f^2 scaling of the PTA signals, induced gravitational waves, and primordial black holes

Takahiro Terada

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Collaborators: **Keisuke Harigaya, Keisuke Inomata**, and **Kazunori Kohri**

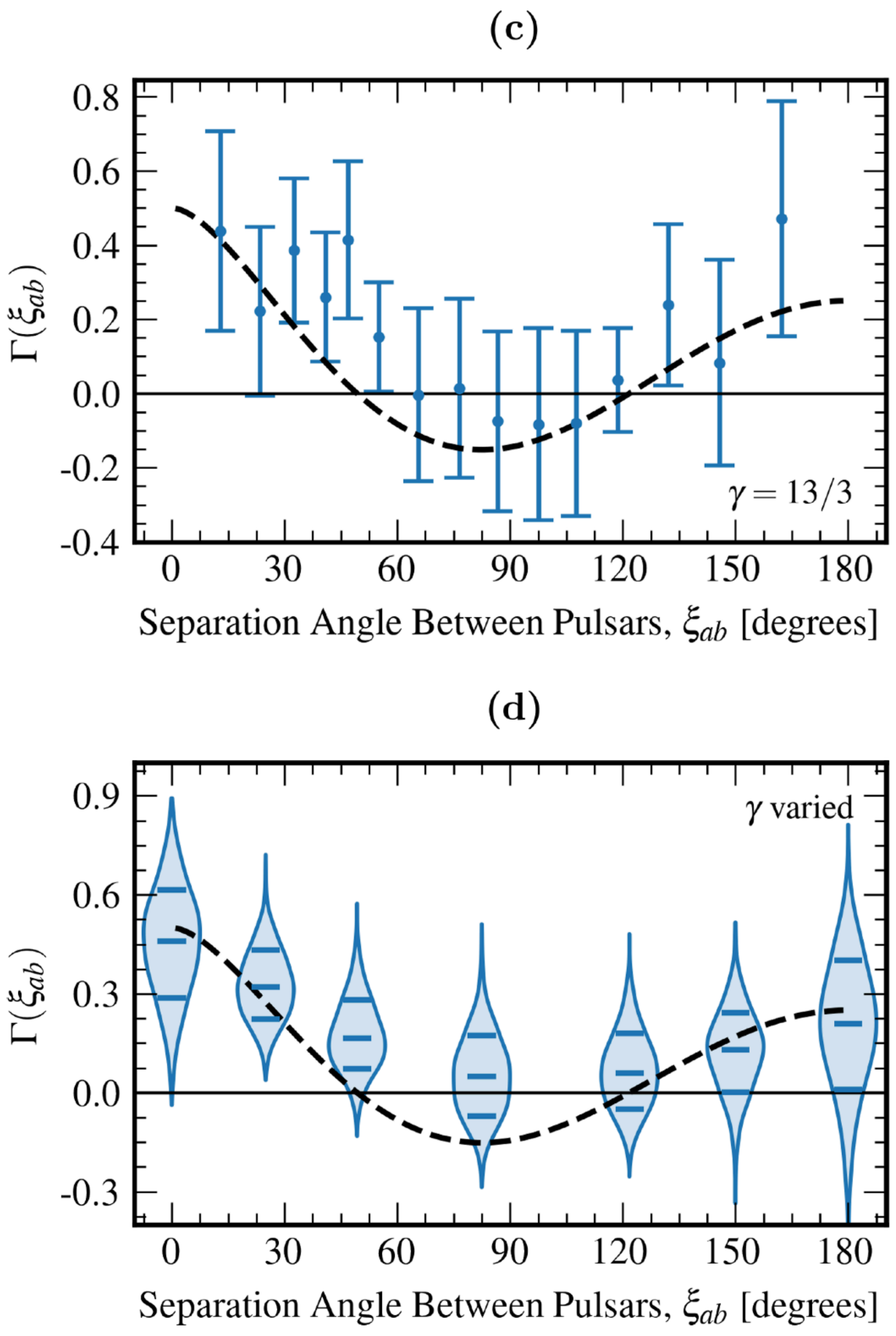
Based on [Inomata, Kohri, Terada, **2306.17834**] and [Harigaya, Inomata, Terada, **2309.00228**]

Gravitational Wave Probes of Physics Beyond Standard Model, Media Center, Osaka Metropolitan University, Nov. 9, 2023

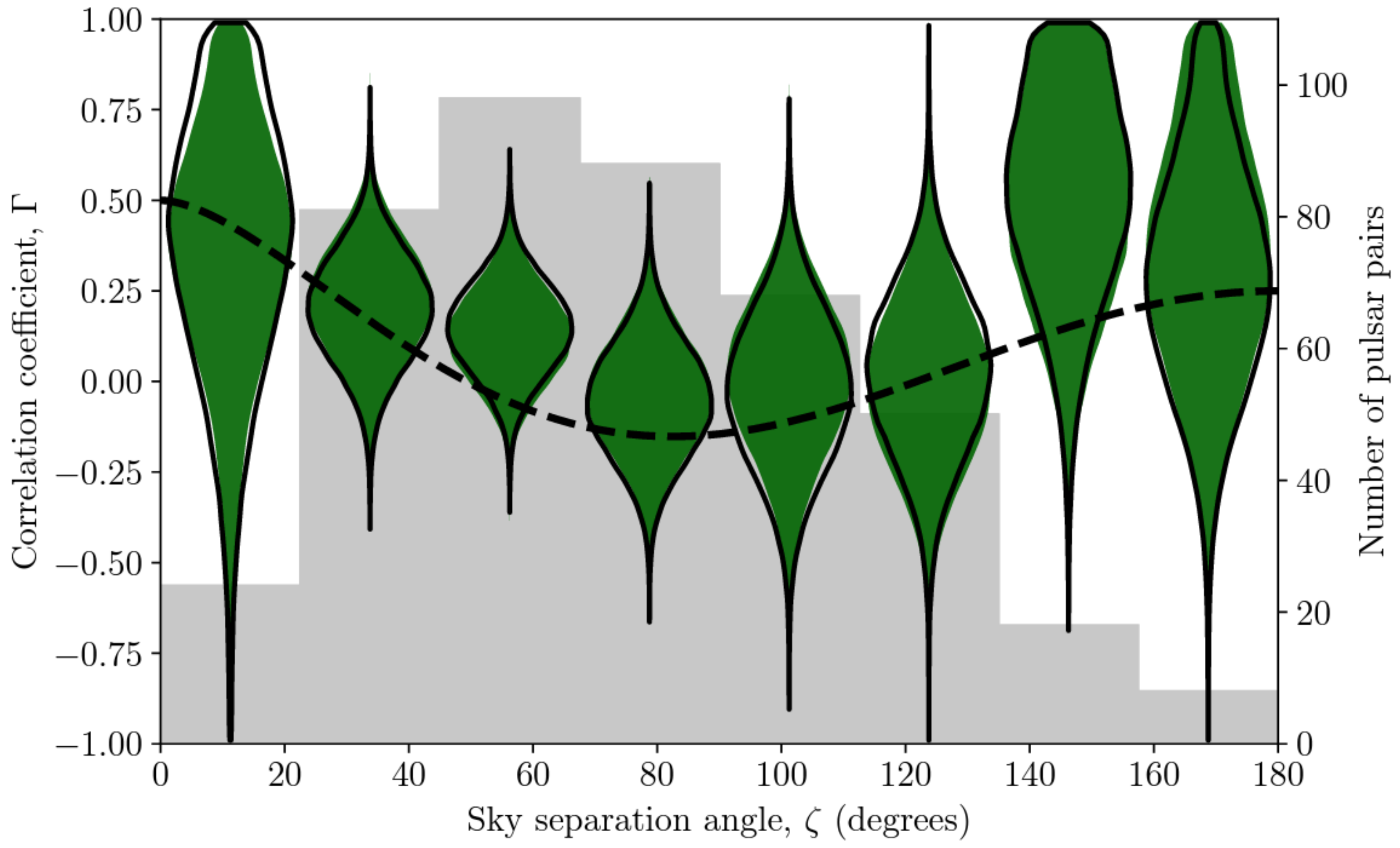
Pulsar Timing Array results

Hellings-Downs Curve

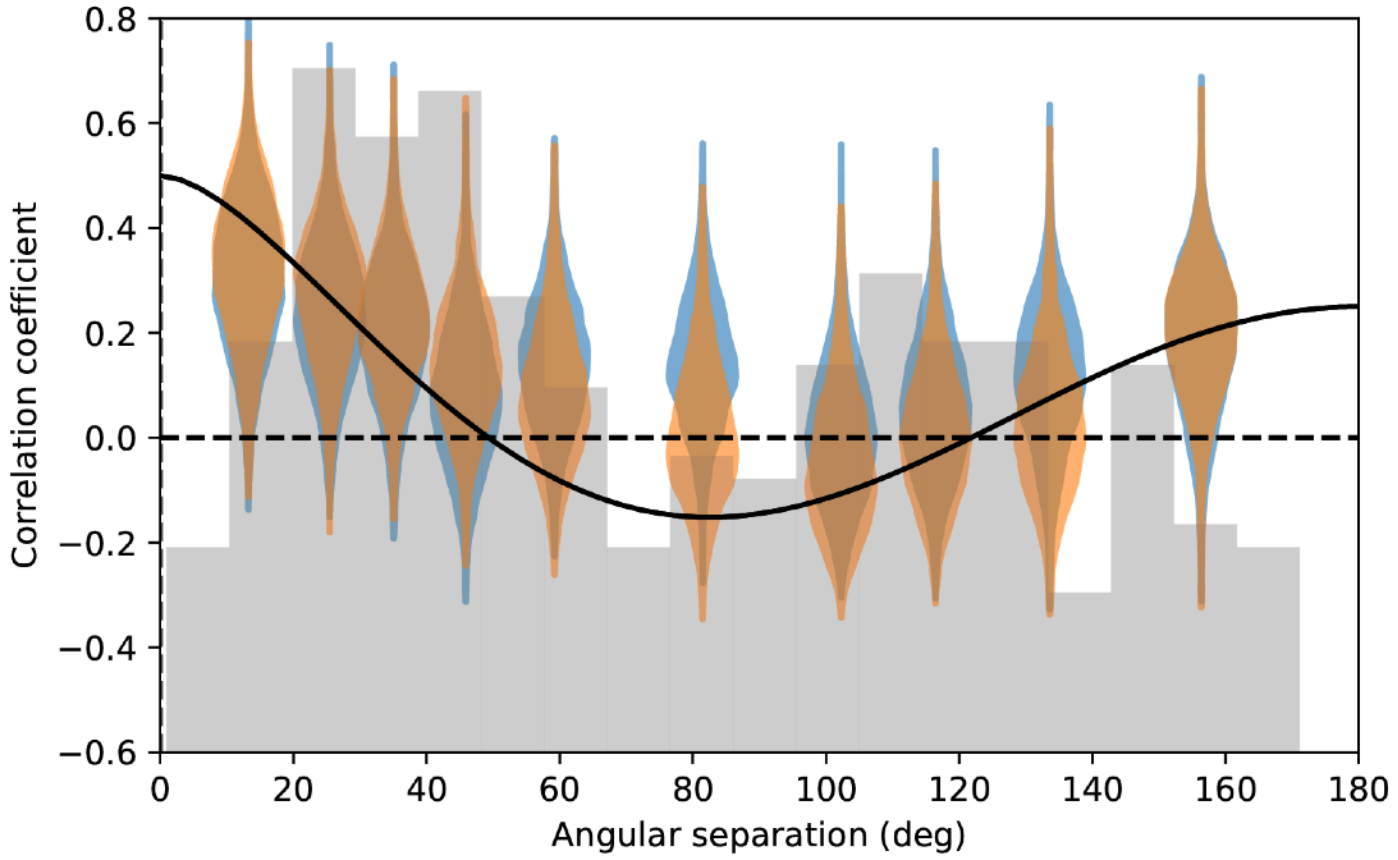
[NANOGrav, 2306.16213]



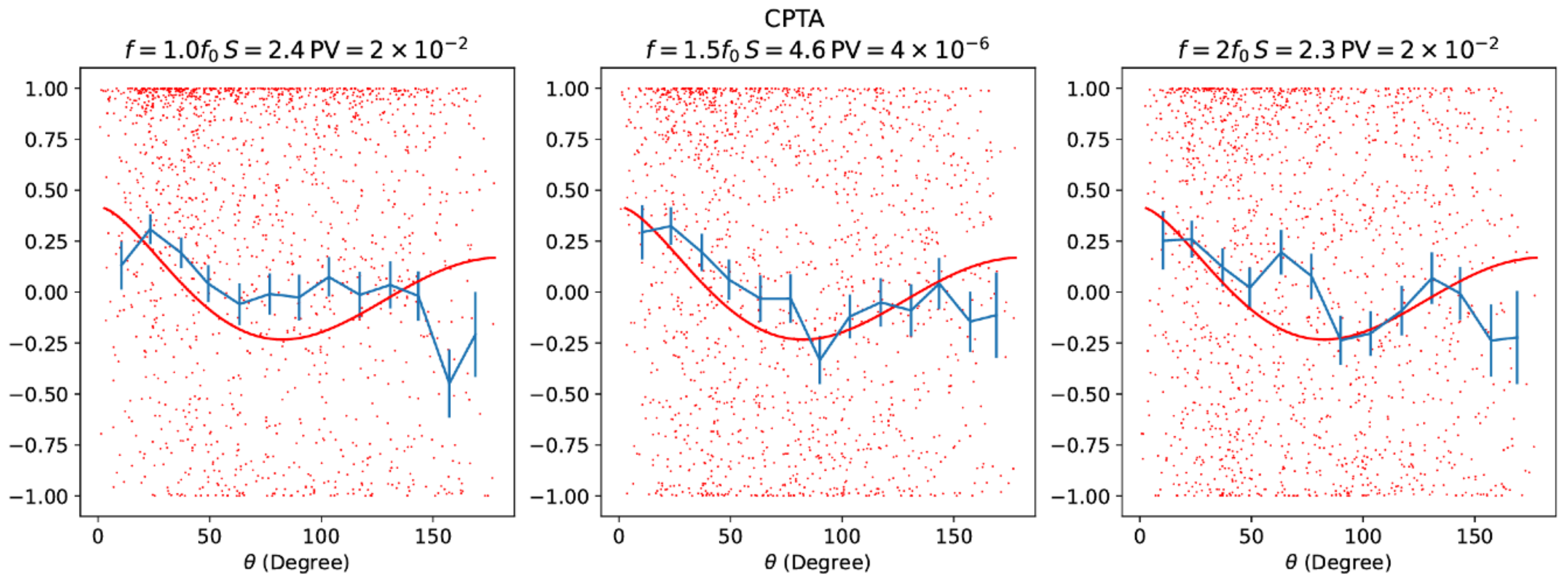
[PPTA, 2306.16215]

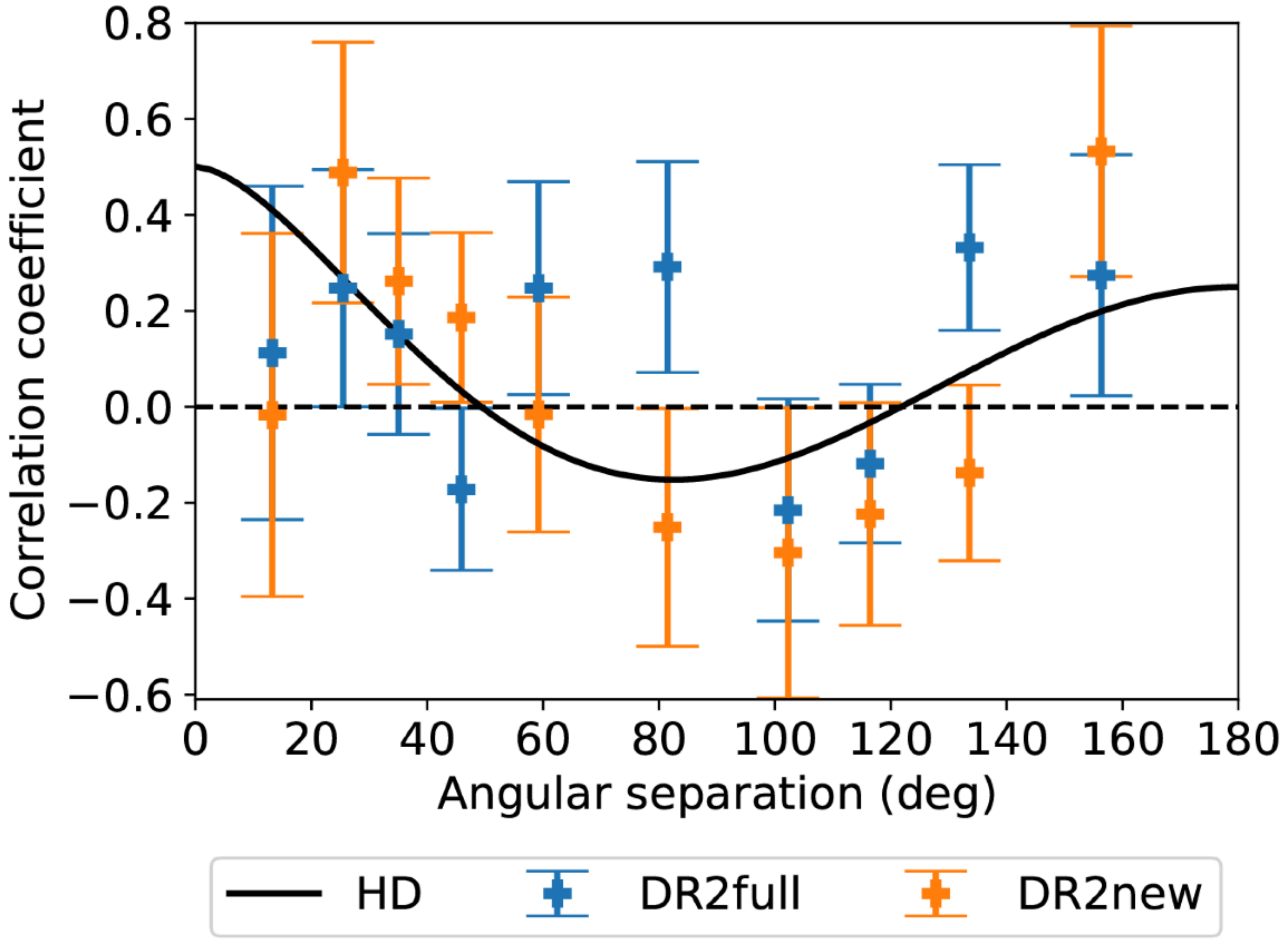


[EPTA/InPTA, 2306.16214]



[CPTA, 2306.16216]





