (domain walls) during inflation.
- Summary and outlook



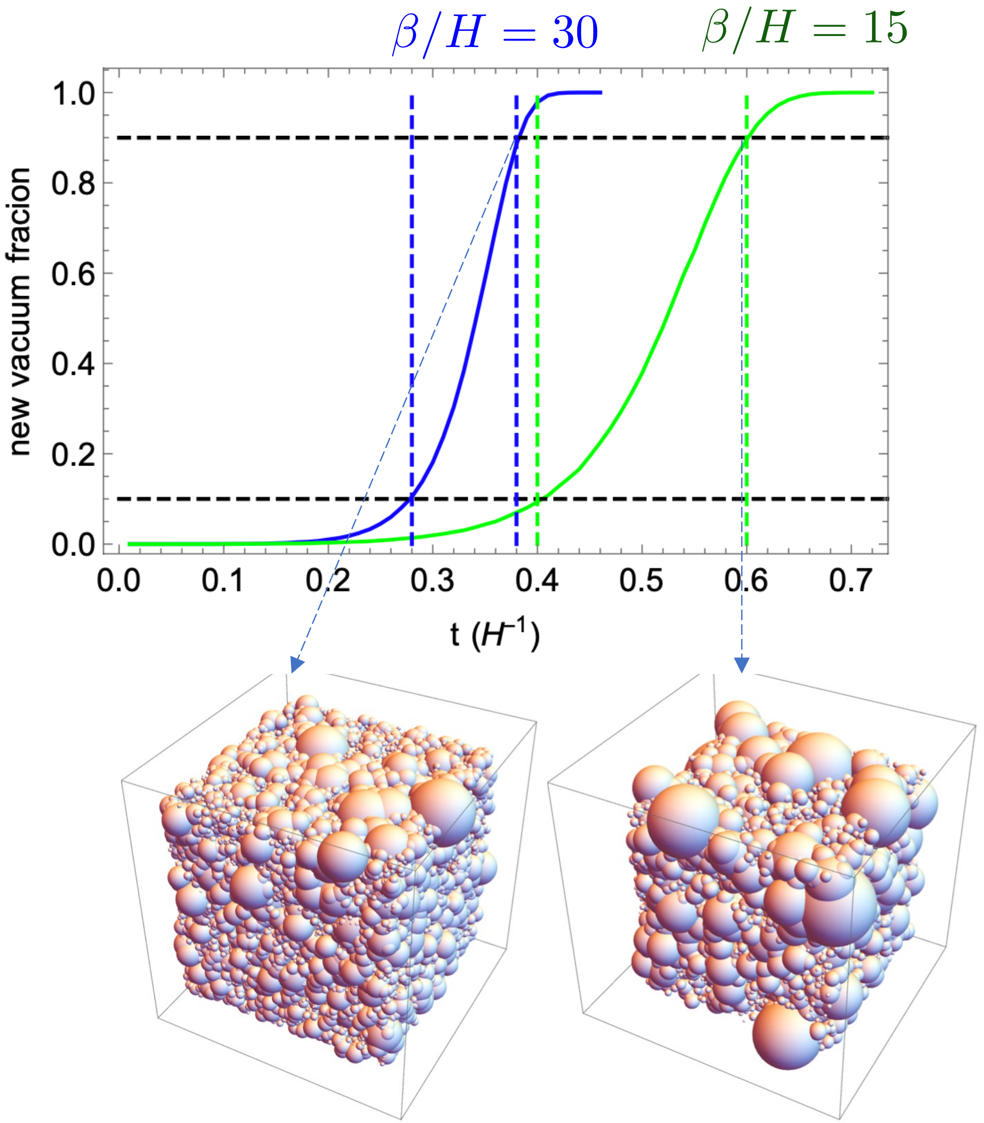




First-order phase transition during inflation



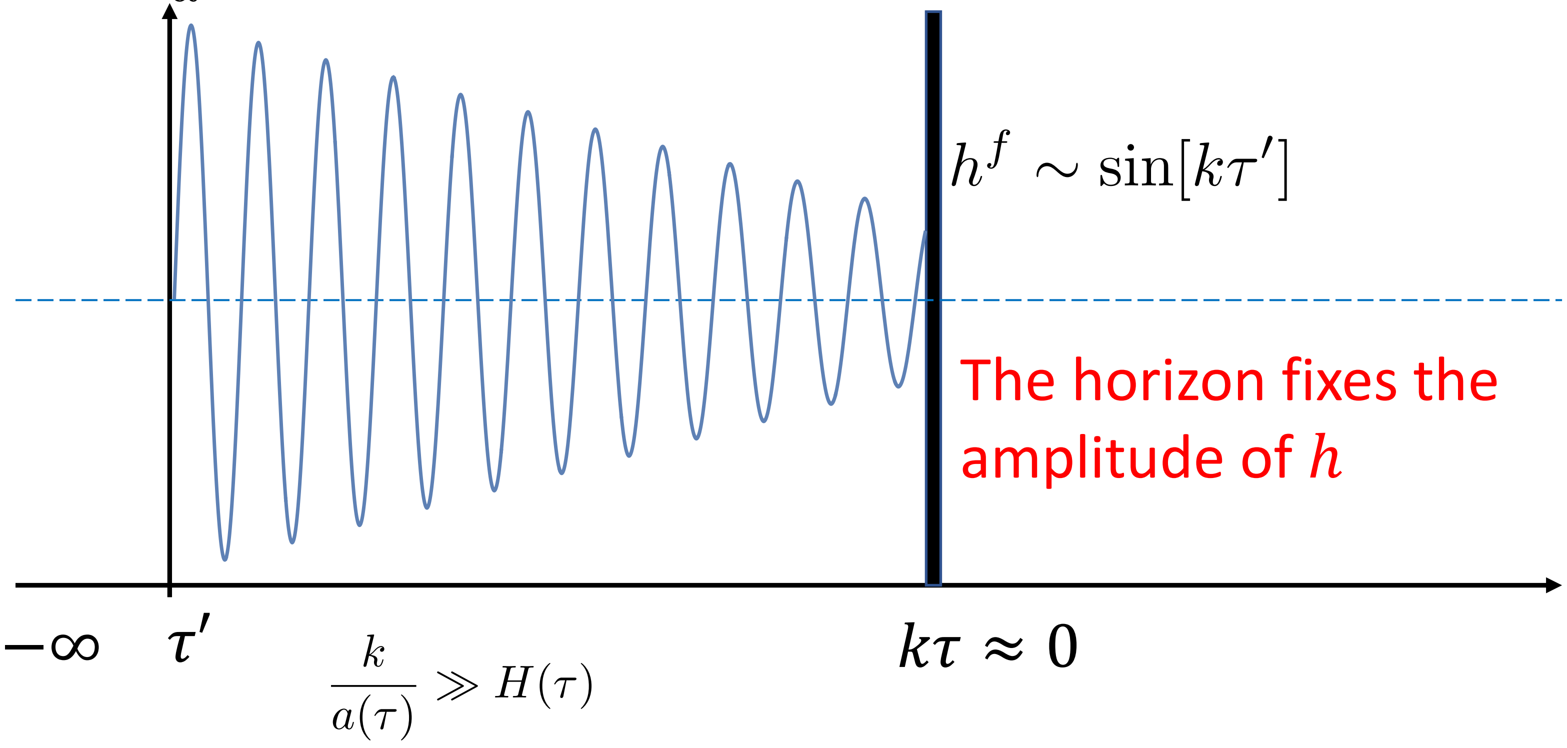
S_4 becomes smaller during



- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

GW from instantaneous sources

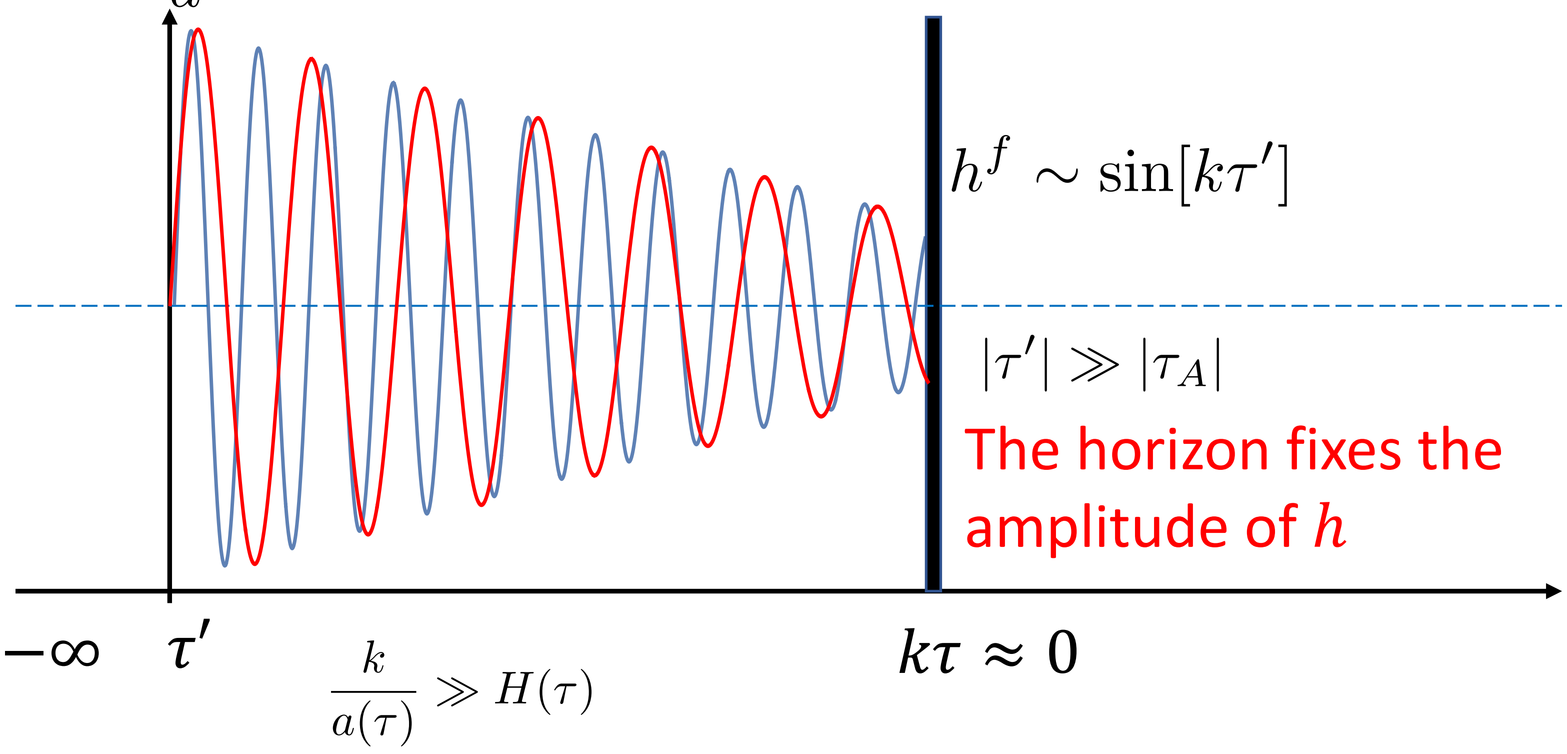
- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$



$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

GW from instantaneous sources

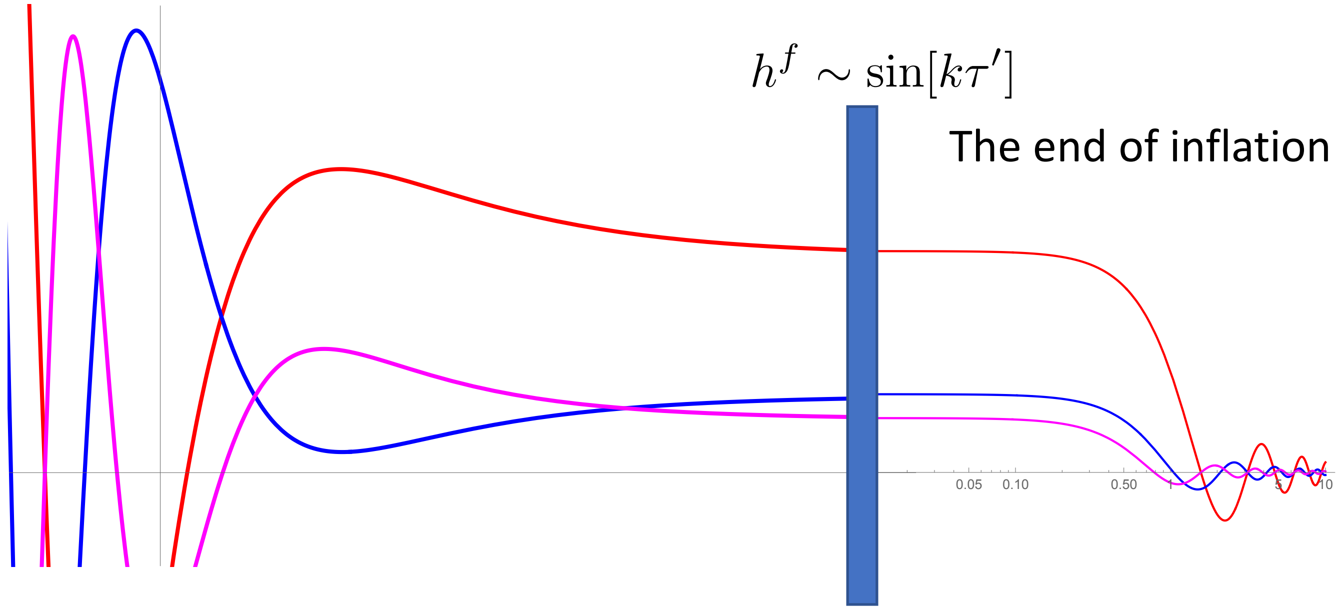
- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$



$$h \sim \frac{\sin k(\tau - \tau')}{a(\tau)}$$

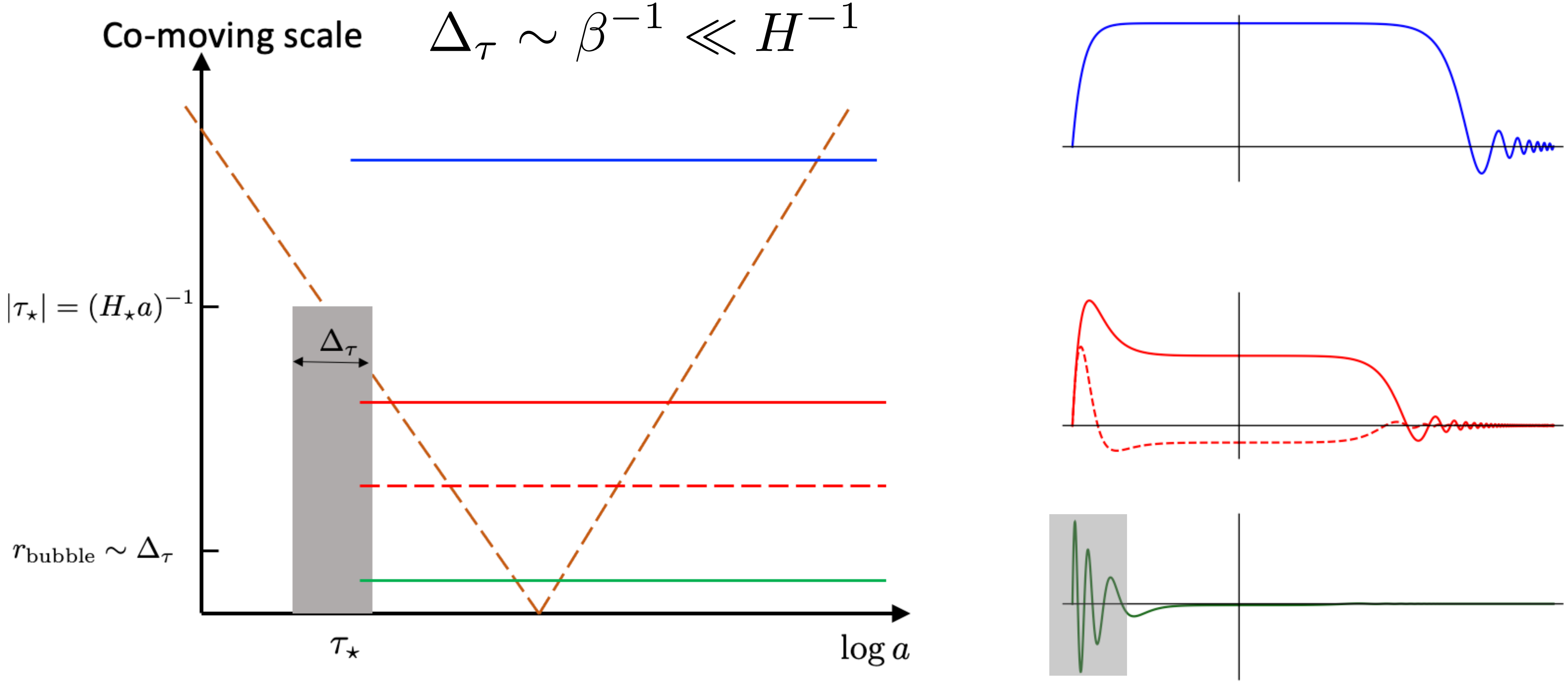
After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$