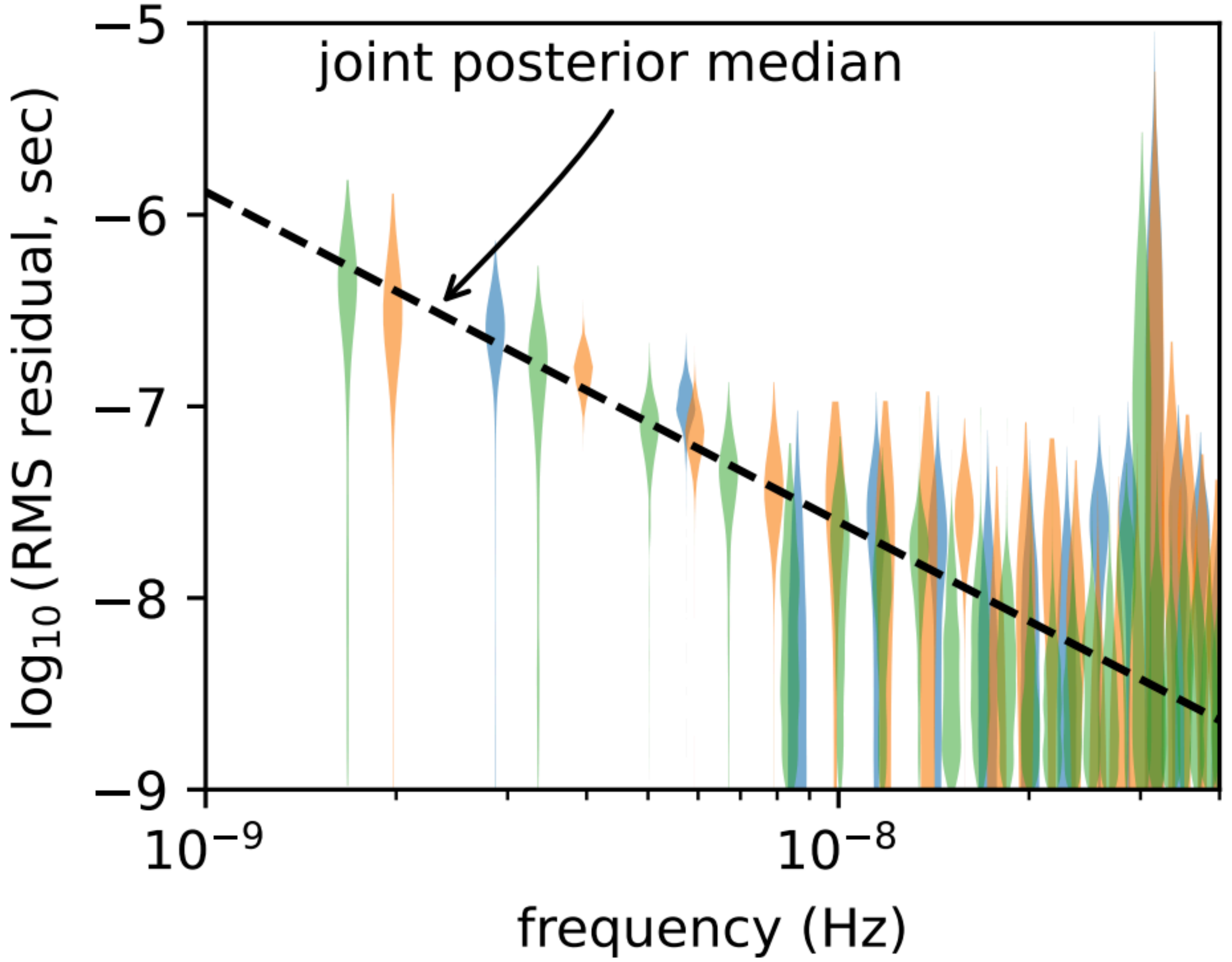
Gravitational-Wave Spectrum

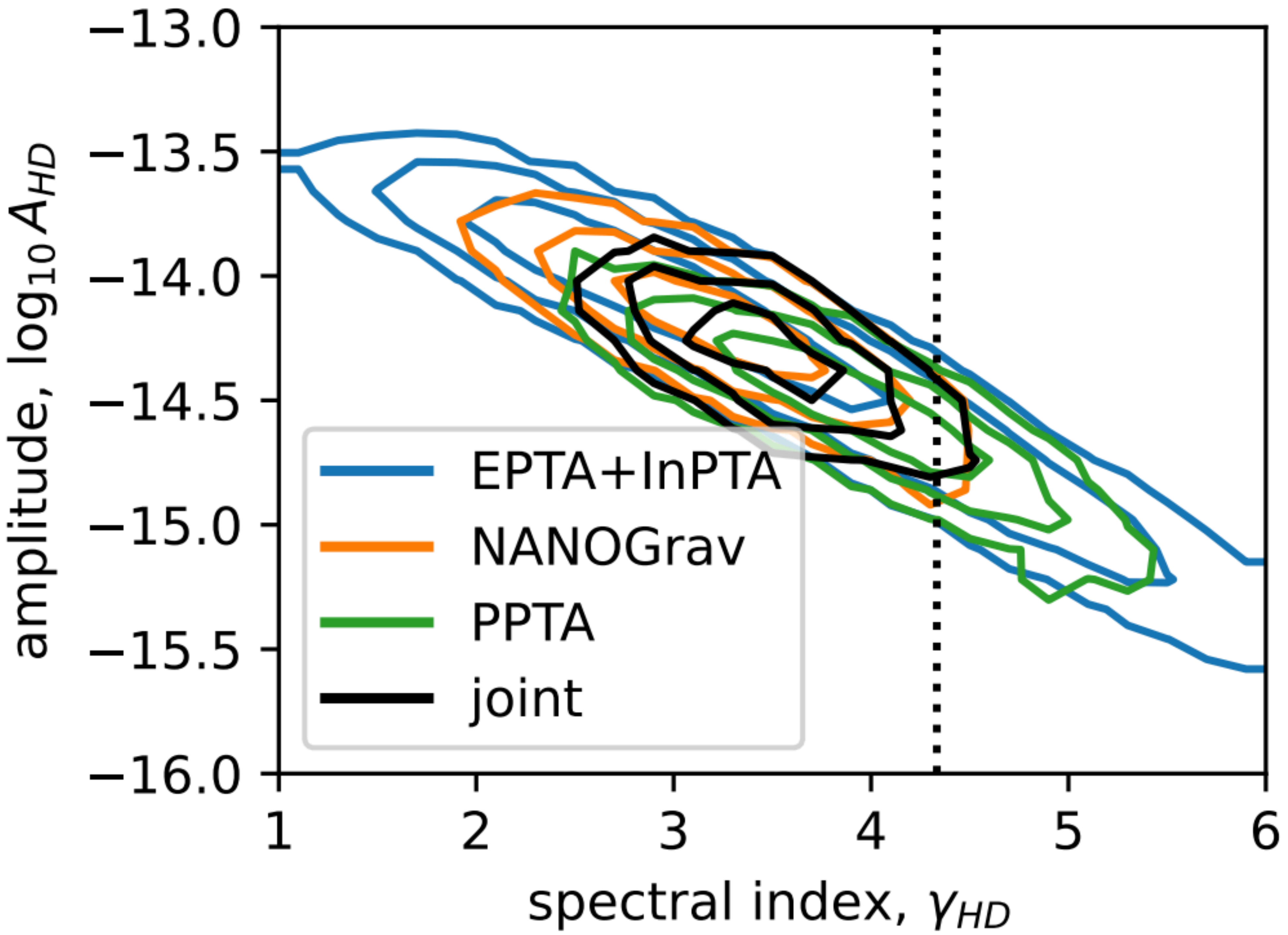
$$\Omega_{\mathrm{GW}}(f) = \frac{2\pi^2 f_*^2}{3H_0^2} A_{\mathrm{GWB}}^2 \left(\frac{f}{f_*}\right)^{5-\gamma}$$

$5 - \gamma = 1.8 \pm 0.6$ (90% credible region)

[NANOGrav, 2306.16213]

[IPTA, 2309.00693]

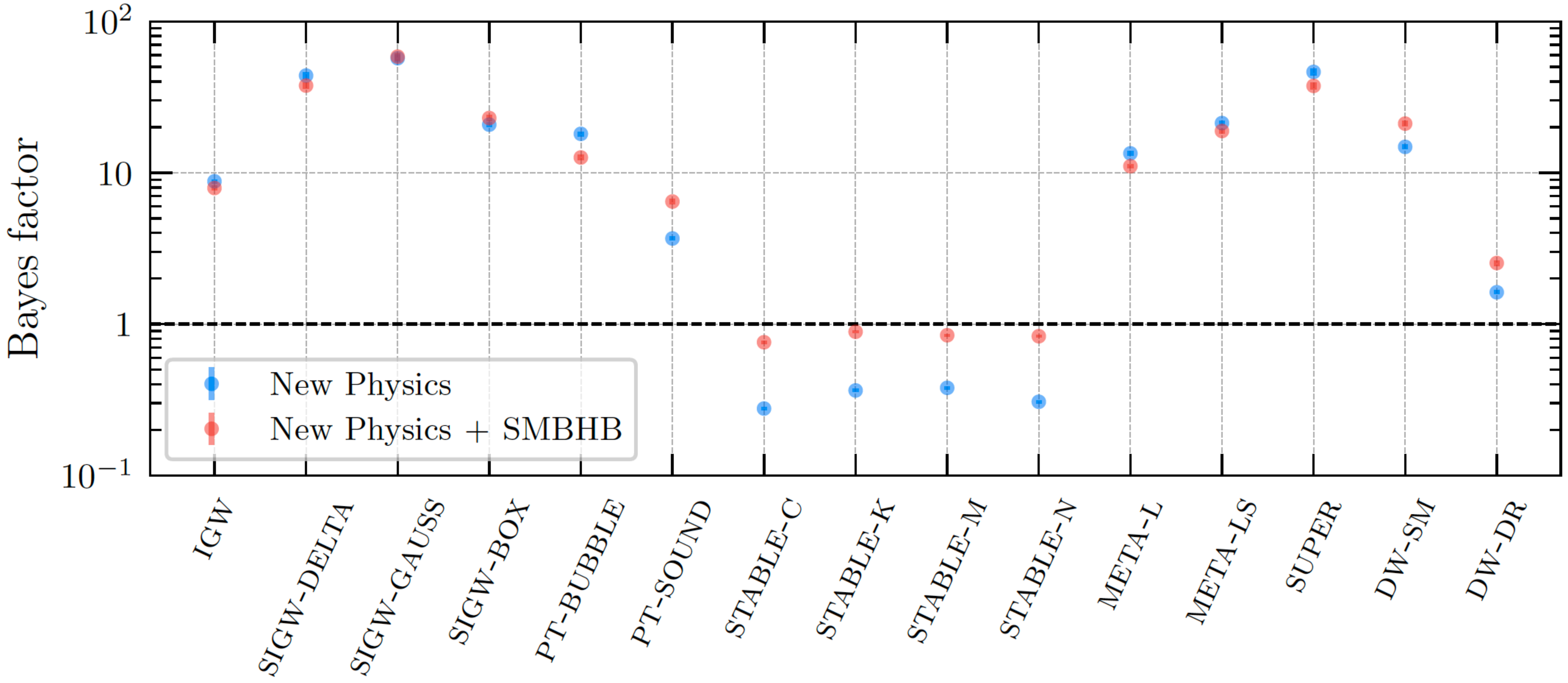


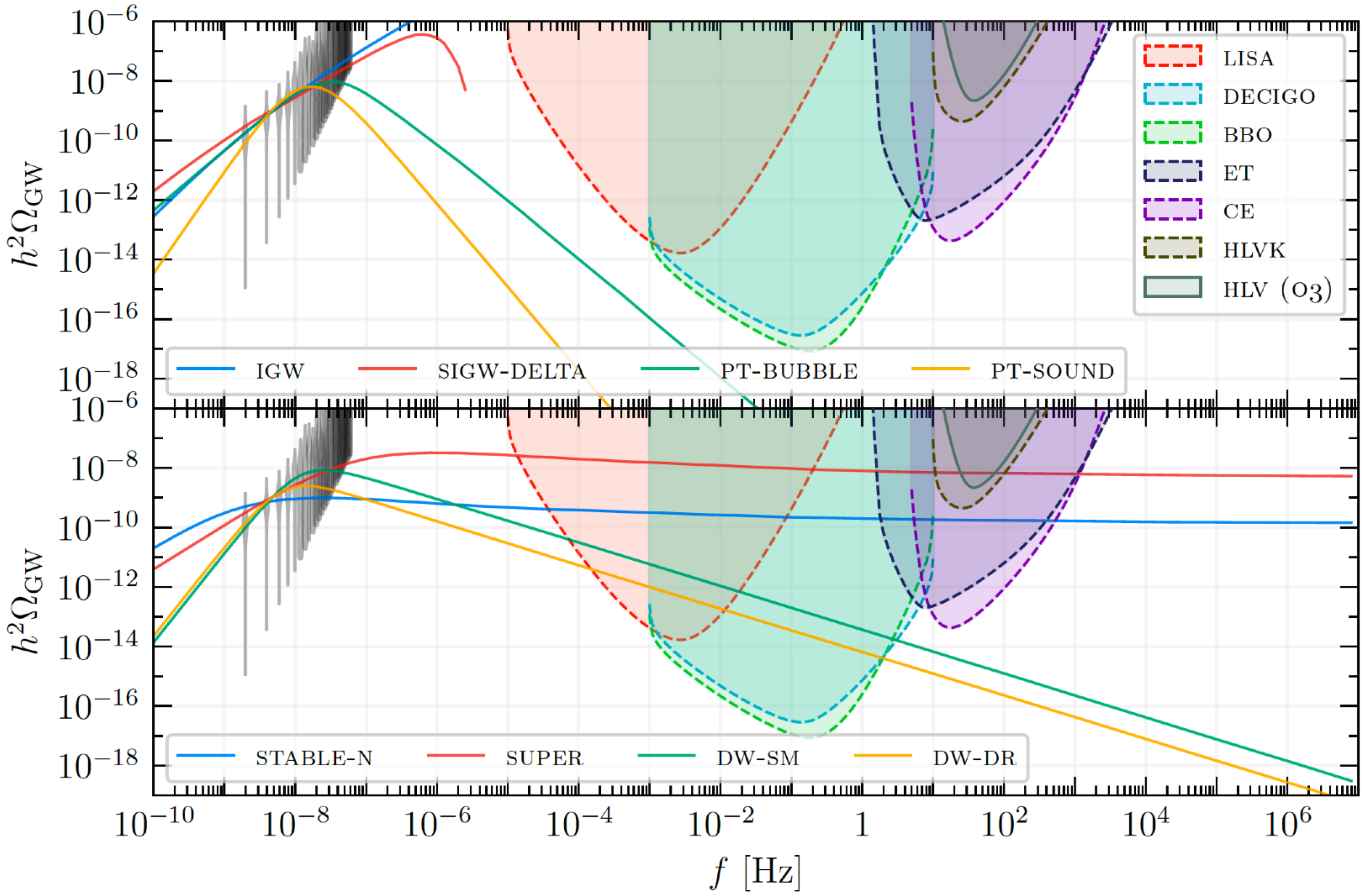


PTA, Induced GW, and PBH

New Physics Interpretations [NANOGrav, 2306.16219]

See also [EPTA/InPTA, 2306.16227], [Bian et al., 2307.02376], [Figueroa, 2307.02399], and [Ellis et al, 2308.08546].





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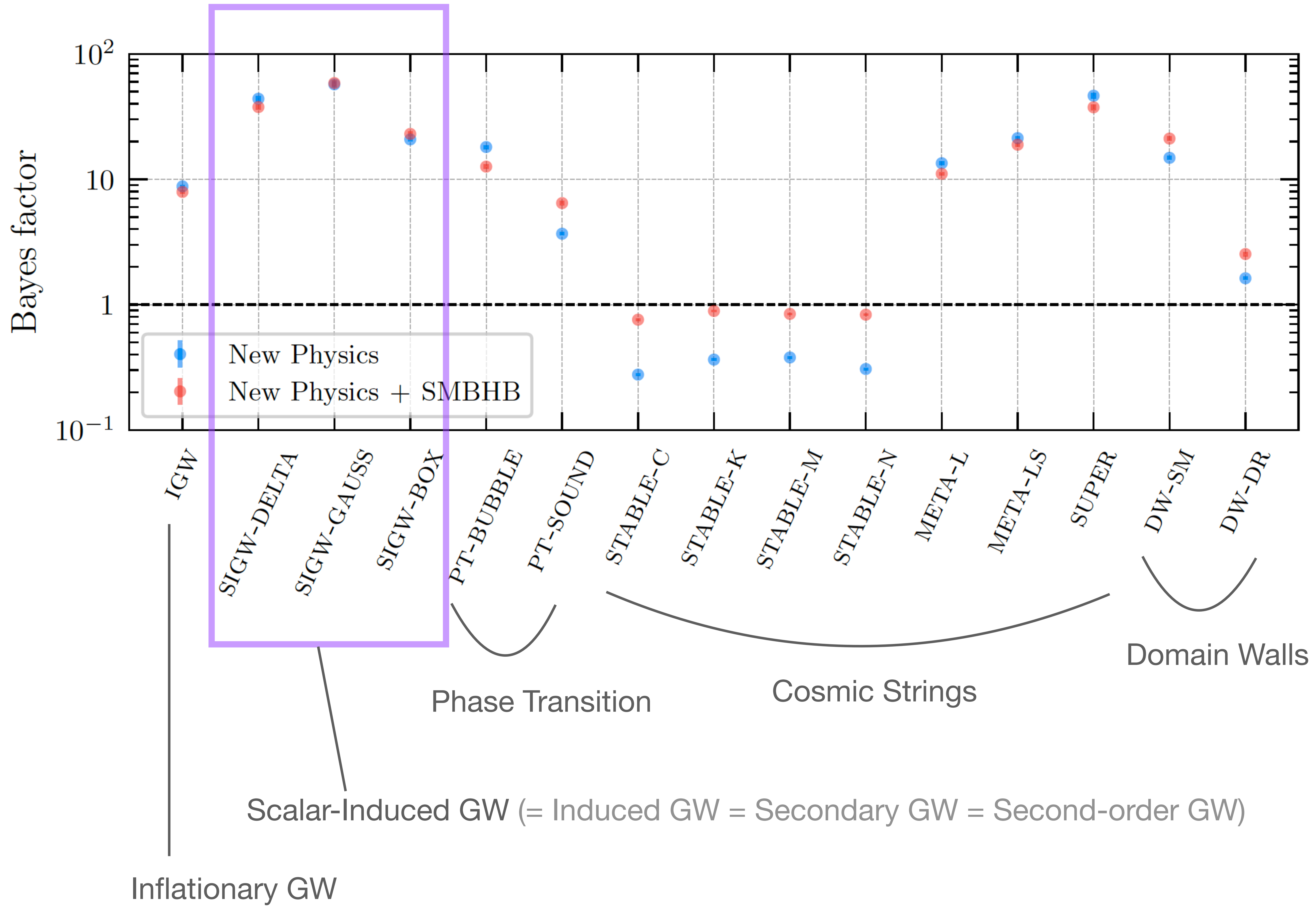

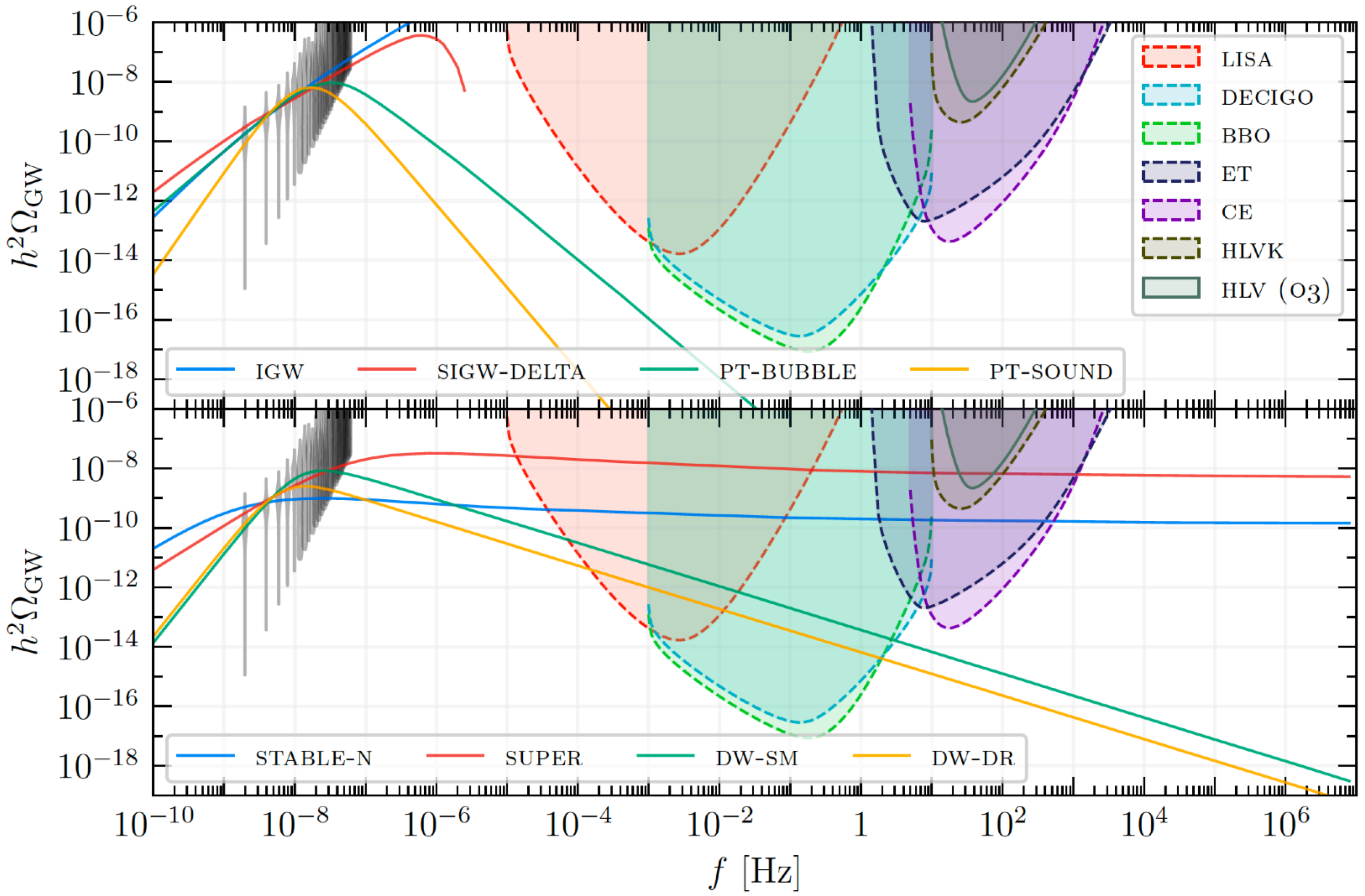

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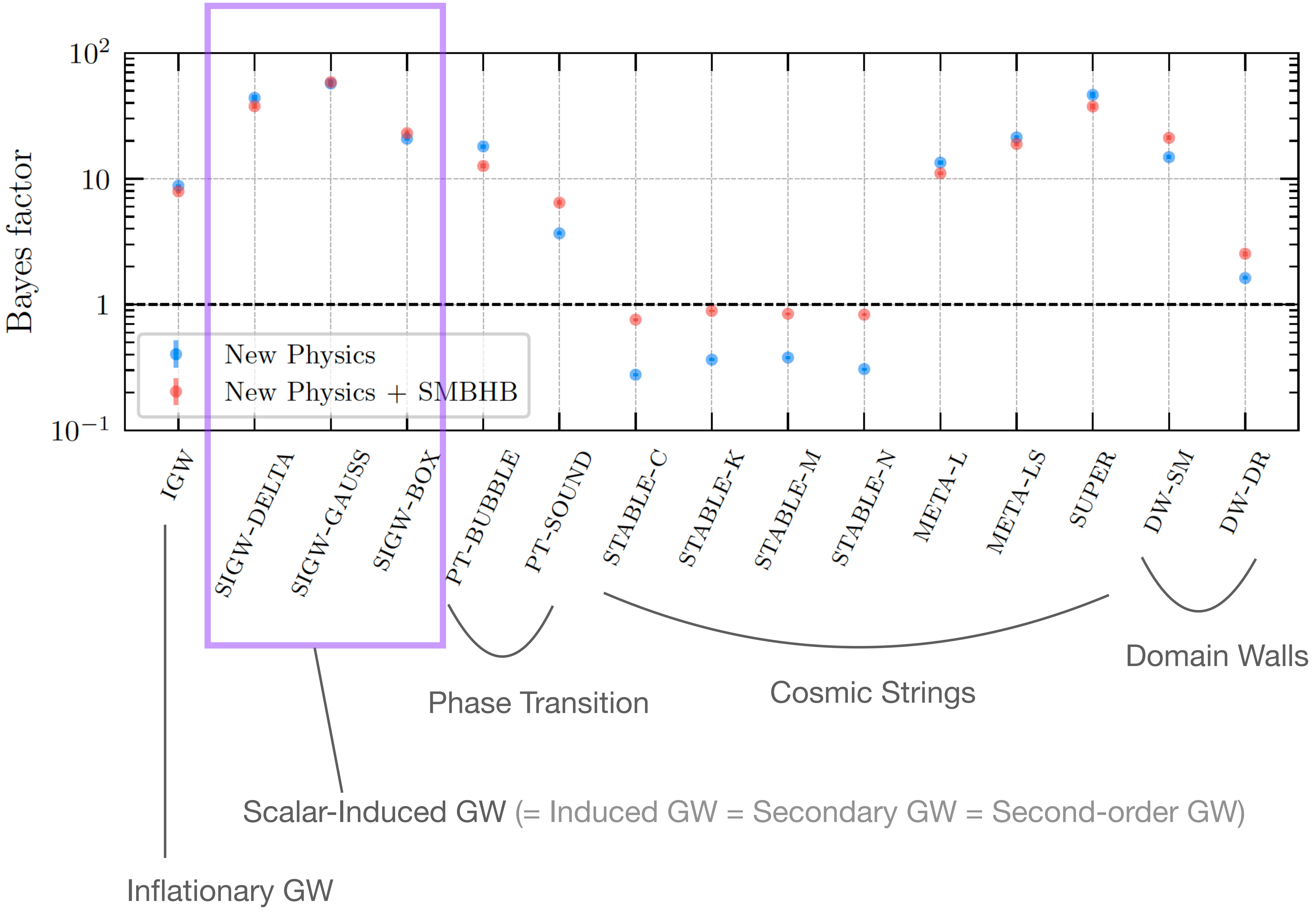
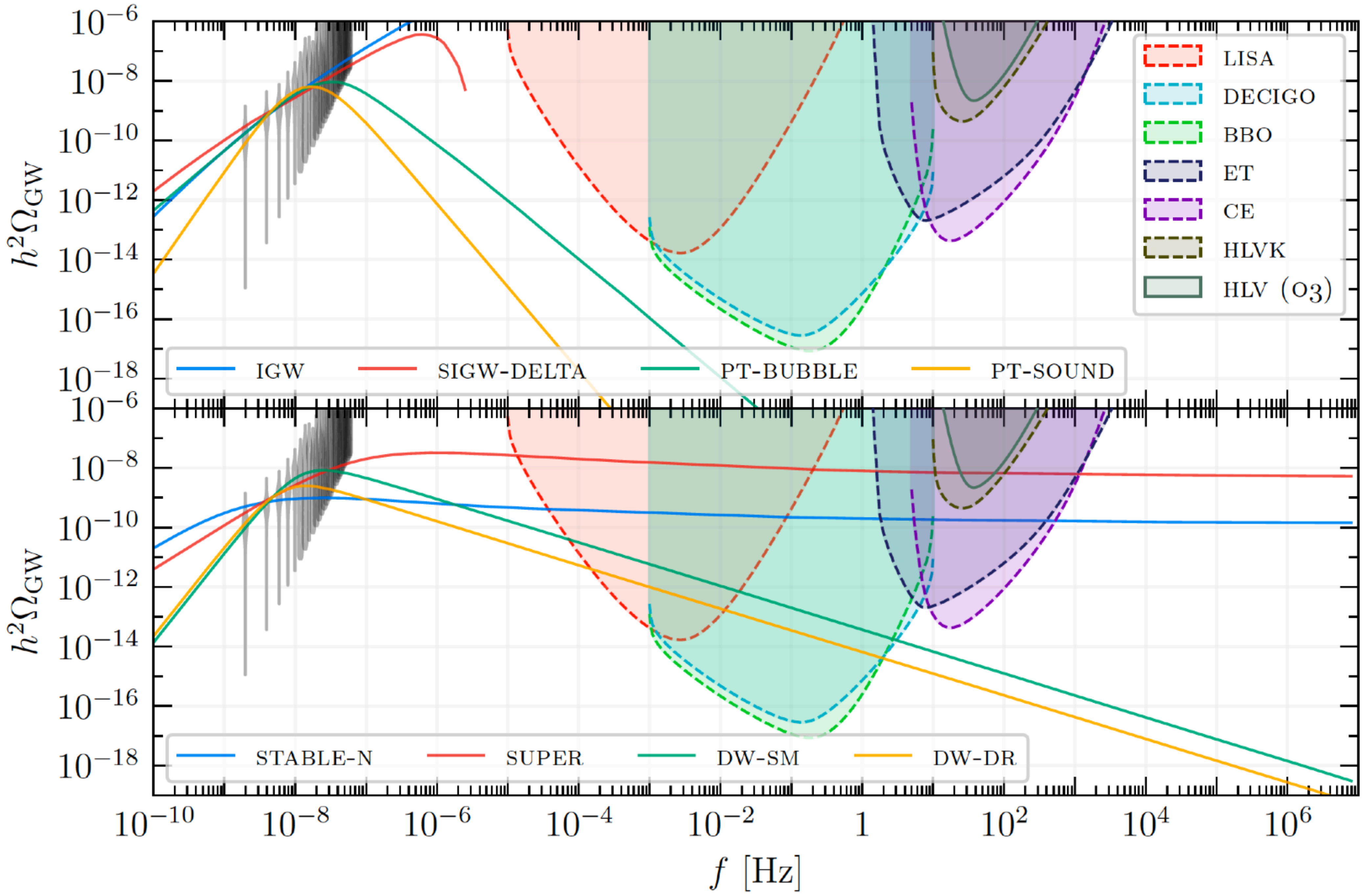


Note on the dark-radiation bound from BBN $\Omega_{\rm GW}h^2 < 1.8\times10^{-6}$.

[Kohri, Terada, 1804.08577]



Scalar-Induced Gravitational Waves

[Ananda, Clarkson, Wands, gr-qc/0612013], [Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290] For reviews, see [Yuan, Huang, 2103.04739], [Domènech, 2109.01398].

What are they?

Gravitational waves induced by (primordial) curvature perturbations via (derivative) interactions in General Relativity.

$$\mathrm{d}s^2 = -\underbrace{a^2(1+2\Phi)}_{\text{Gravitational potential}}\mathrm{d}\eta^2 + a^2\left(\underbrace{(1-2\Psi)}_{\text{Curvature perturbations}}\delta_{ij} + \frac{1}{2}\underbrace{h_{ij}}_{\text{GW (tensor mode)}}\right)\mathrm{d}x^i\mathrm{d}x^j$$

(In the absence of anisotropic stress, $\Phi = \Psi$.)

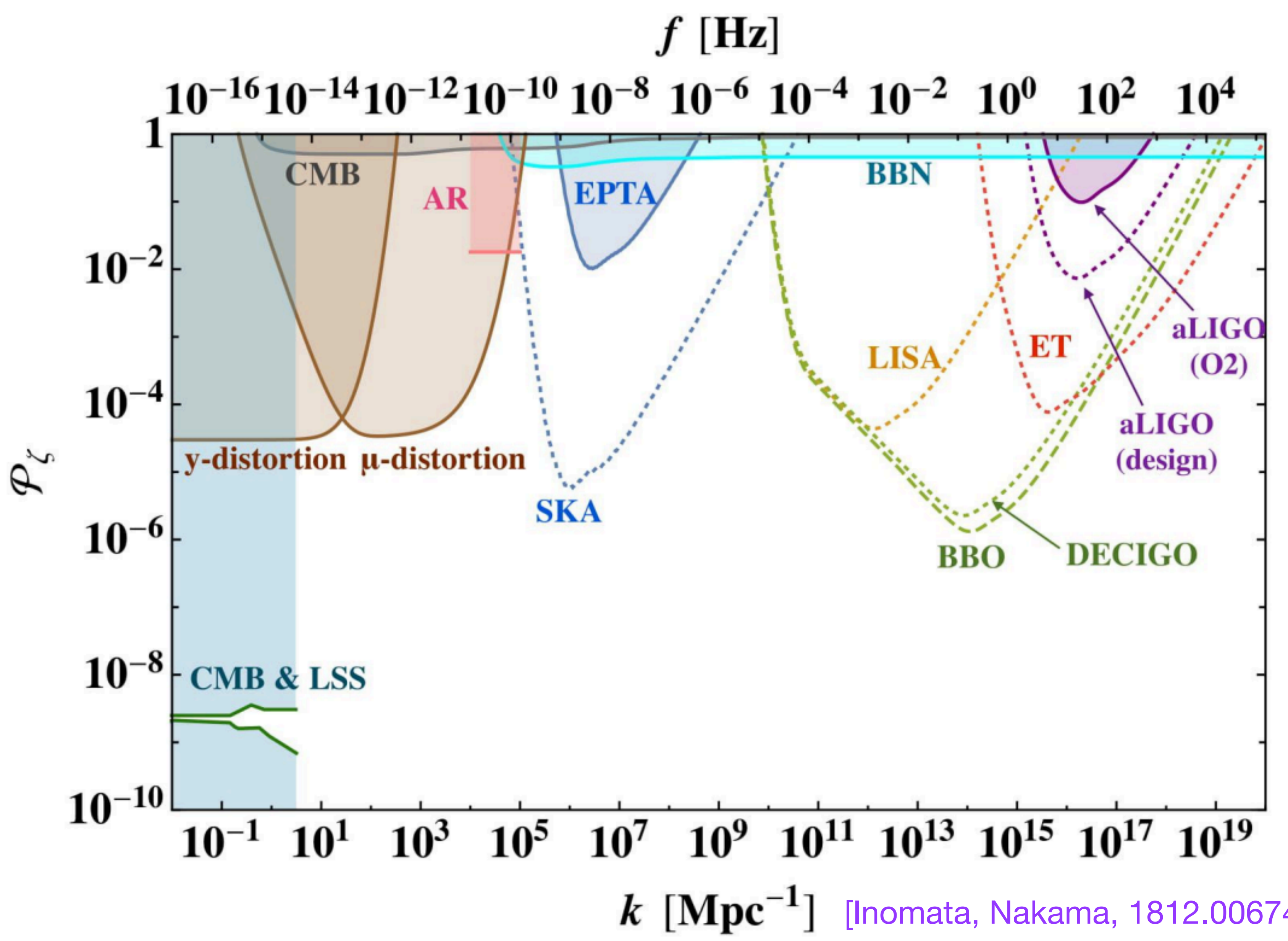
Equation of motion $\quad h''_{\mathbf{k}}(\eta) + 2\mathscr{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$

where $\mathscr{H} = aH$ is the conformal Hubble, and the source term is

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k})q_iq_j\left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}})(\mathscr{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}})\right)$$

Why important?

- They give us some information on small-scale cosmological perturbations and the underlying inflation model.
- They give us some hints on the equation of state and reheating dynamics of the early Universe. See, e.g., [Domènech, 1912.05583], [Inomata, Kohri, Nakama, Terada, 1904.12878; 1904.12879].
- There is also a strong connection to the primordial-black-hole scenario. [Saito, Yokoyama, 0812.4339; 0912.5317]
- They can fit the nHz SGWB found by PTAs!