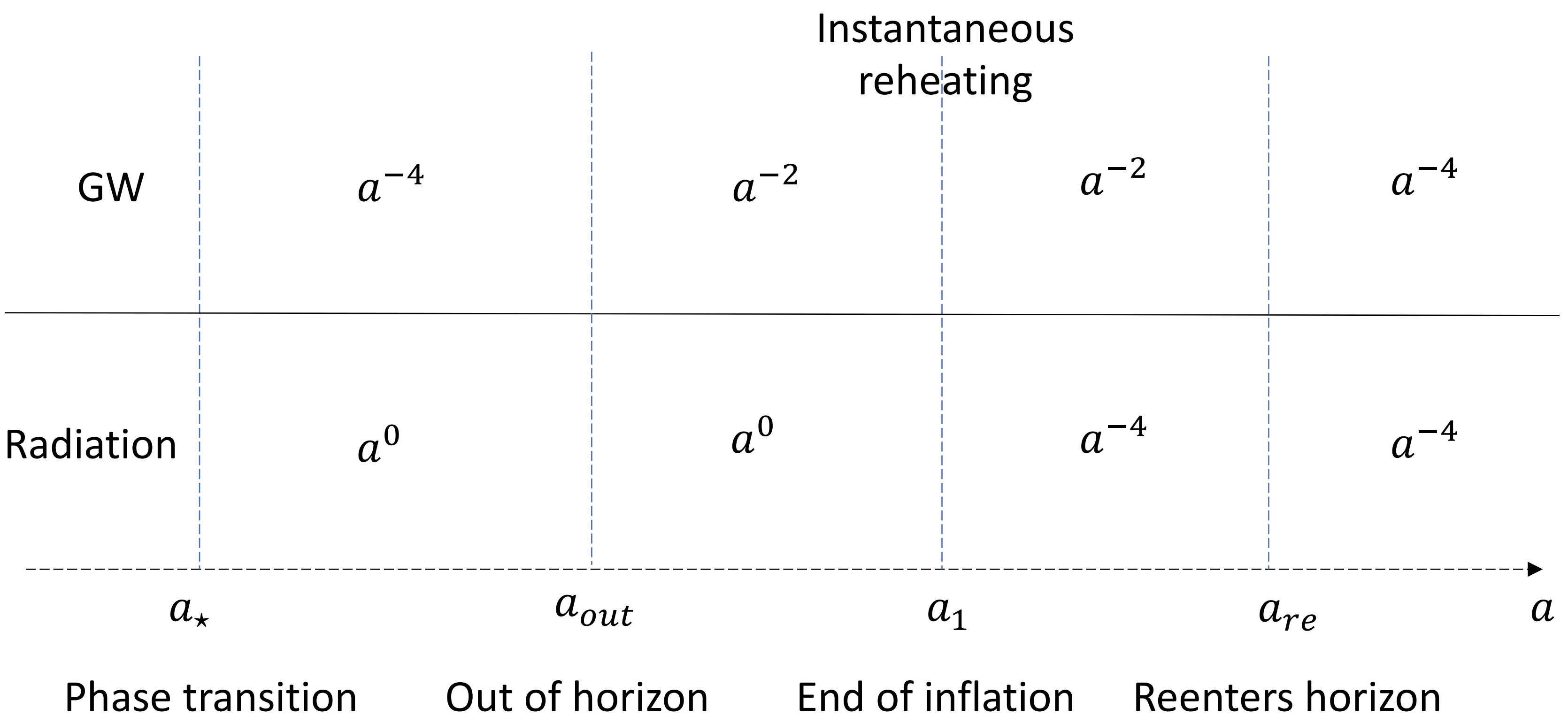


Spectrum of GW from a real source

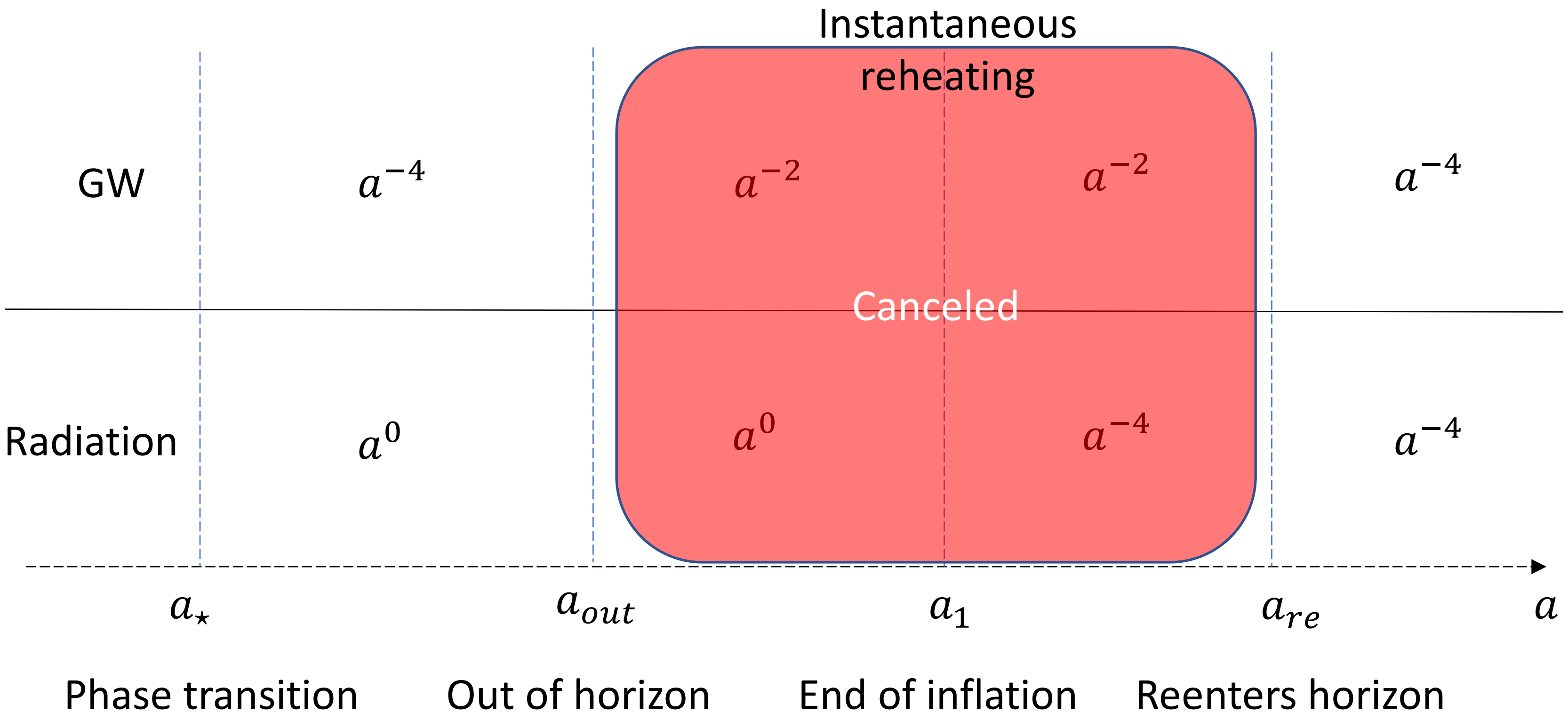
$$\Delta_\tau \sim \beta^{-1} \ll H^{-1}$$