[Inomata, Nakama, 1812.00674]

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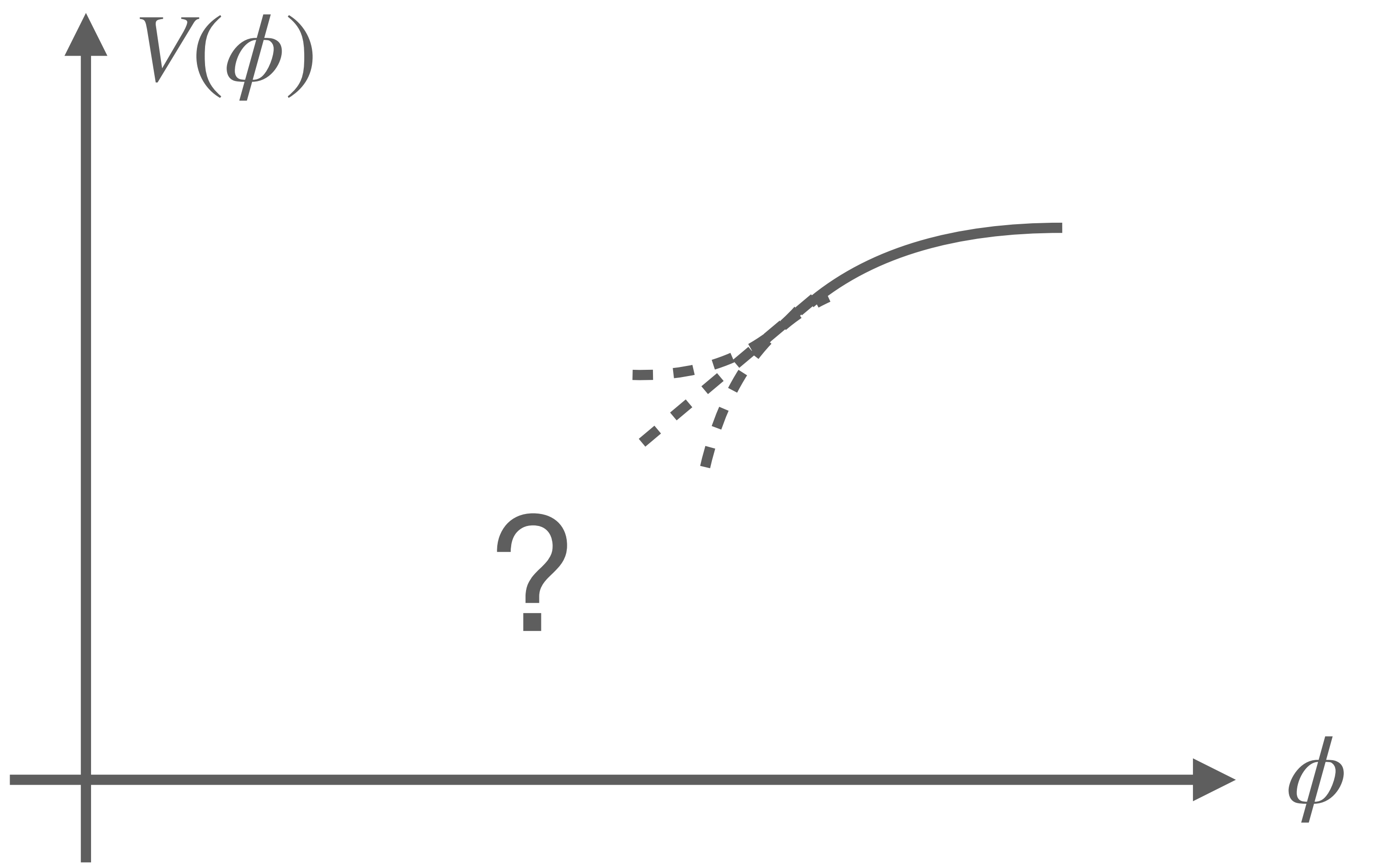
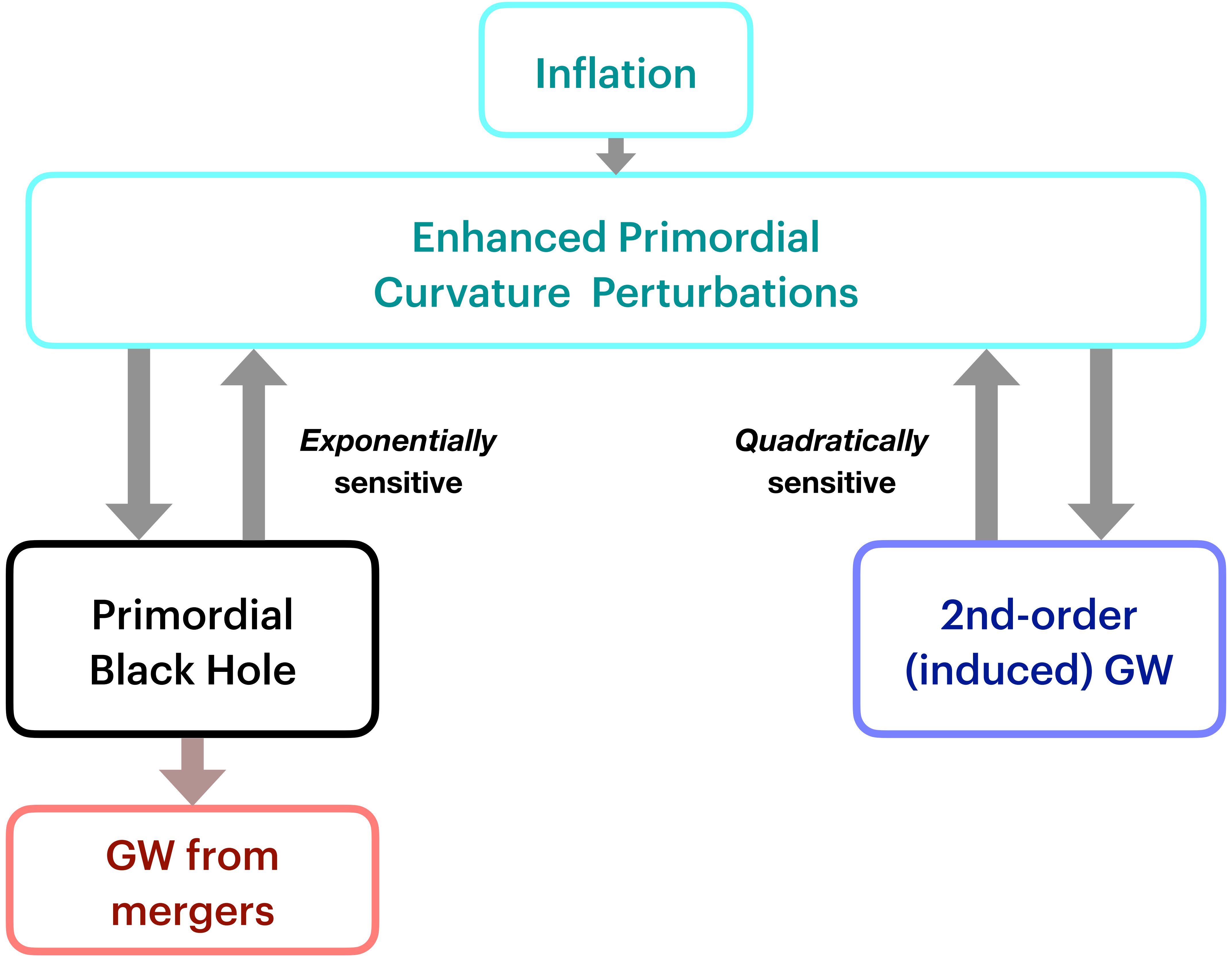


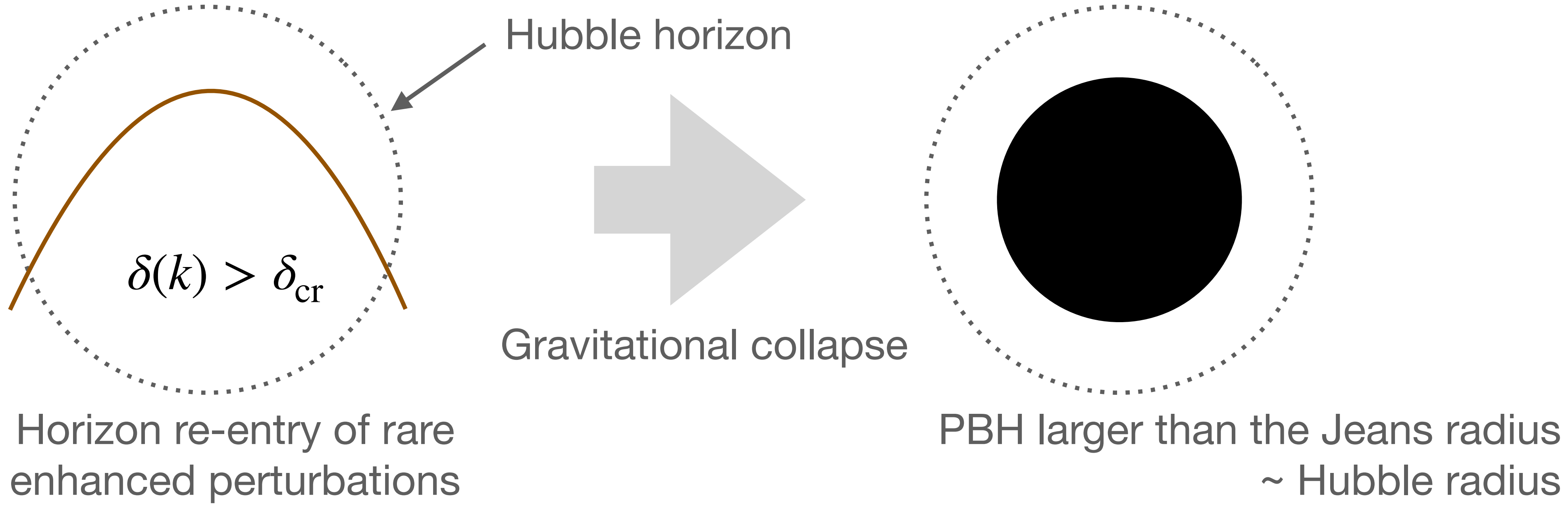
[Inomata, Nakama, 1812.00674]

Relation to Primordial Black Holes



$$M = 6.1 \times 10^{-4} M_\odot \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*(T)}{80}\right)^{-1/6} \left(\frac{6.5 \times 10^{7}\,\mathrm{Mpc}^{-1}}{k}\right)^{2}$$

$$= 6.1 \times 10^{-4} M_\odot \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*(T)}{80}\right)^{-1/6} \left(\frac{1.0 \times 10^{-7}\,\mathrm{Hz}}{f}\right)^{2}$$



Previous studies

[Saito, Yokoyama, 0812.4339; 0912.5317]
[Bugaev, Klimai, 0908.0664]
[Chen, Yuan, Huang, 1910.12239]
…

After NANOGrav-12.5

[Vaskonen, Veermäe, 2009.07832]
[De Luca, Franciolini, Riotto, 2009.08268]
[Kohri, Terada, 2009.11853]
[Zhou, Jiang, Cai, Sasaki, Pi, 2010.03537]
[Domènech, Pi, 2010.03976]
[Inomata, Kawasaki, Mukaida, Yanagida, 2011.01270]
[Dandoy, Domcke, Rompineve, 2302.07901]
…

After June 29
(including works related not to induced GWs but to PBHs)

[Guo, Khlopov, Liu, Wu, Wu, Zhu, 2306.17022]
[Franciolini, Iovino, Vaskonen, Veermäe, 2306.17149]
[Cai, He, Ma, Yan, Yuan, 2306.17822]
[Depta, Schmidt-Hoberg, Tasillo, 2306.17836]
[Inomata, Kohri, Terada, 2306.17834]
[Gouttenoire, Vitagliano, 2306.17841]
[Huang, Cai, Jiang, Zhang, Piao, 2306.17577]
[Wang, Zhao, Li, Zhu, 2307.00572]
[Liu, Chen, Huang, 2307.01102]
[Gouttenoire, Trifinopoulos, Valogiannis, Vanvlasselaer, 2307.01457]
[Jhurani, Gunhal, 2307.02677]
[Unal, Papageorgiou, Obata, 2307.02322]
[Figueroa, Pieroni, Ricciardone, Simakachorn, 2307.02399]
[Zhu, Zhao, Wang, 2307.03095]
[Firouzjahi, Talebian, 2307.03164]
[Gow, Miranda, Nurmi, 2307.03078]

f^2 spectrum from Induced-GW

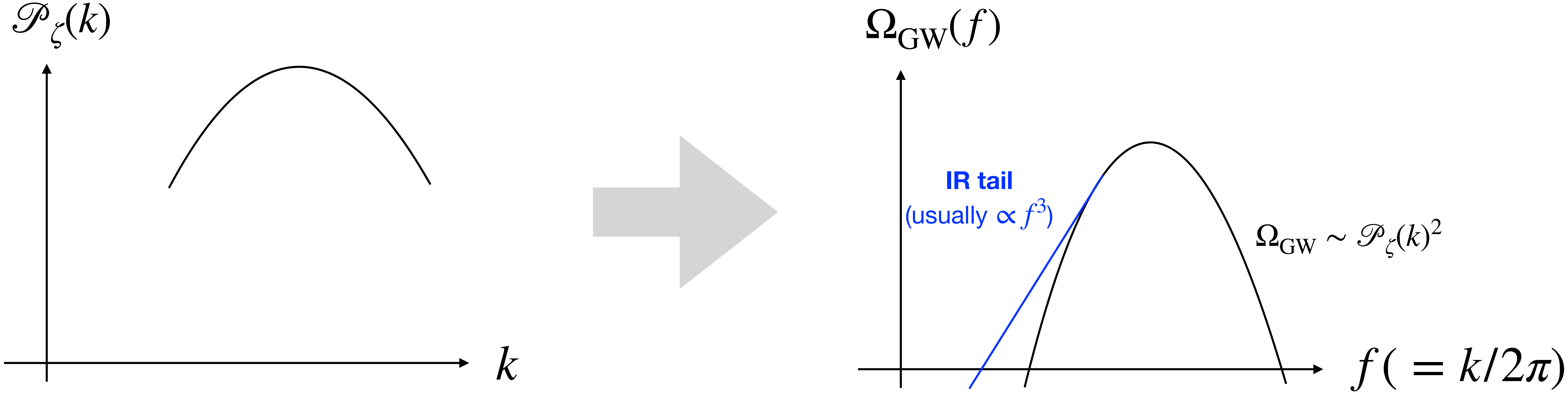
IR tail of the induced GWs

$$\Omega_{\mathrm{GW}}^{\mathrm{ind}}(f) = \Omega_{\mathrm{r}} \left(\frac{g_*(f)}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}(f)}\right)^{4/3} \bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f)$$

$$\bar{\Omega}_{\mathrm{GW}}^{\mathrm{ind}}(f) = \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\, \mathcal{K}(u,v)\, \mathcal{P}_{\mathcal{R}}(uk)\, \mathcal{P}_{\mathcal{R}}(vk)$$

$$\mathcal{K}(u,v) = \frac{3\left(4v^2-(1+v^2-u^2)^2\right)^2 (u^2+v^2-3)^4}{1024\, u^8 v^8}\left[\left(\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right| - \frac{4uv}{u^2+v^2-3}\right)^2 + \pi^2\Theta(u+v-\sqrt{3})\right]$$

[Espinosa, Racco, Riotto, 1804.27732]
[Kohri, Terada, 1804.08577]

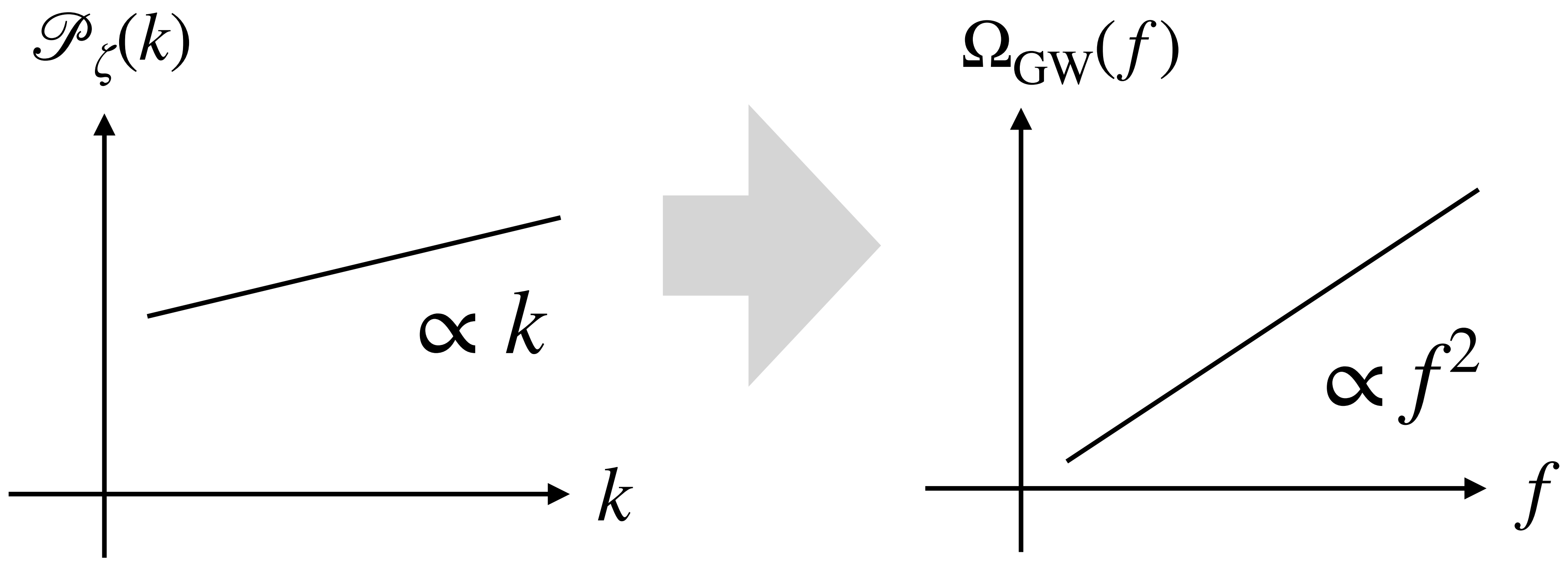


[Cai, Pi, Sasaki, 1909.13728]
[Yuan, Chen, Huang, 1910.09099]
[Domènech, Pi, Sasaki, 2005.12314]

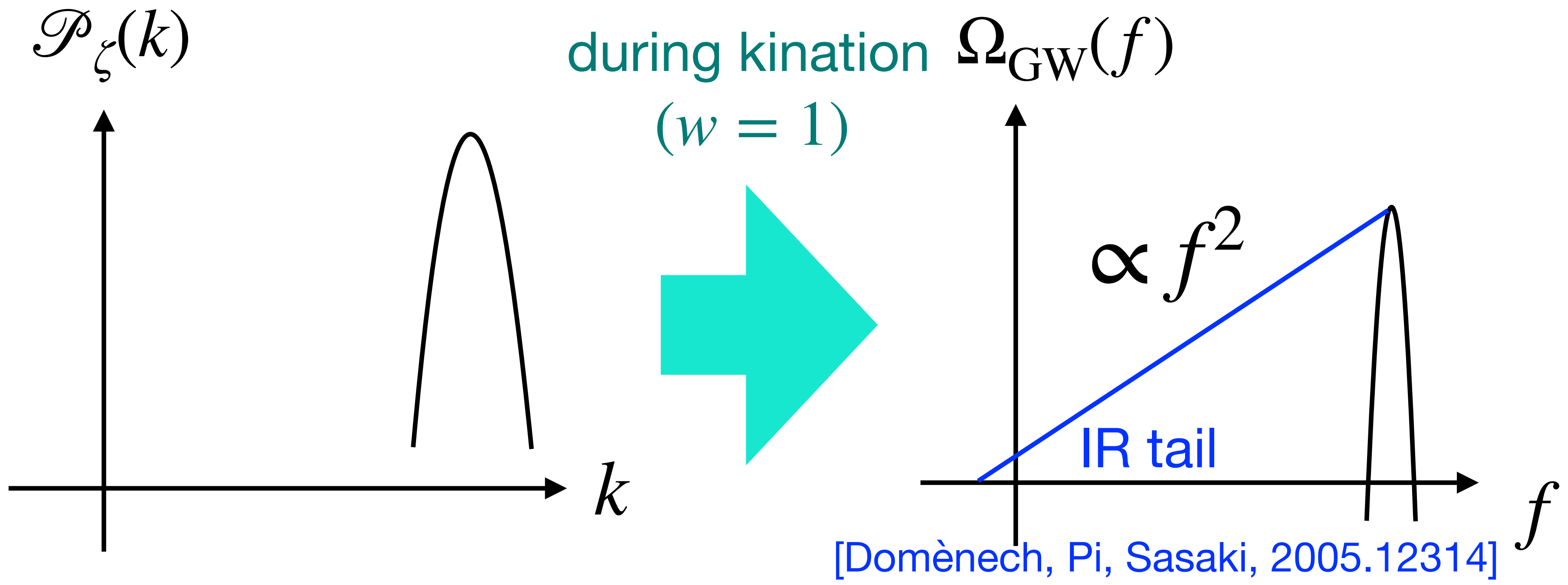
Explaining $\Omega_{\mathrm{GW}} \propto f^2$ Scaling

[Inomata, Kohri, Terada, 2306.17834]

Possibility 1

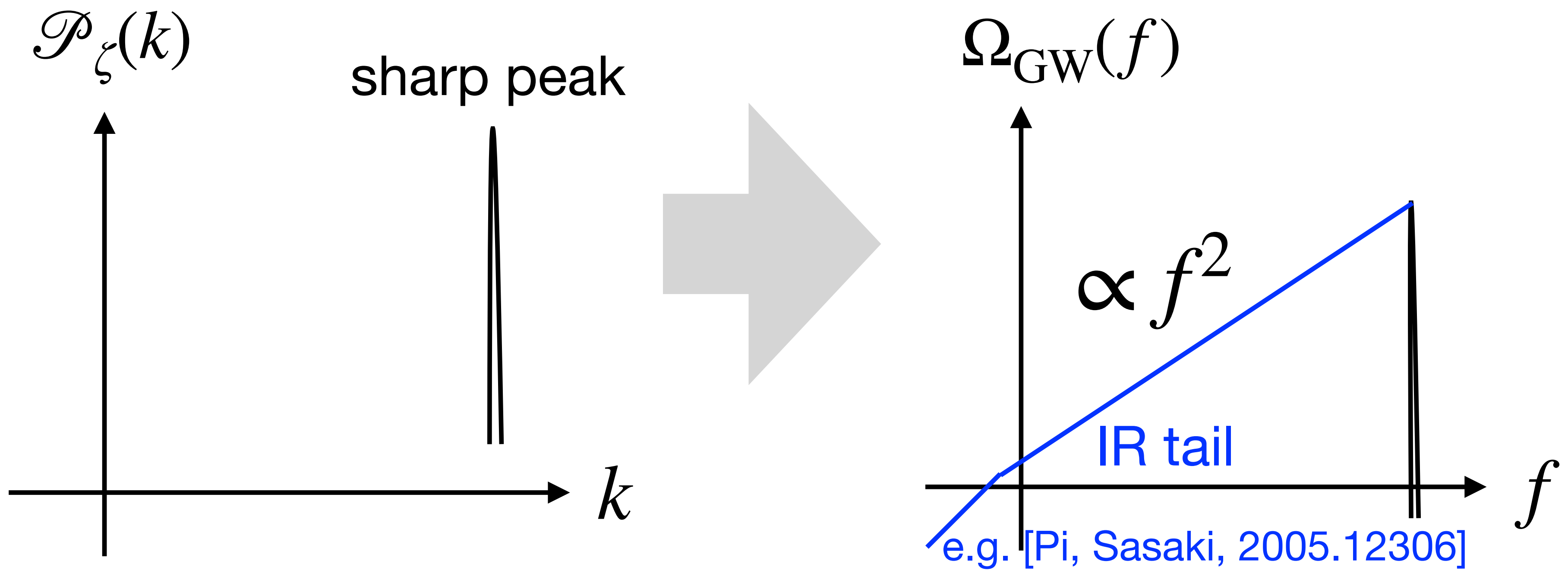


Possibility 2



[Harigaya, Inomata, Terada, 2309.00228]

Possibility 3



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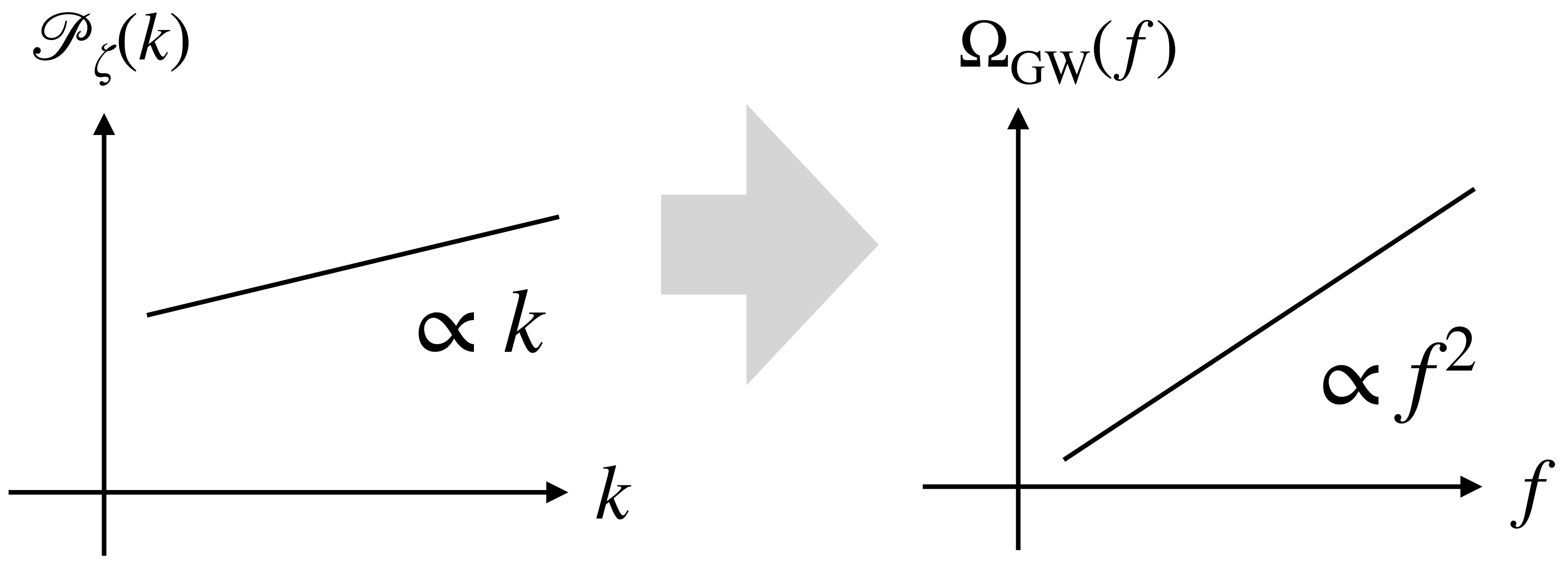
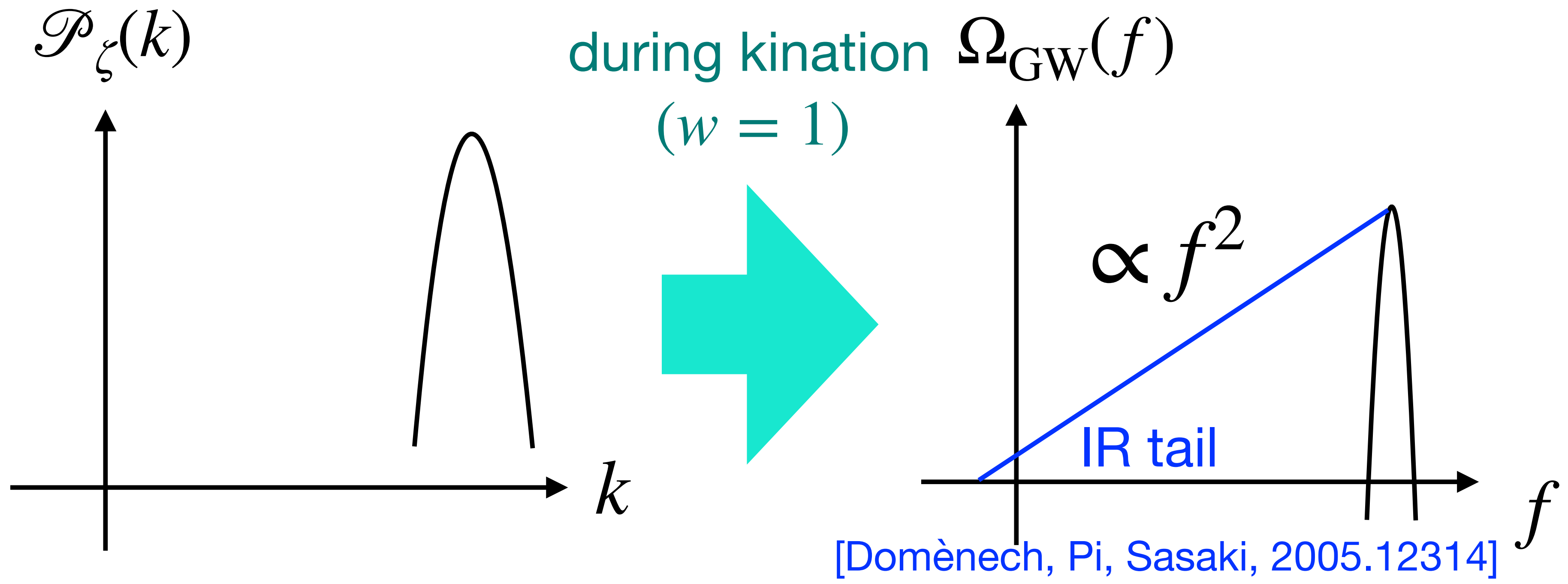


Possibility 2



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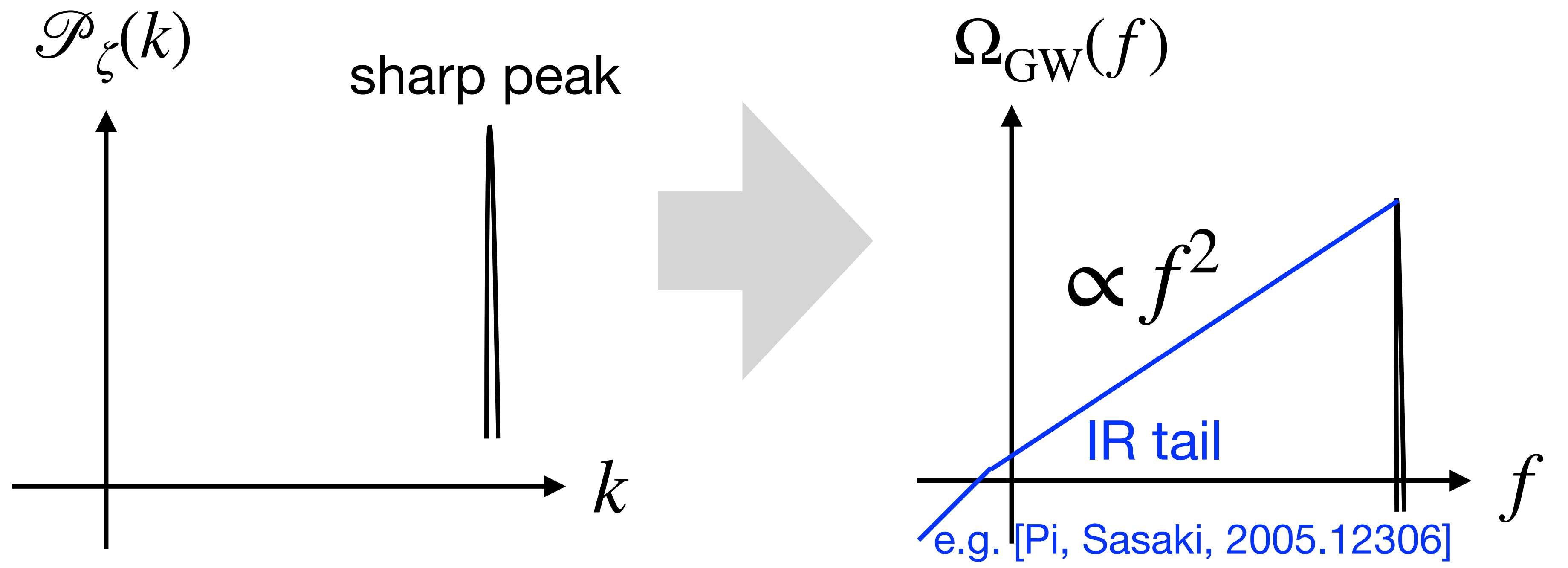

[Inomata, Kohri, Terada, 2306.17834]

The peak is
on a **smaller scale**
than the PTA range.

→ **SUB-solar mass PBHs**

Explaining $\Omega_{\rm GW} \propto f^2$ Scaling

[Inomata, Kohri, Terada, 2306.17834]

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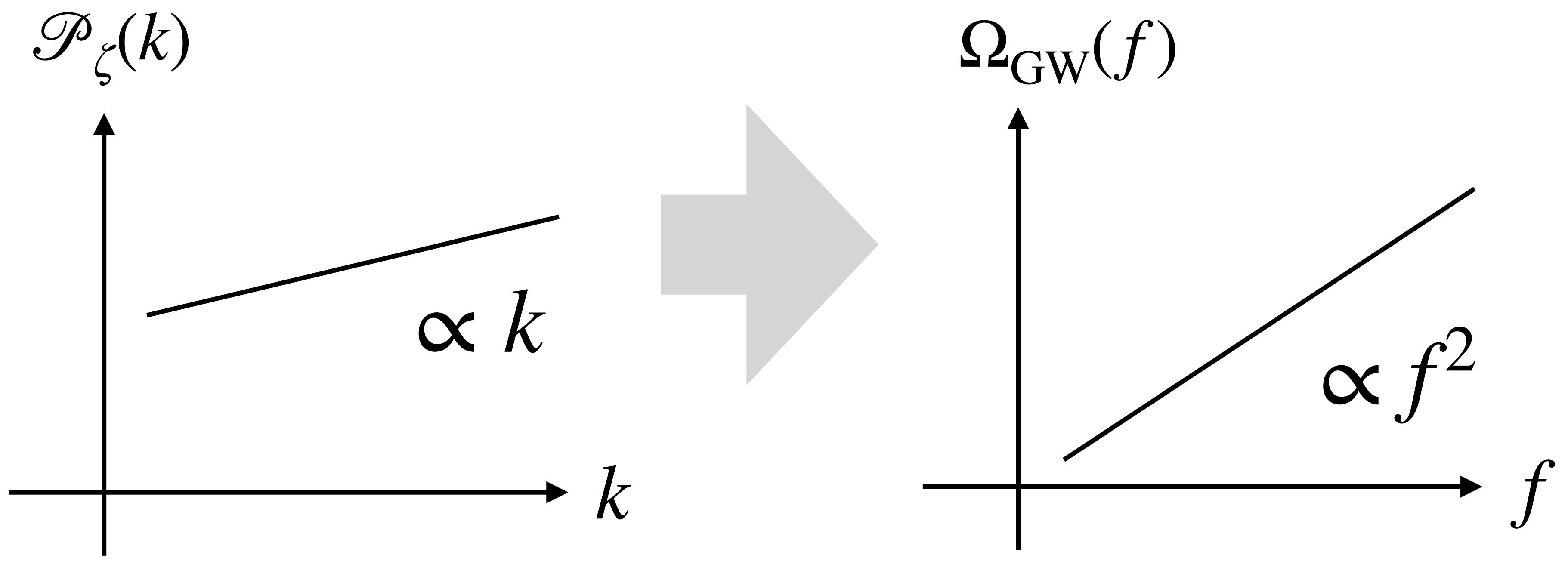

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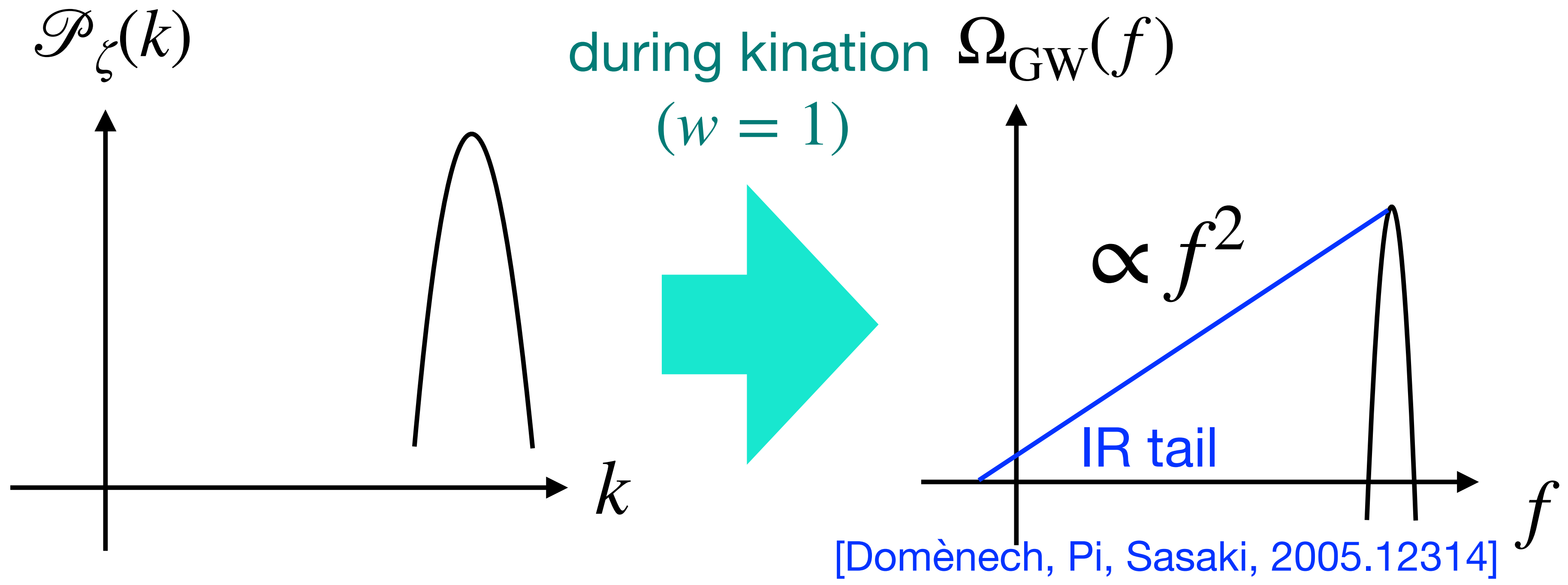

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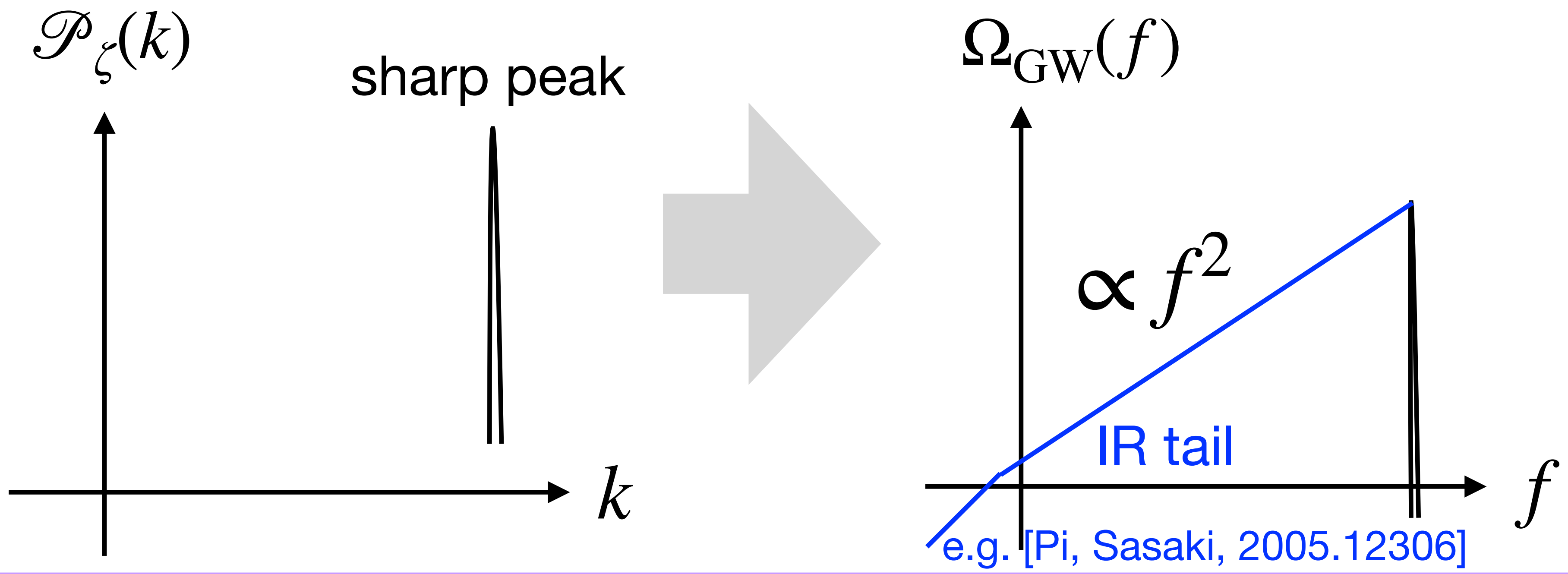

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The peak is on a **smaller scale** than the PTA range.