Redshifts of the GW signal

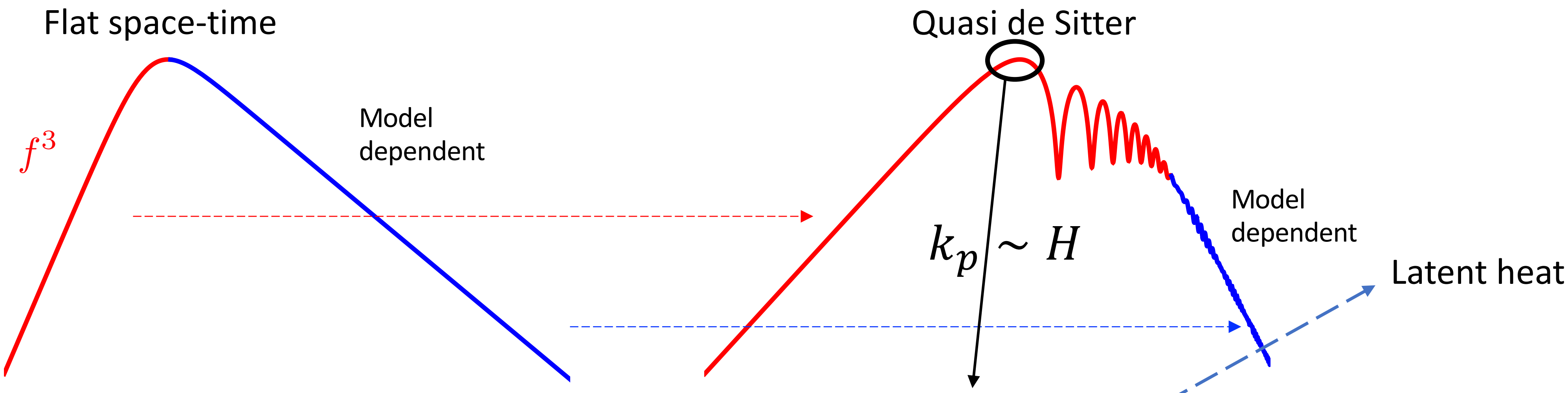


Redshifts of the GW signal



$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

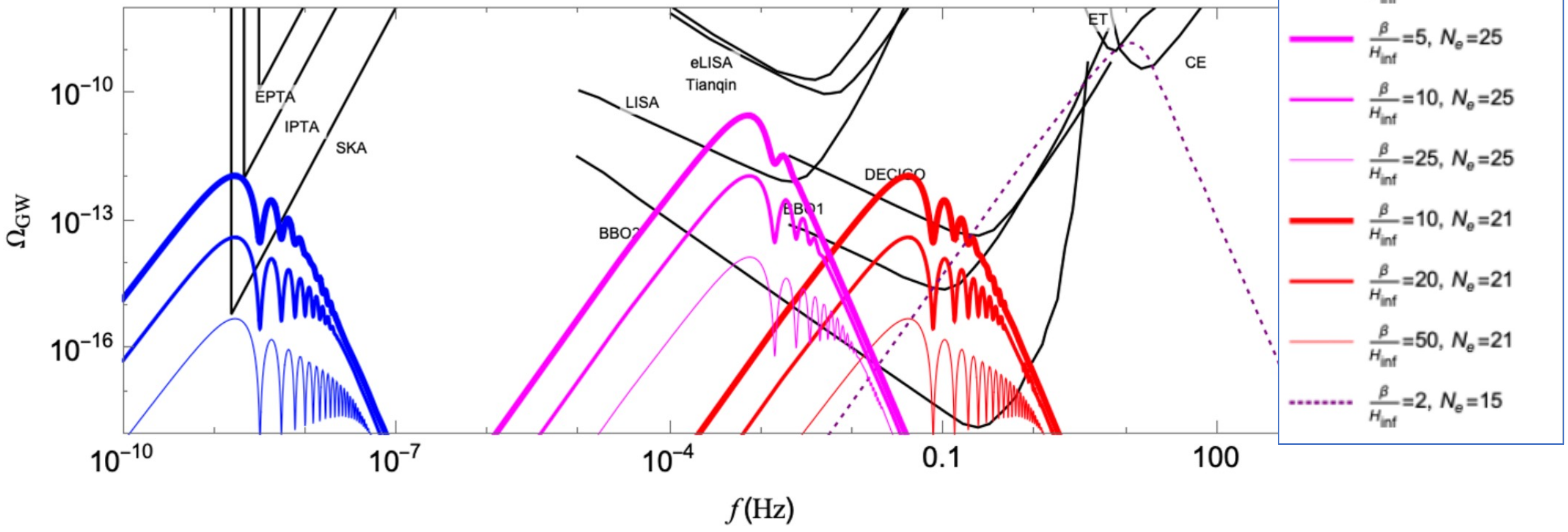
$$\approx 10^{-17} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

First-order phase transition during inflation

- Primordial stachastic GW signals

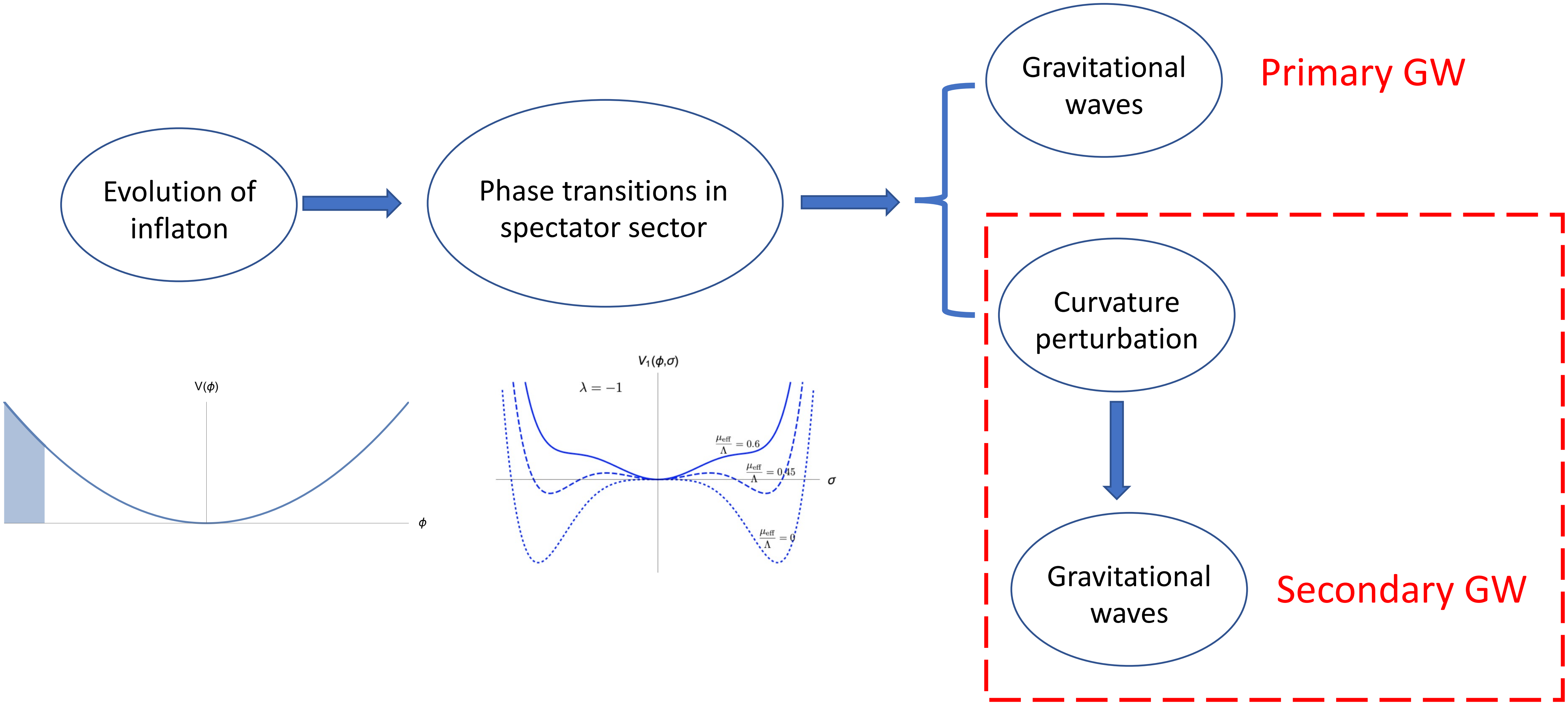
Instantaneous reheating

$$H_{\rm inf} = 10^{12}\ {\rm GeV}$$
$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.3$$



HA, K.-F. Li, L.-T. Wang, S. Zhou, 2009.12381, 2201.05171

Induced phase transition in spectator sectors



Induced curvature perturbation ζ

- We solve the following equations of motion numerically with a $1000\times1000\times1000$ lattice

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} \ ,$$

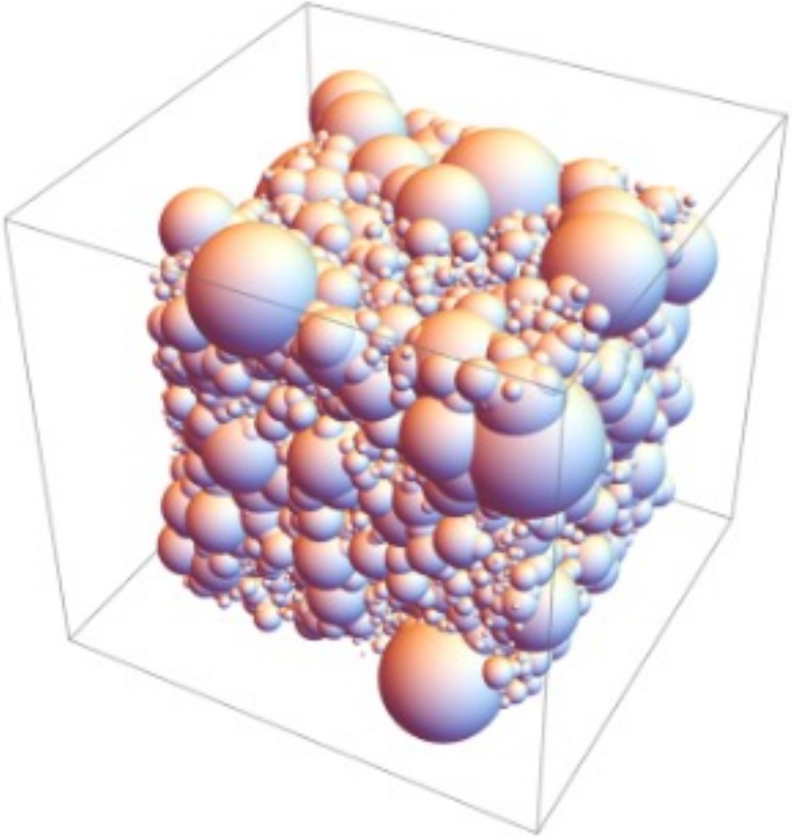
$$\mathcal{S}_{\mathbf{q}} \ = \ -\frac{1}{H^2\tau^2}\left[\frac{\partial V_1}{\partial\phi}\right]_{\mathbf{q}} - \left\{\frac{2\Phi_{\mathbf{q}}}{H^2\tau^2}\left(\frac{\partial V_0}{\partial\phi_0} + \left[\frac{\partial V_1}{\partial\phi}\right]_0\right) + \frac{\dot{\phi}_0}{H\tau}\left(3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}\right)\right\}$$

$$\tilde{\Psi}'_{\mathbf{q}} - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N\left(\frac{\dot{\phi}_0\delta\tilde{\phi}_{\mathbf{q}}}{H_{\rm inf}\tau} + \left[\frac{\partial_i}{\partial^2}(\sigma'\partial_i\sigma)\right]_{\mathbf{q}}\right)$$

$$\tilde{\pi}^S_{\mathbf{q}} = -\frac{3}{2}H_{\rm inf}^2\tau^2 q_i q_j q^{-4}\left[(\partial_i\sigma\partial_j\sigma)^{\rm TL}\right]_{\mathbf{q}}$$

- Conserved quantity

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\rm inf}\delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

- After the collision of the bubbles, σ field oscillates and keeps producing ζ.
- The production of ζ lasts about H^{-1}, longer than β^{-1}.

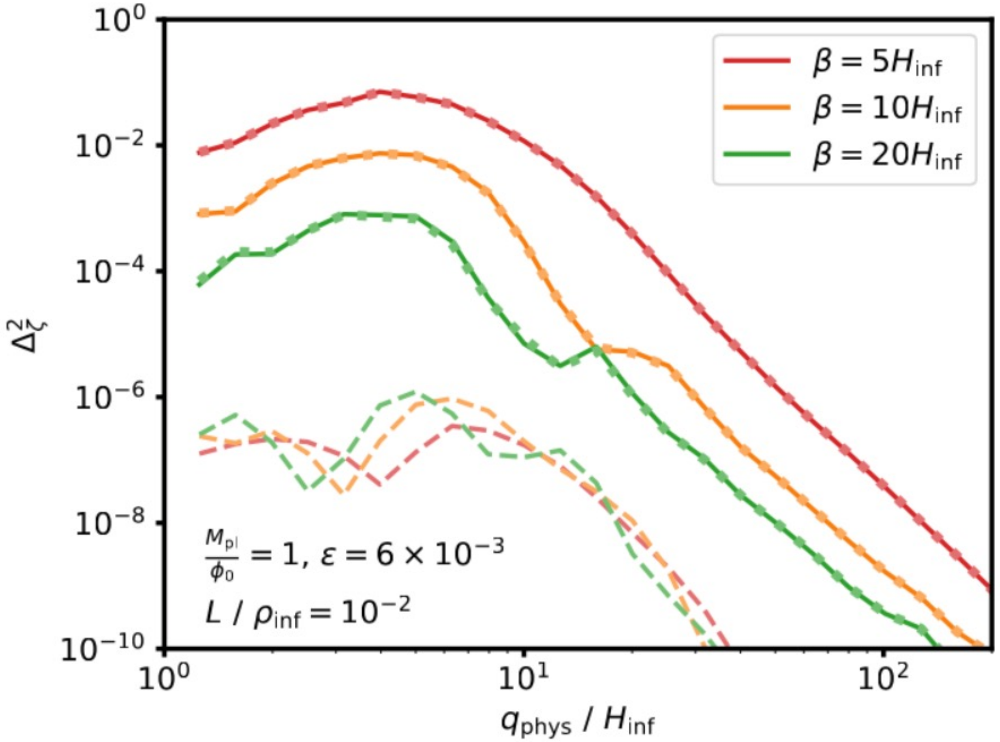
$$\zeta_{\mathbf{q}} \approx \frac{H_{\rm inf}}{\dot{\phi}_0 q^2} \int \frac{d\tau'}{\tau'} \left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'} \right) \frac{c_m \phi_0 [\sigma^2(\tau')]_{\mathbf{q}}}{H_{\rm inf}^2 \tau'^2}$$

$$\Delta_\zeta^{2\,(\rm emp)}(q) = A_{\rm ref} \mathcal{F}\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

$$\mathcal{F}(x) = \frac{x^3}{1 + (\alpha_1 x)^4 + (\alpha_2 x)^9}$$

$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\rm pl}}{\phi_0}\right)^2 \left(\frac{H_{\rm inf}}{\beta}\right)^3 \left(\frac{L}{\rho_{\rm inf}}\right)^2$$

$\mathcal{A} \approx 24$ $L \equiv \Delta\rho$ $\alpha_1 \approx 0.31, \alpha_2 \approx 0.17$



Secondary GWs

- After inflation $\zeta \to \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}^{\;\;lm}\mathcal{S}_{lm}\,,$$

$$\begin{aligned}\mathcal{S}_{ij} \;\equiv\;& 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i\left(\Psi' + \mathcal{H}\Phi\right)\partial_j\left(\Psi' + \mathcal{H}\Phi\right) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)\,.\end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278
Mollerach, Harari, and Matarrese, astro-hp/0310711
Ananda, Clarkson, and Wands, gr-qc/0612013
Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290

Secondary GWs

$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

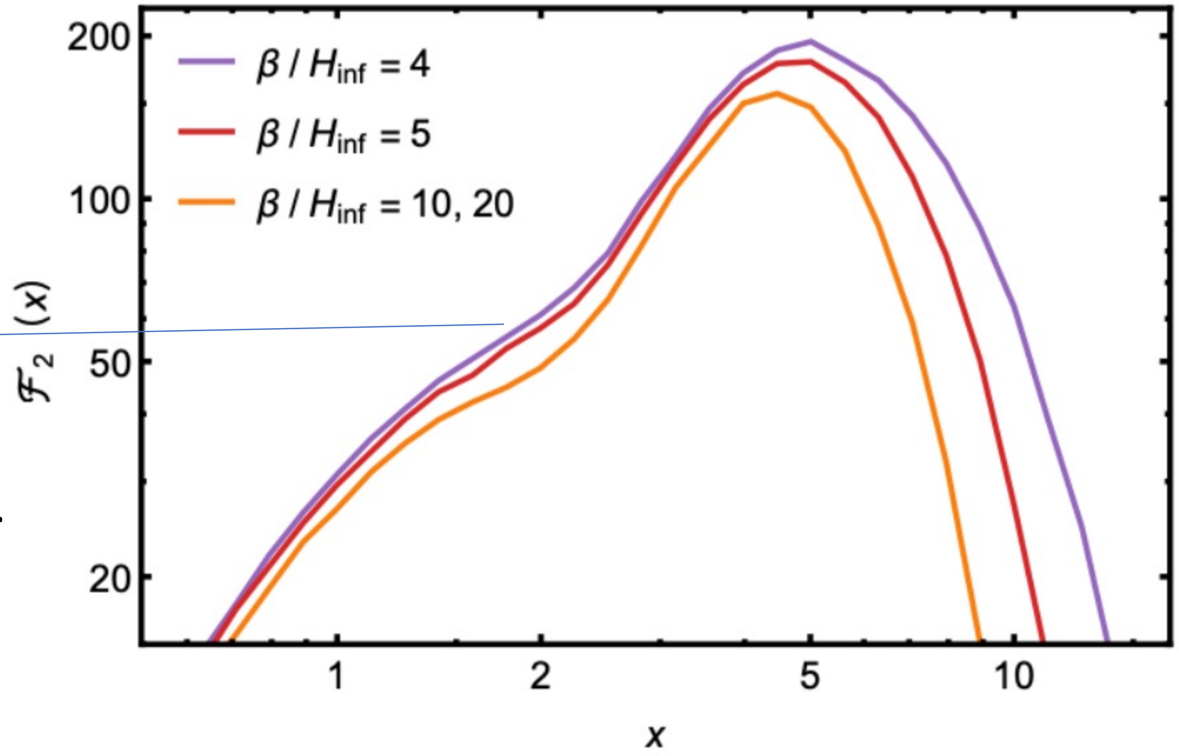
$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{L}{\rho_{\rm inf}}\right)^2$$

$$f = \frac{q}{2\pi a_0} = f_{\rm ref}\times\frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ {\rm Hz}\times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\mathcal{F}_2^{\rm max} \approx 200$$

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$



F_2 Collects information of the transfer functions.