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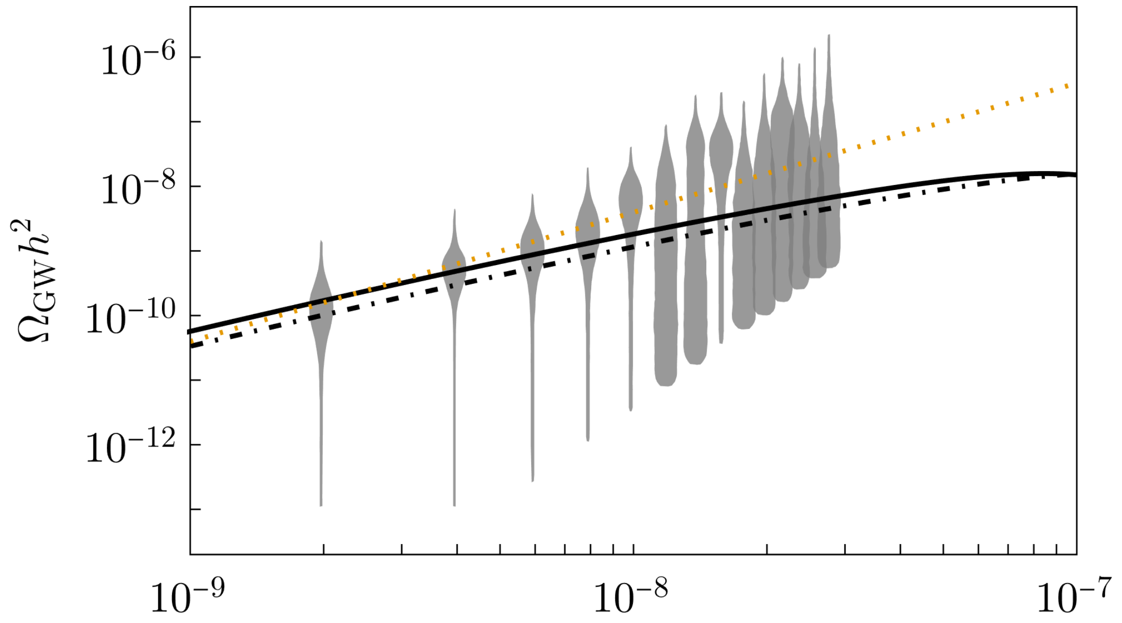
Implications for Primordial Black Holes

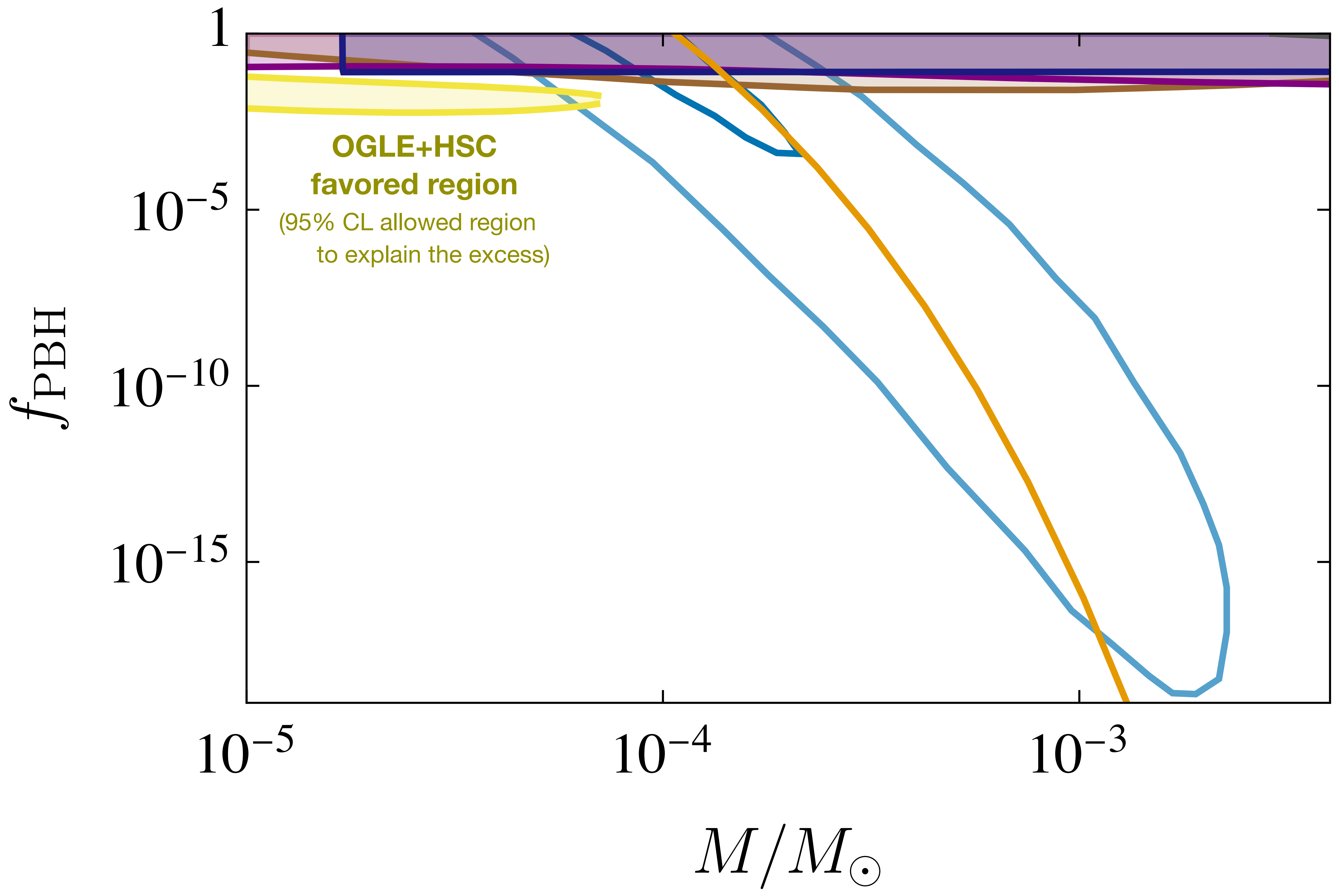
[Inomata, Kohri, Terada, 2306.17834]

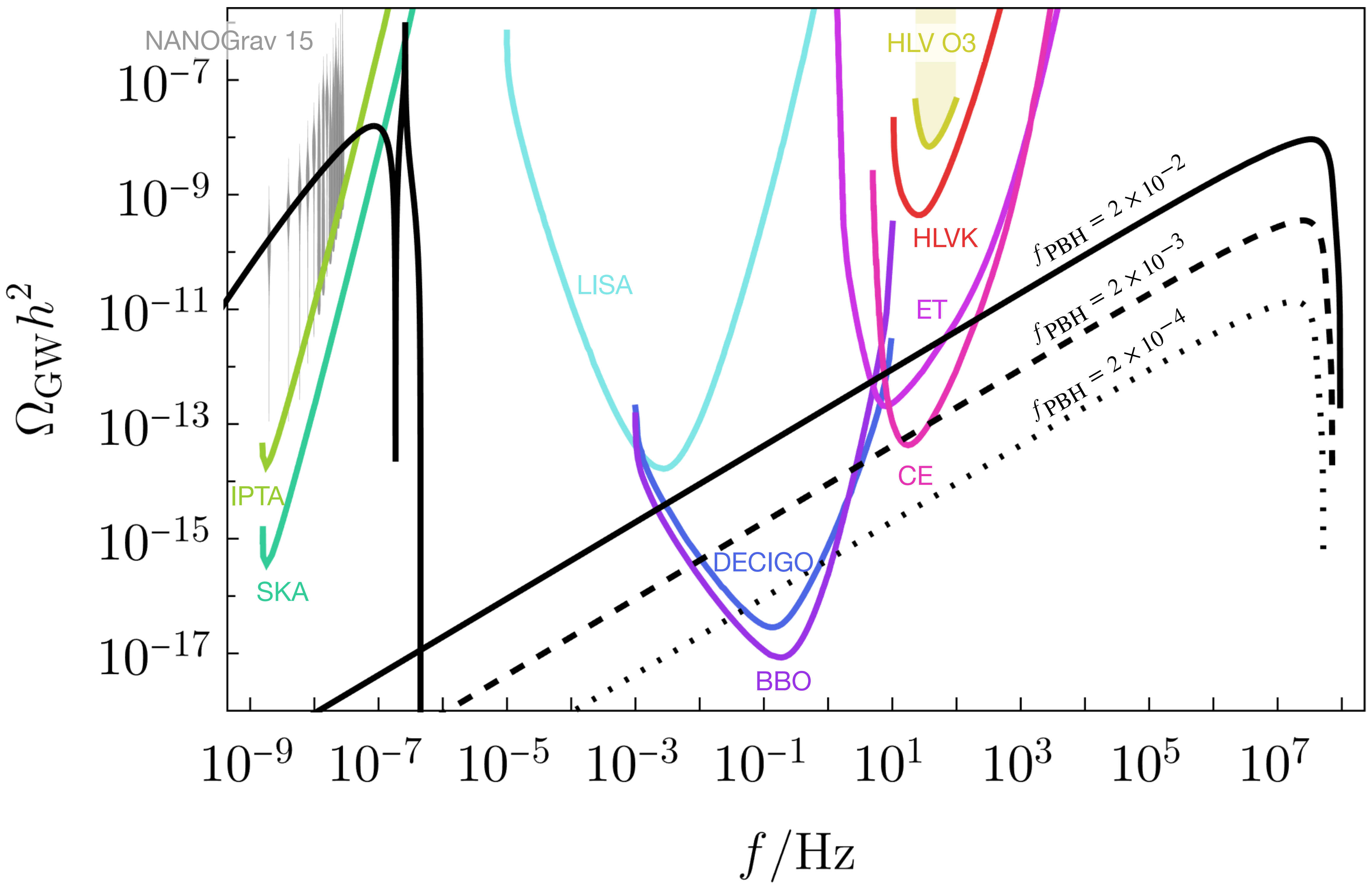
See also [Franciolini et al., 2306.17149]



The excess events of microlensing at OGLE: [Mróz et al., 1707.07634]

Interpretation by PBHs: [Niikura et al., 1901.07120]





$M/M_\odot = 1.2\times10^{-4}, 1.6\times10^{-4}$, and 2.2×10^{-4} from top to bottom.

The sensitivity curves were taken from [Schmitz, 2002.04615].

The HLV O3 constraint is from [Abbott et al. (LIGO-Virgo-KAGRA), 2101.12130].

Explaining $\Omega_{\mathrm{GW}} \propto f^2$ Scaling

[Inomata, Kohri, Terada, 2306.17834]

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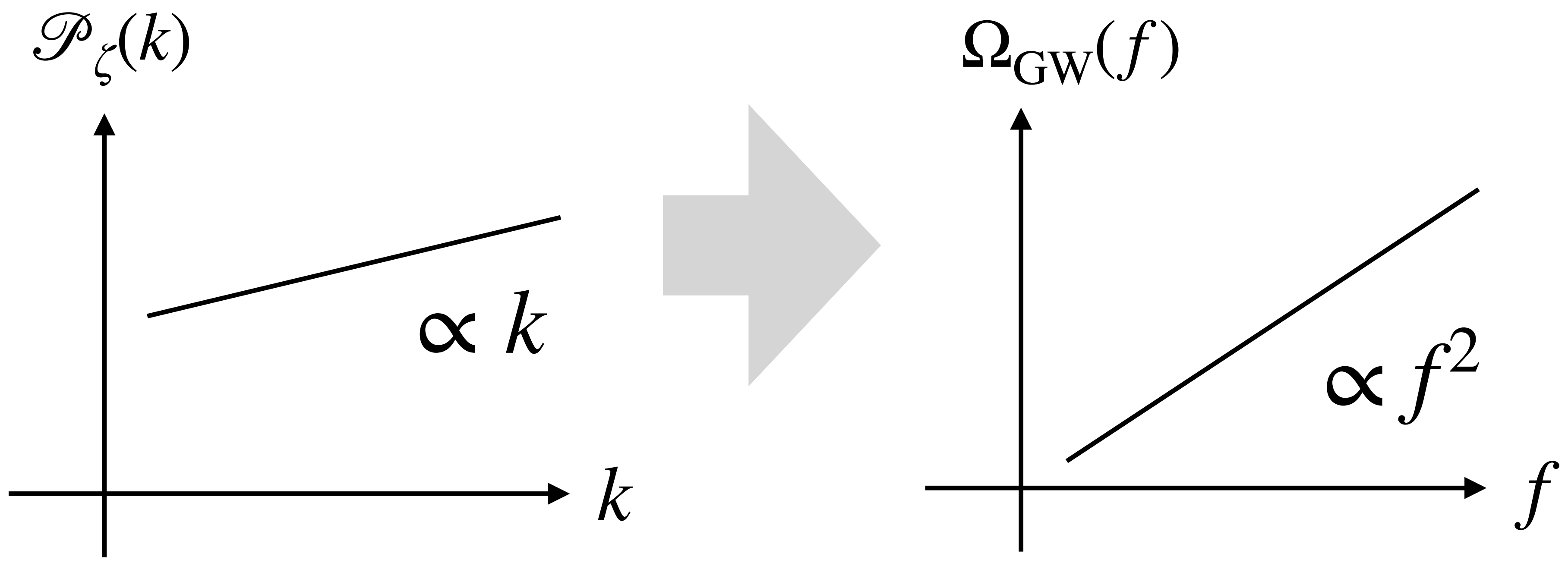

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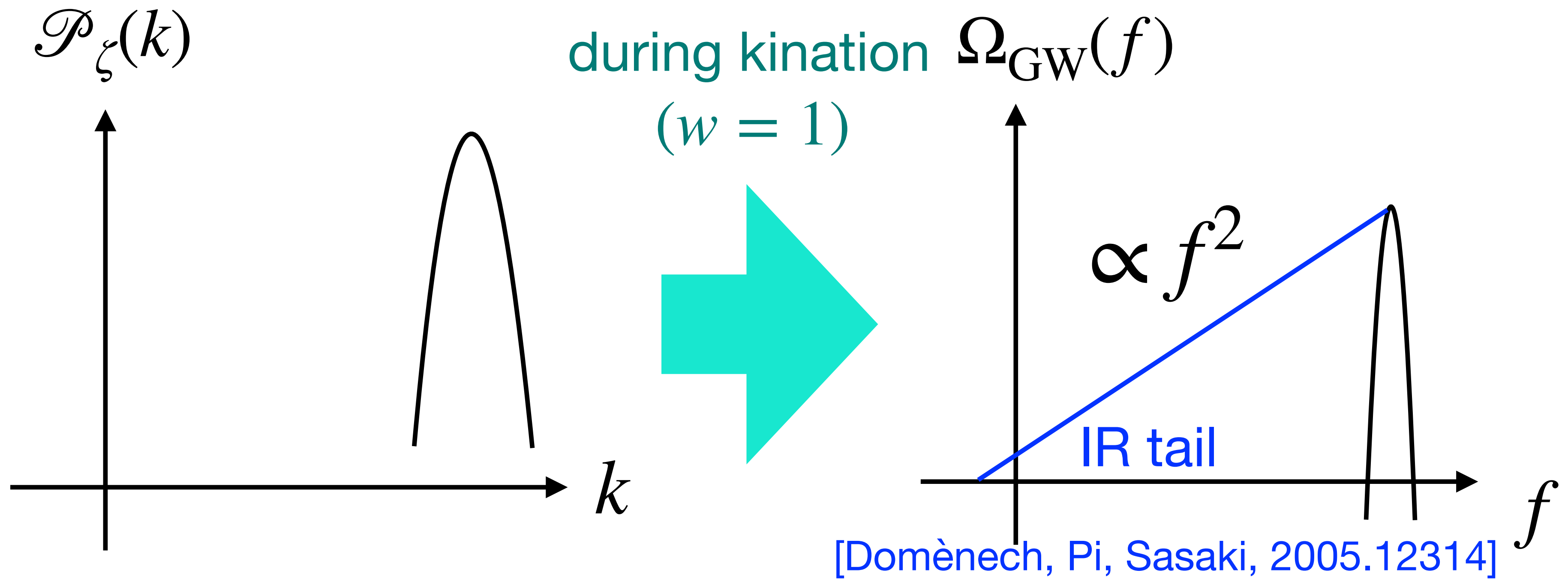

[Harigaya, Inomata, Terada, 2309.00228]