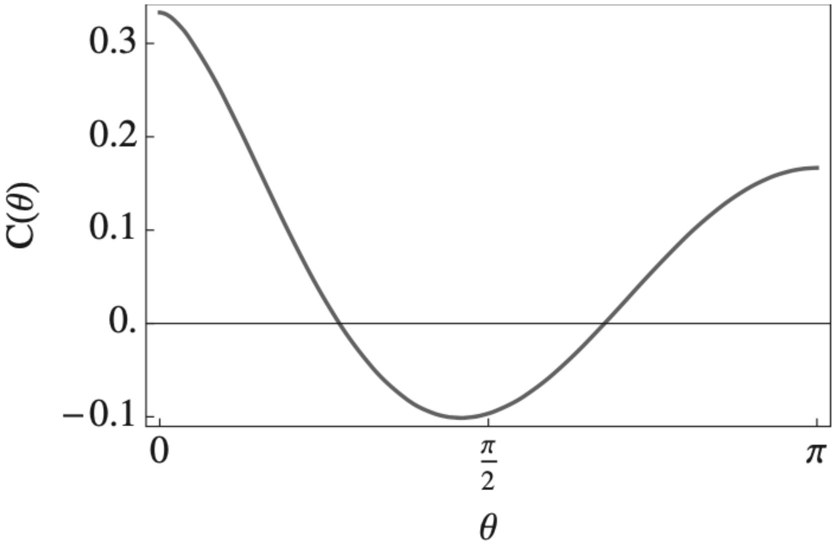
Observation from PTAs

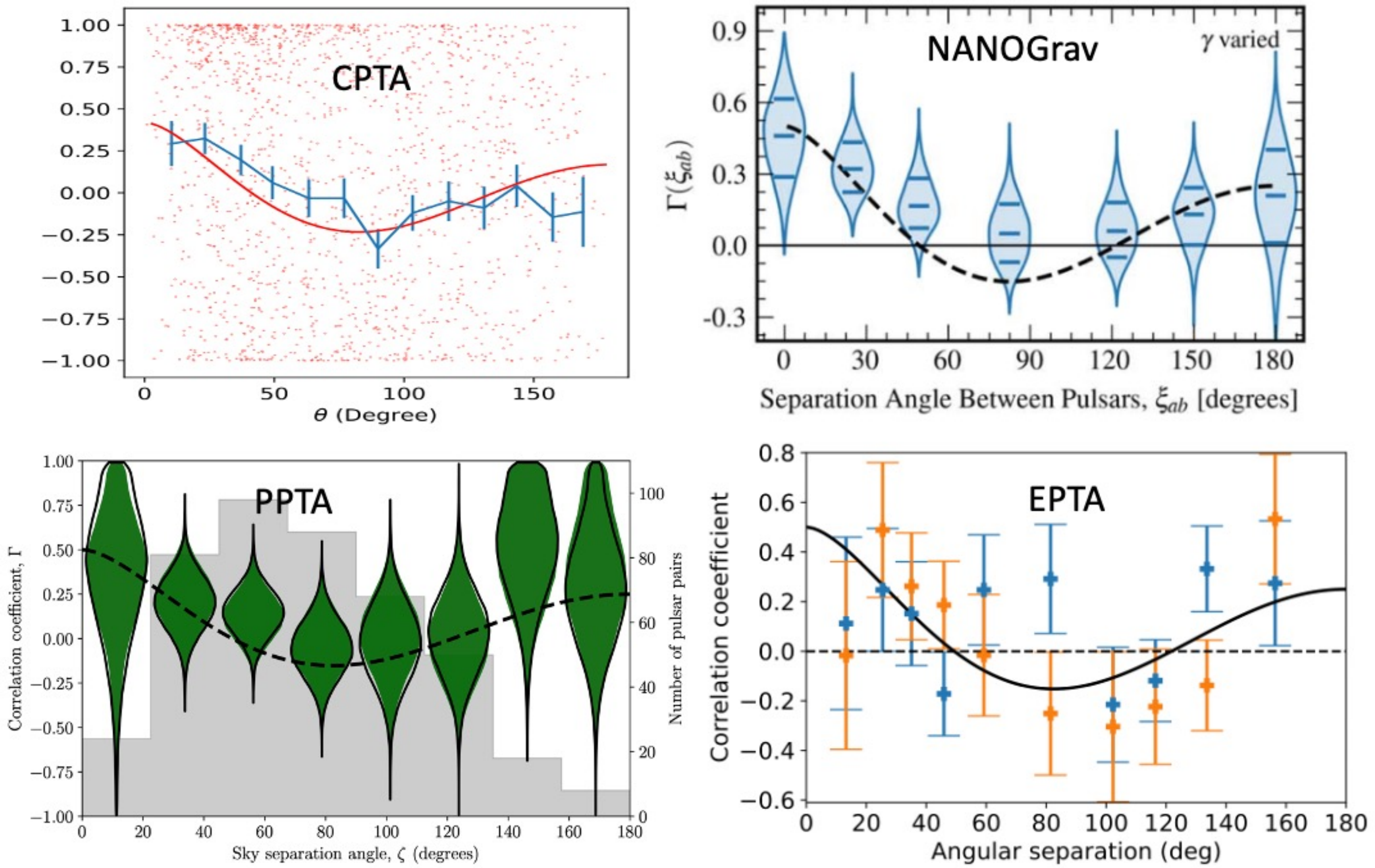
- Hellings-Downs curve

$$\langle z_a(t) z_b(t) \rangle = C(\theta_{ab}) \int_0^\infty df\, S_h(f)$$

Angular correlation

$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a / T_a$$

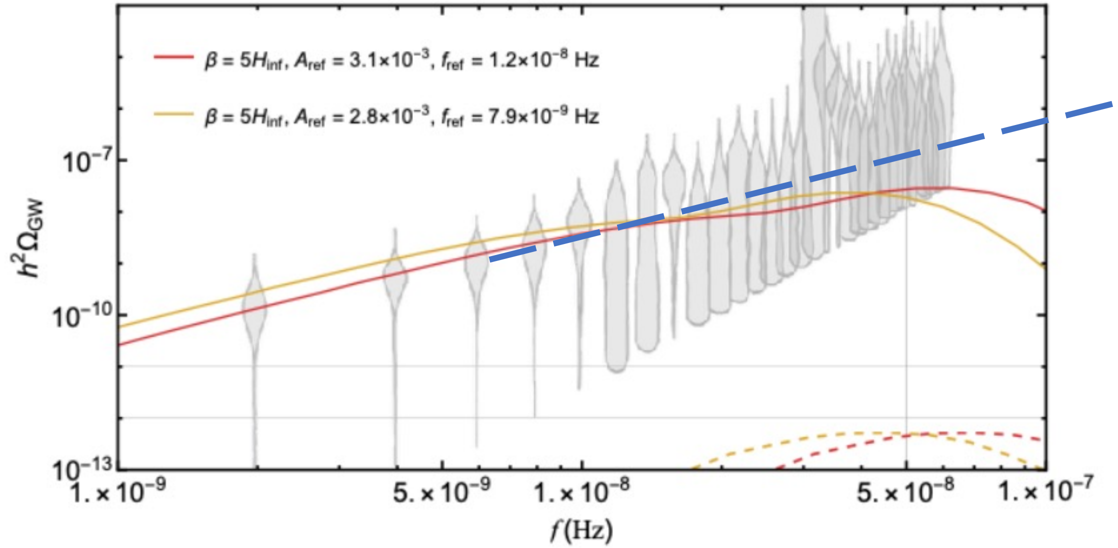


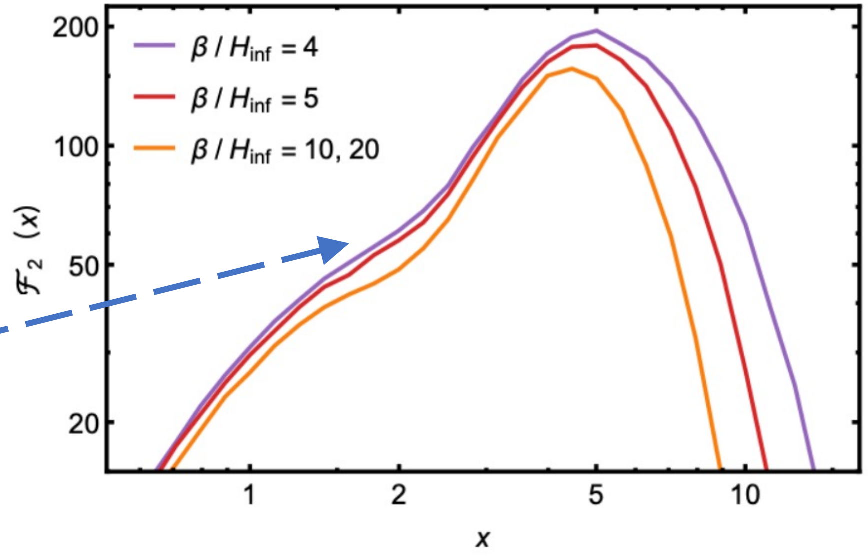


Observation from PTAs

- The slope is around 2 in the IR region

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$





$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2 \left(\frac{q_{\rm phys}}{H_{\rm inf}} \right)$$