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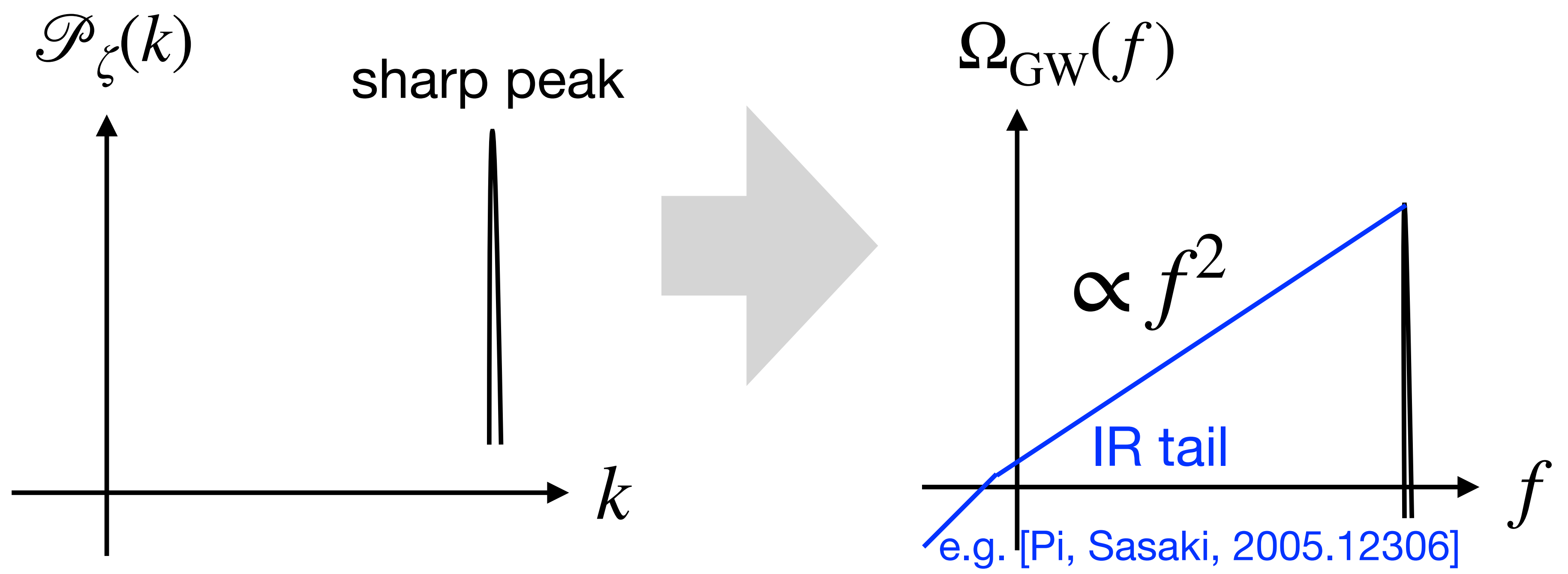

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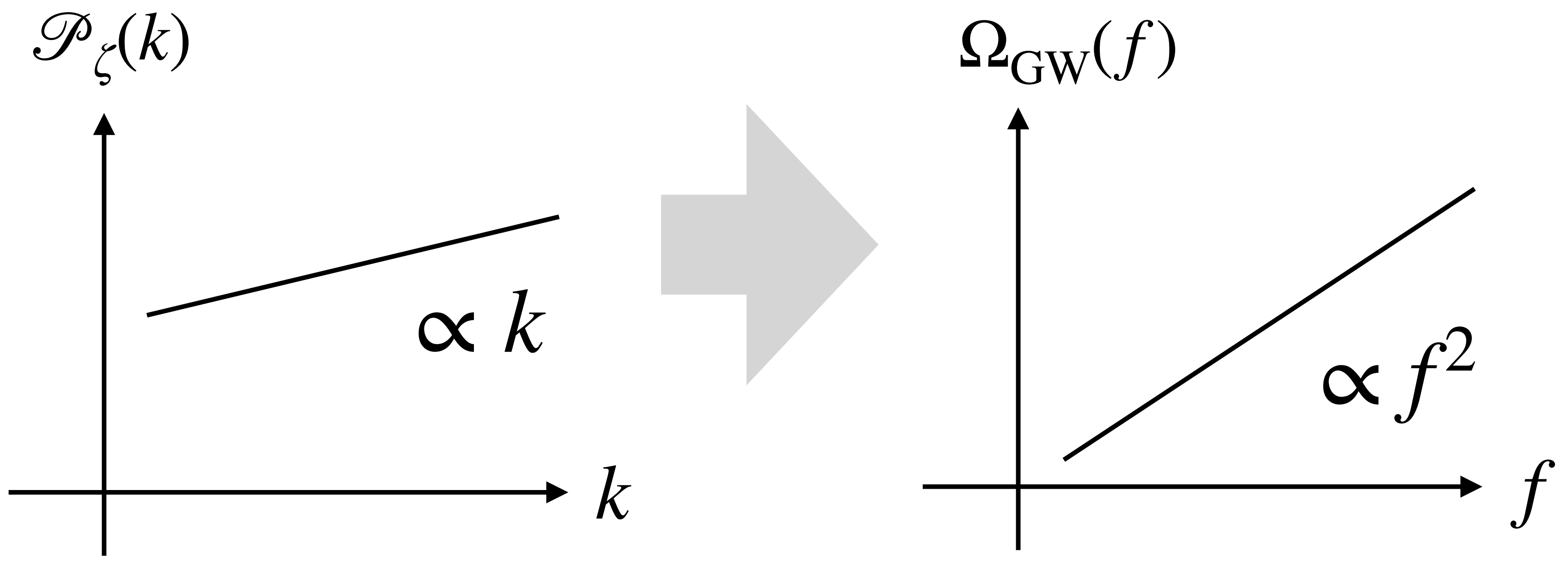

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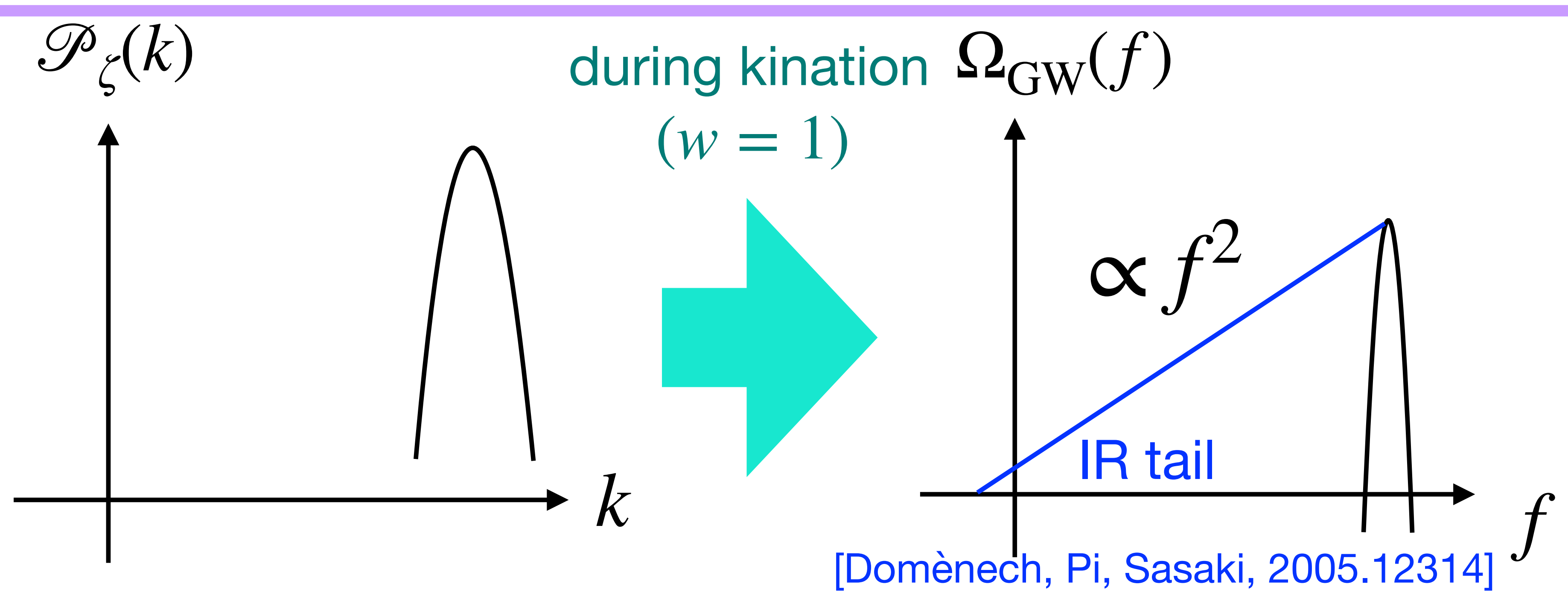

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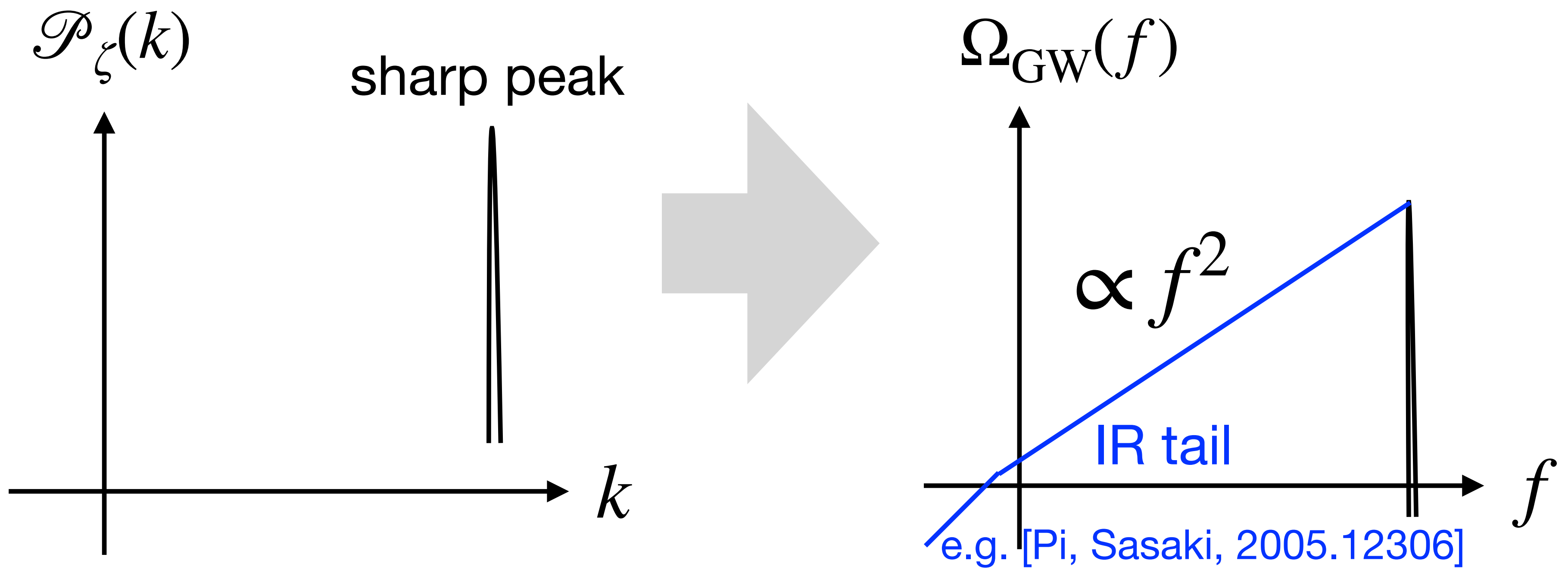


[Inomata, Kohri, Terada, 2306.17834]

f^2 Spectrum in the Kination Scenario

$$w := \frac{P}{\rho} = 1 \quad \rho \propto a^{-6}$$

- Growth factor for superhorizon modes from growing subhorizon density perturbations

  The source term decreasing slower than the Hubble scale

  an additional factor of $\left(\frac{a(k)}{a_{\text{fixed}}}\right)^4 \sim f^{-2}$

- Relative redshift factor for subhorizon modes during kination

  an additional factor of $\left(\frac{a_{\text{fixed}}}{a(k)}\right)^2 \sim f$

During an era with $w = 1$,

$$2\pi f = k = \mathscr{H} \propto a^{-2},$$

$$a \propto \eta^{1/2},$$

η: conformal time

$\mathscr{H}$: conformal Hubble parameter

Multiplying the above factors to the standard one (f^3), we obtain $f^3 \cdot f^{-2} \cdot f = f^2$.

More generally, it nontrivially depends on the equation-of-state parameter w:

$$\Omega_{\text{GW}}(f) \sim f^{3-2\,(1-3w)/(1+3w)}$$

for the IR tail part of the spectrum. [Domènech, Pi, Sasaki, 2005.12314]

Induced GW scenario with kination

$$w := \frac{P}{\rho} = 1 \quad \rho \propto a^{-6}$$

The PBH abundance is exponentially suppressed compared to the standard scenario.

$$f_{\mathrm{PBH}} \equiv \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{DM}}} \sim \exp\left(-\frac{\delta_{\mathrm{c}}^2}{2\mathscr{P}_\zeta(k(M))}\right)$$

1. Smaller curvature perturbation is required to fit the PTA data.

   This is because the GW fraction is enhanced during kination.

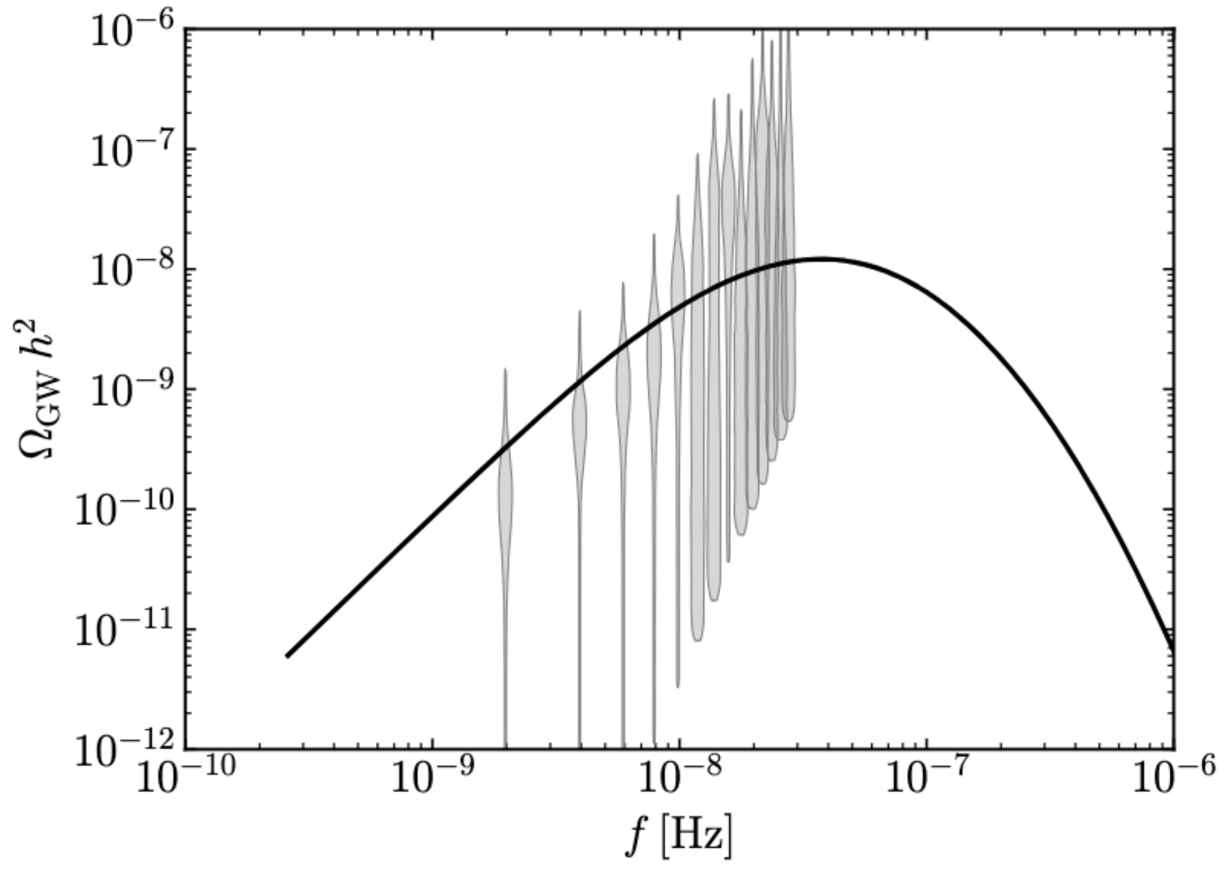
$$\Omega_{\mathrm{GW}} \propto a^2$$

2. It will be harder for a PBH to form during kination.

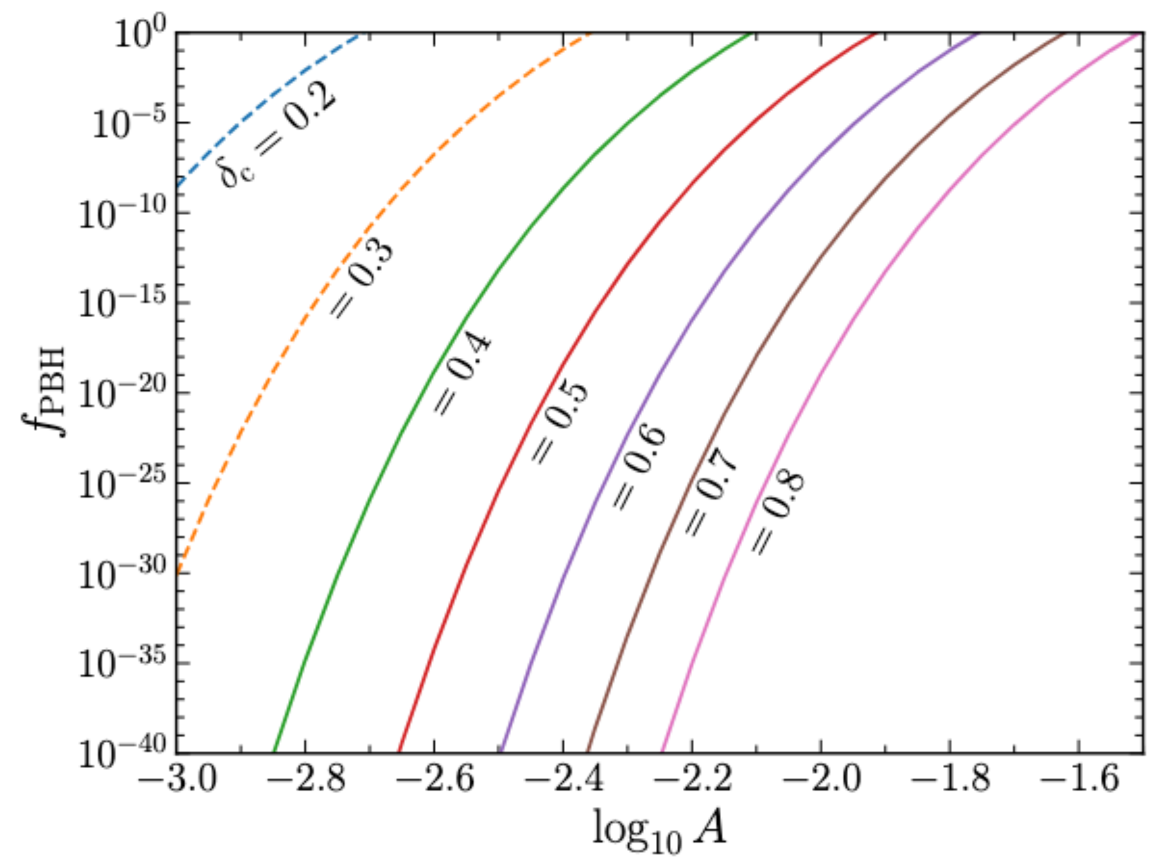
$$\delta_{\mathrm{c}} \approx 0.4 - 0.75$$

See, e.g., [Escrivà et al., 2007.05564] and references therein.