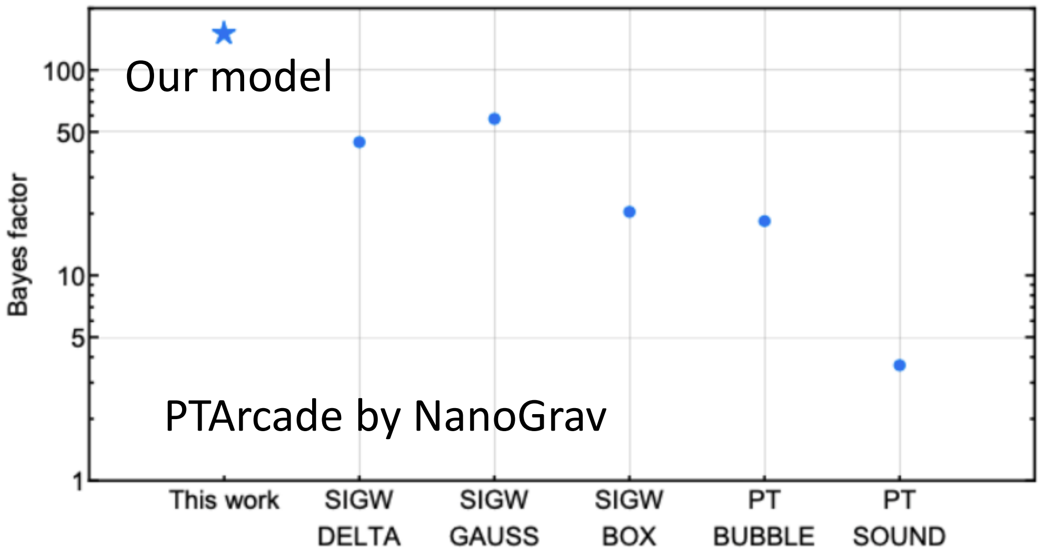
$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

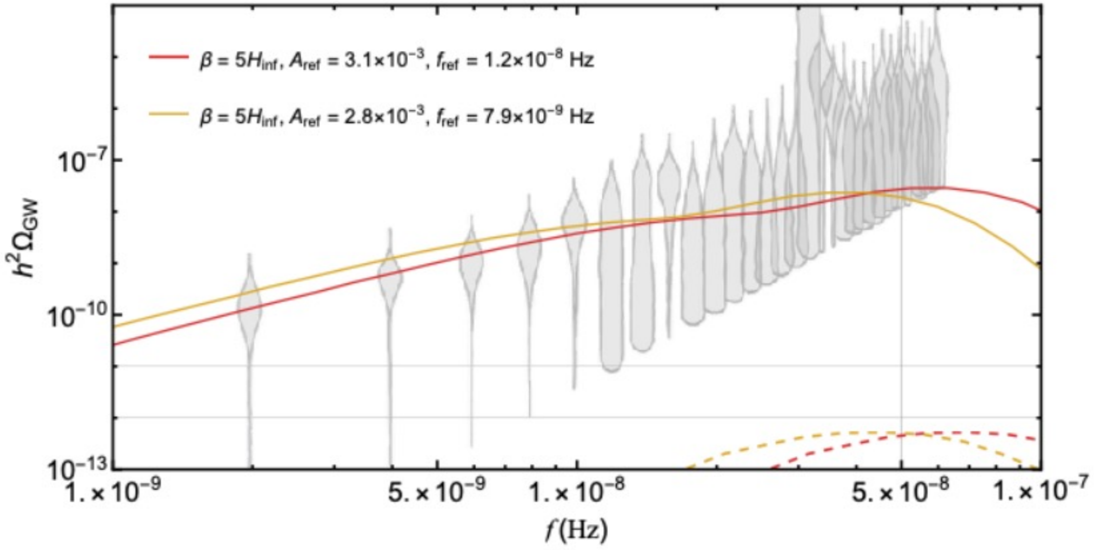
$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e} \left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}} \right)^{1/2}$$

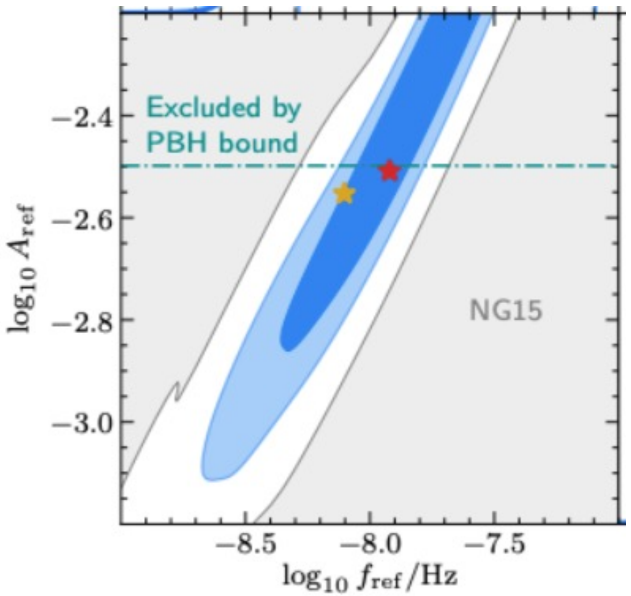
HA, B. Su, H. Tai, L.-T. Wang, C. Yang, 2308.00070

Observation from PTAs

- Bayes factor against SMBHB



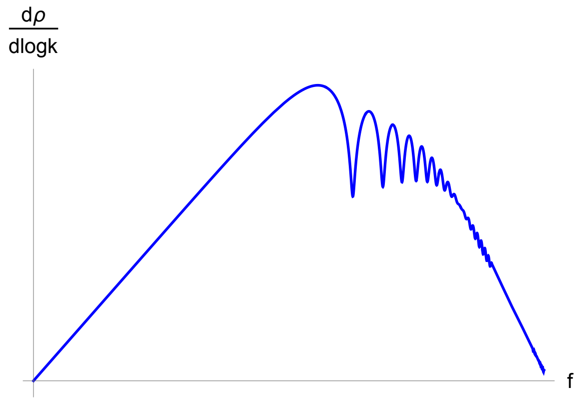


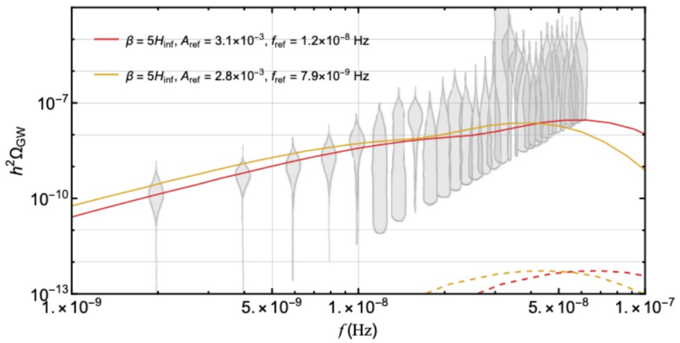


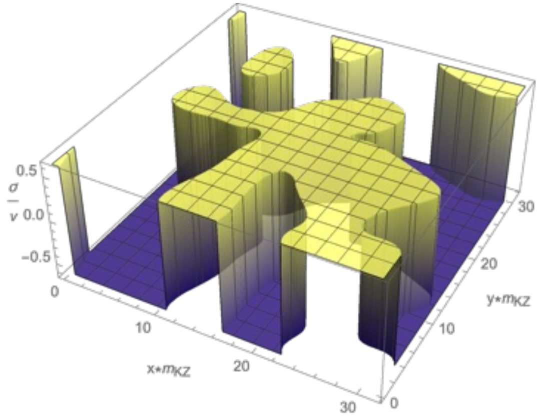
HA, B. Su, H. Tai, L.-T. Wang, C. Yang, 2308.00070

Outline

- GWs from first-order phase transitions during inflation.
  - Primary GWs
  - Curvature perturbation and secondary GWs
- GWs from second-order phase transitions (domain walls) during inflation.
- Summary and outlook

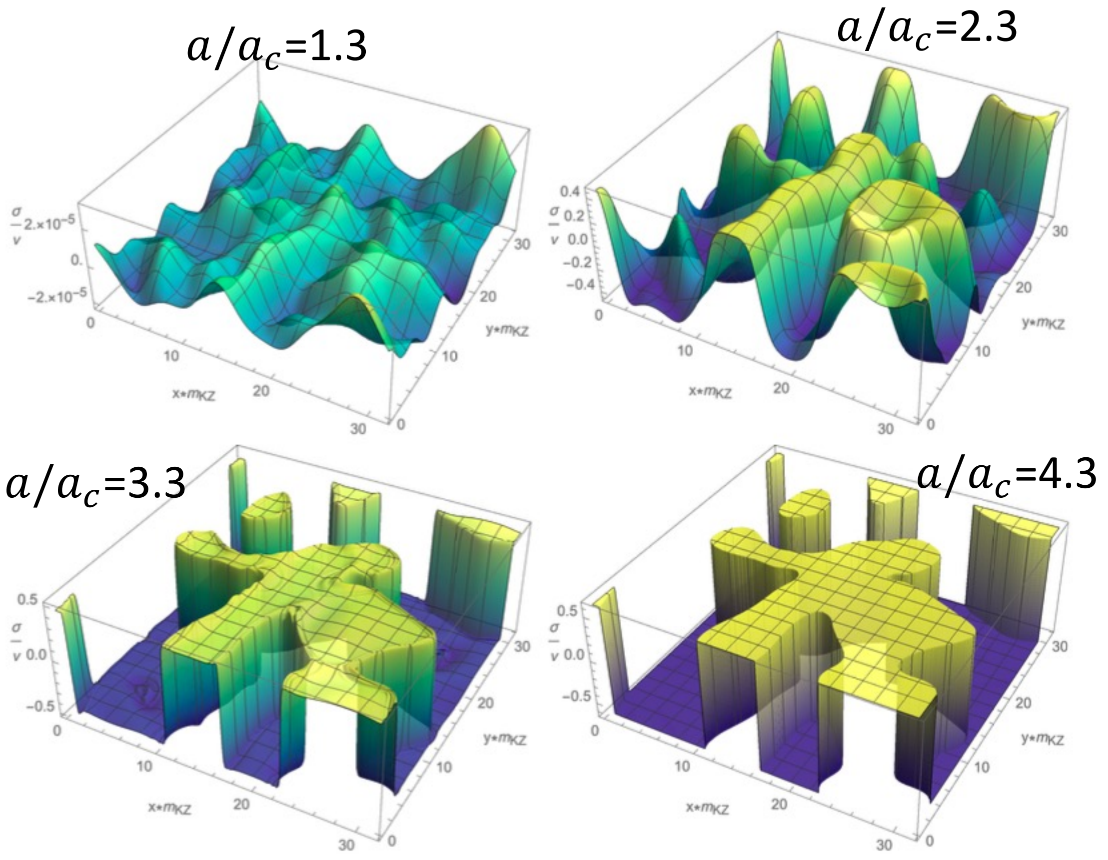






Formation of domain walls

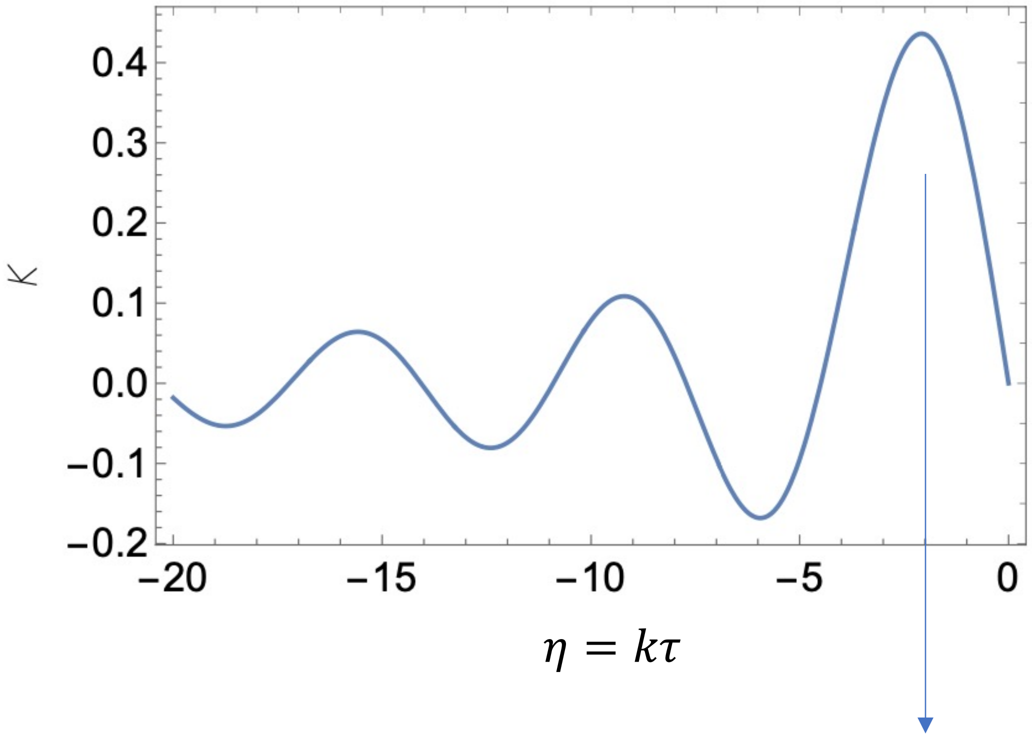
- We numerically solve the nonlinear evolution of σ field with $1000\times 1000\times 1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.

$$\tilde{h}^{f}_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k}\int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau')\tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$



The dominant contribution

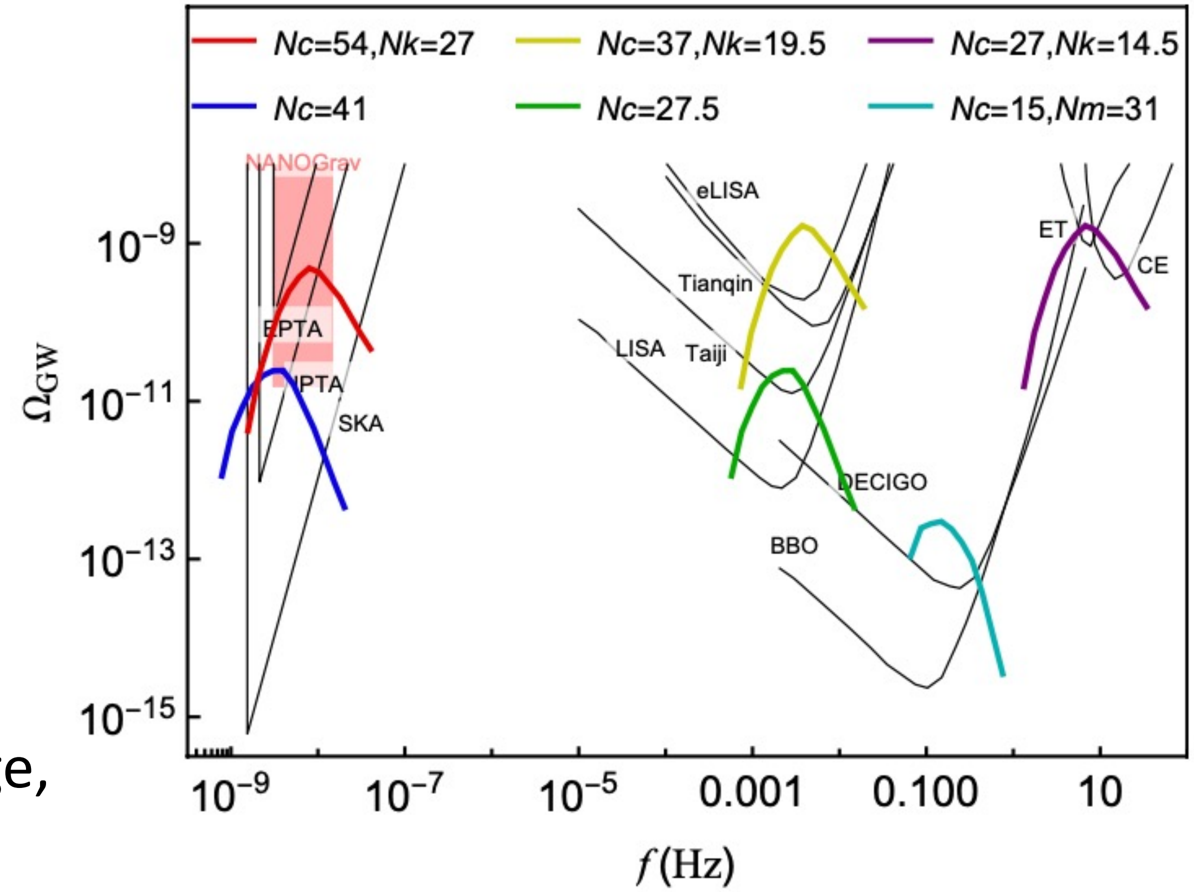
Numerical results for GWs

$$\Omega_{\rm GW}(f) = \Omega_R \times \rho_R^{-1} \left.\frac{d\rho_{\rm GW}}{d\ln f}\right|_{\rm today}$$

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

The detailed shape and strength also depends on the evolution of the universe.

- Instantaneous reheating,
- Matter dominated intermediate stage,
- Kination dominated intermeditate stage.



HA, C. Yang, 2304.02361

Summary

- Phase transitions can happen in a spectator sector during inflation.
- We show that there is an oscillatory feature in the GW spectrum if it is produced by first-order phase transition during inflation.
- We show that the secondary GW can be strong enough to explain the signals observed by PTAs
- Static topological defects can produce GWs during inflation.