Example GW spectrum



PBH abundance



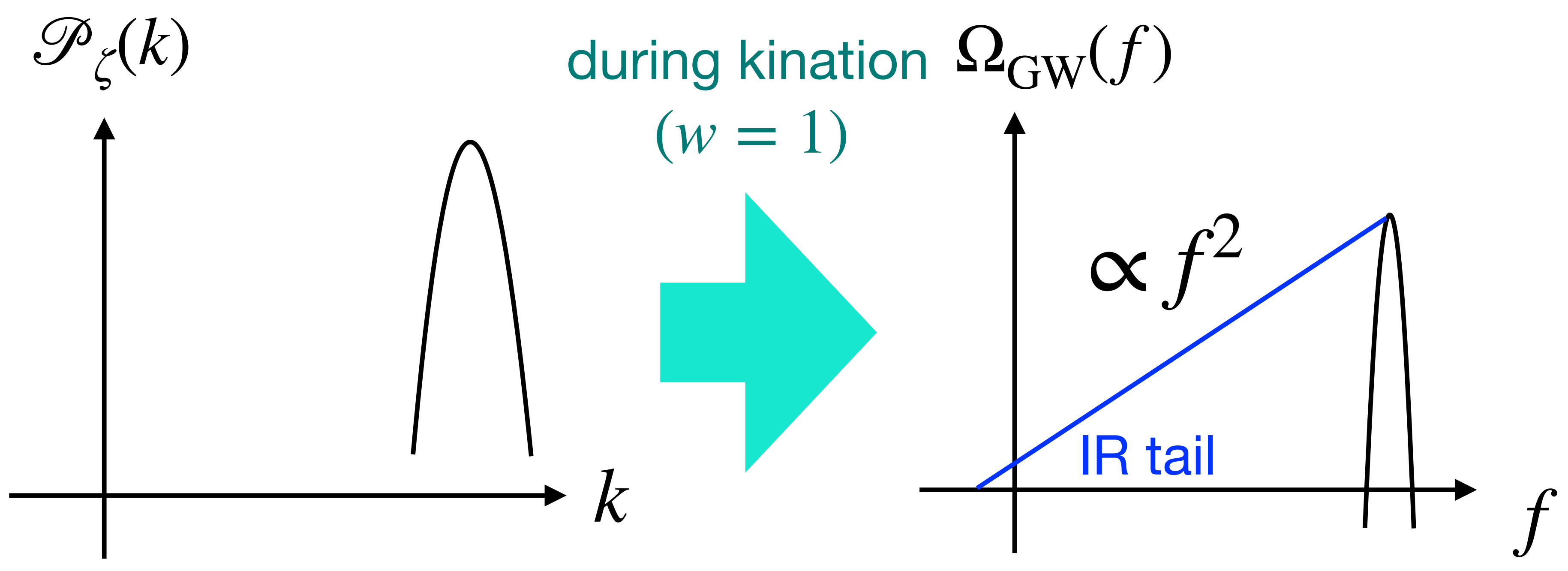
The PTA data can be fit without PBH overproduction.

[Harigaya, Inomata, Terada, 2309.00228]

See also [Balaji et al., 2307.08552] for a similar scenario.

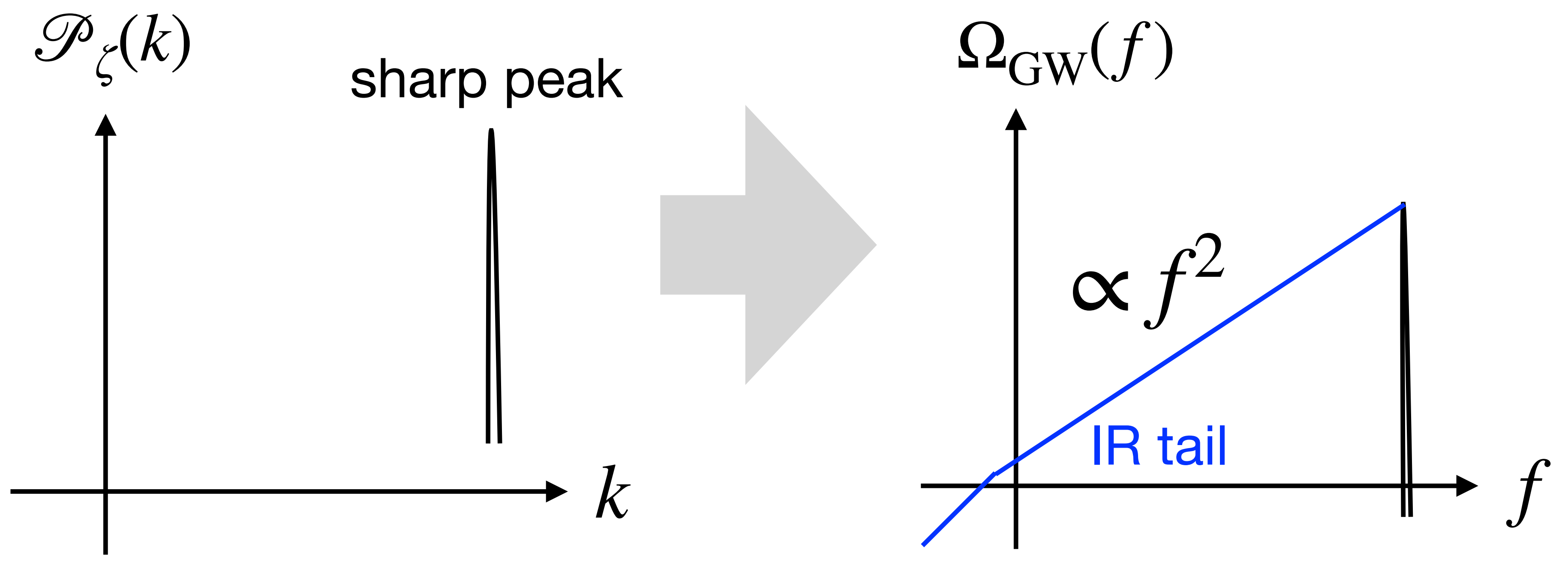
Summary and Conclusion

The PTA data may be indicating $\Omega_{\rm GW} \propto f^2$ spectrum, which can be interpreted in terms of (the IR tail of) the scalar-induced GWs.



[Harigaya, Inomata, Terada, 2309.00228]

- Fitting the PTA data well.
- No PBH overproduction.



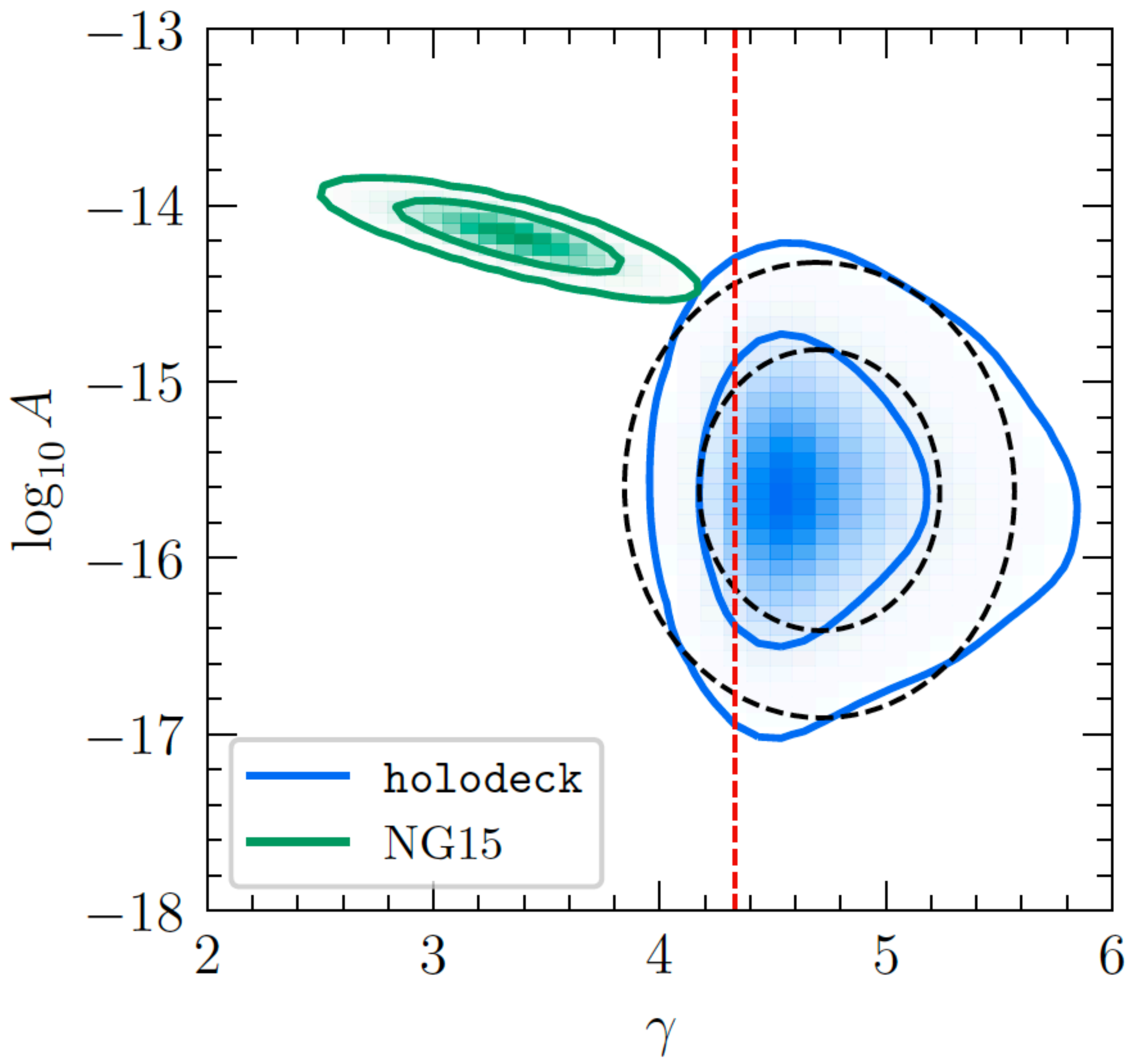
[Inomata, Kohri, Terada, 2306.17834]

- Fitting the PTA data well.
- Associated with $\mathcal{O}(10^{-4})\, M_\odot$ PBHs.
  - Their binary mergers lead to additional GW signals.
  - Small parameter region explaining microlensing data too.

Appendix

Astrophysical Interpretation

Supermassive Black Hole Binary Mergers



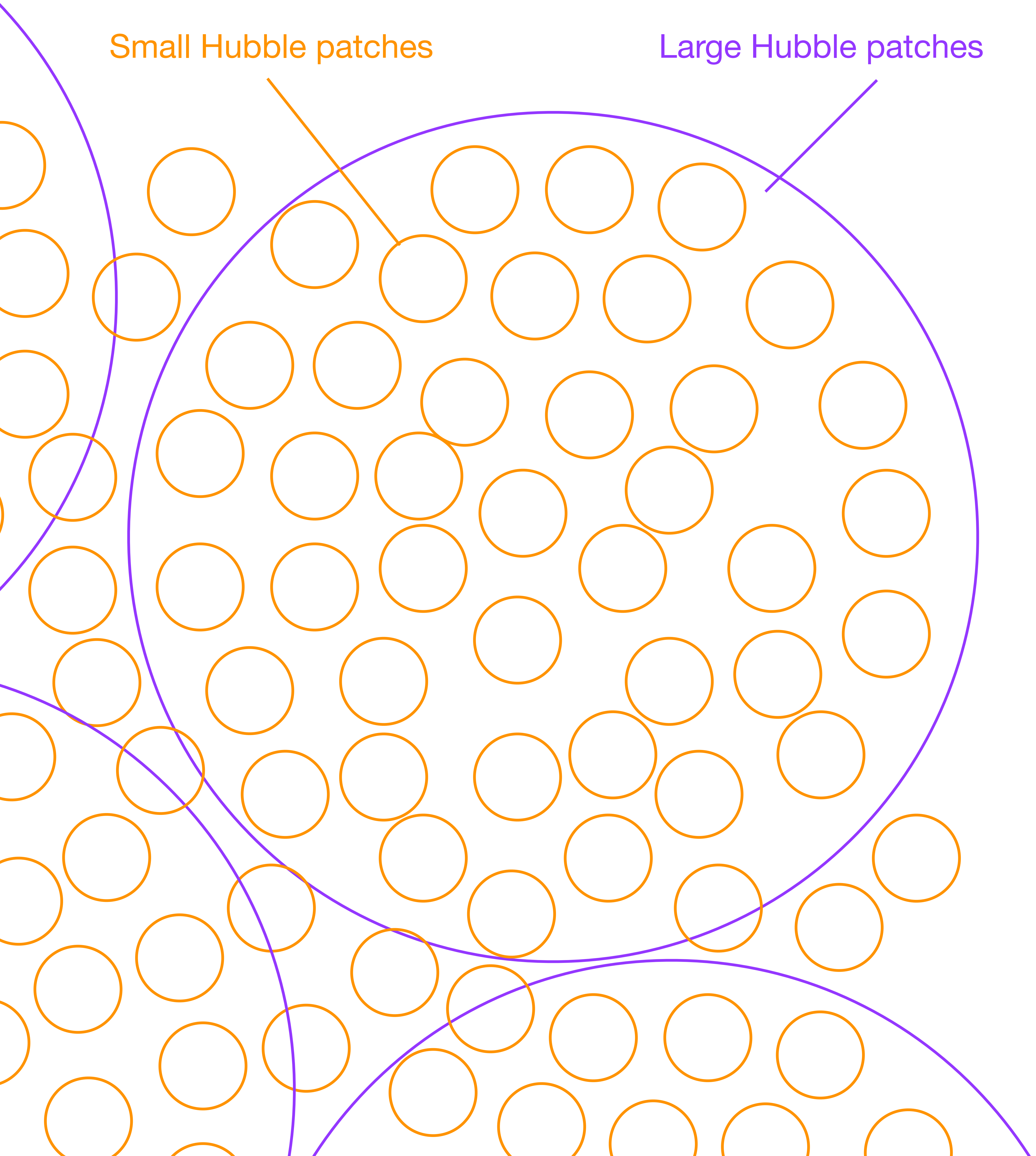
The simplest model doesn't work well.

- Circular orbit
- Energy loss only due to GW emission

Interactions with the environment are important.

Universal Infrared f^3 scaling

[Cai, Pi, Sasaki, 1909.13728]
[Hook, Marques-Tavares, Racco, 2010.03568]



- Finite duration of GW generation on subhorizon scales

Central Limit Theorem $\mathscr{P}_h(k_L) \propto \frac{1}{N_{\rm patch}} = \left(\frac{k_L}{k_S}\right)^3$

- Radiation-dominated era

no further redshift factors $\Omega_{\rm GW}(f) \propto f^3$

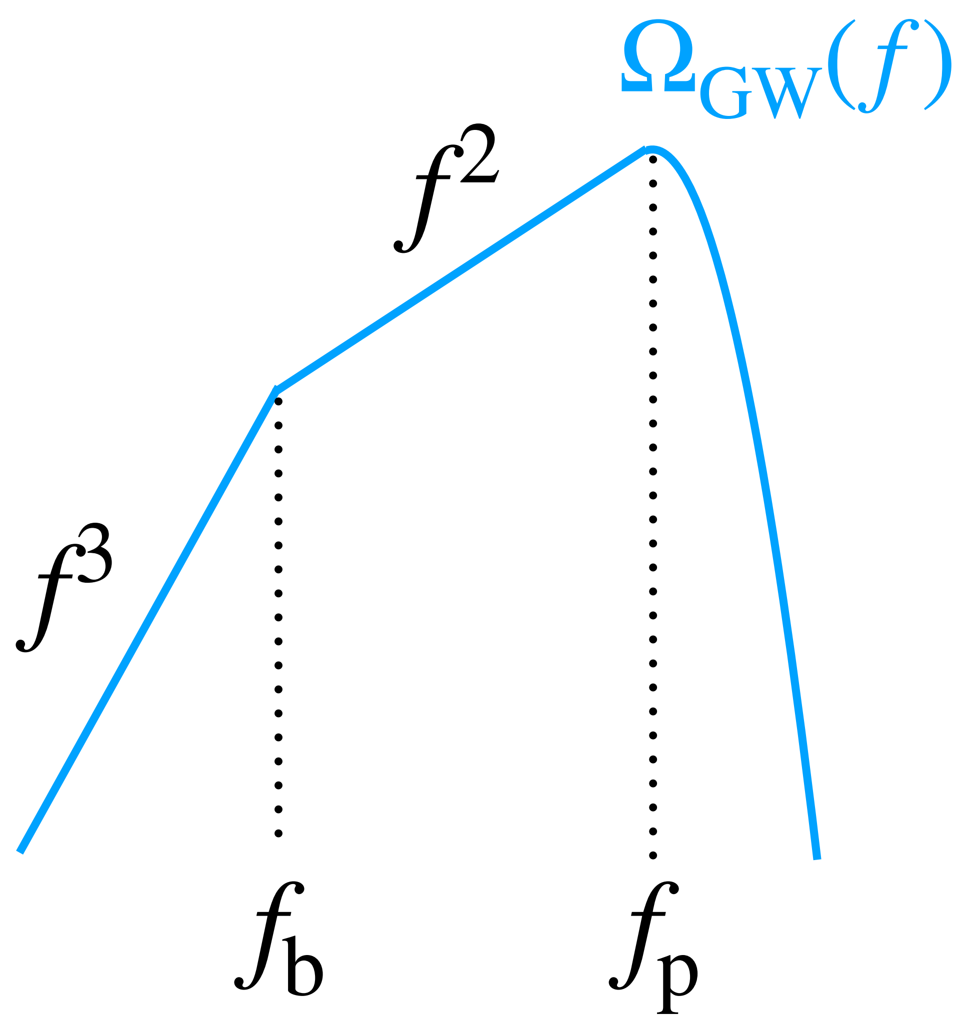
f^2 Spectrum from a sharp peak

An analysis for the lognormal curvature perturbations in [Pi, Sasaki, 2005.12306] is useful.

$$\mathcal{P}_\zeta = \frac{A_\zeta}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2} \right)$$

- For a narrow peak: $\Delta \ll 1$

  The range of the f^2 part is controlled by Δ.



$$f_\mathrm{p} = (2/\sqrt{3}) \times 2\pi k_*$$

$$f_\mathrm{b} \approx \sqrt{3}\Delta f_\mathrm{p}$$

- For a broad peak: $\Delta \gg 1$

  No f^2 tail. Ω_{GW} has a lognormal peak with a width $\Delta/\sqrt{2}$.

Our Recipe for a PBH (in [Inomata, Kohri, Terada, 2306.17834])

We have basically followed the recipe in the NANOGrav-15 paper [Afzal et al. (NANOGrav), 2306.16219], which is relatively simple.

- Carr's formula (a.k.a. the Press-Schechter formalism)
- Critical density $\delta_c = 0.45$
- The ratio between the PBH mass and the horizon mass $\gamma = 0.2$
- The relativistic degrees of freedom $g_* = g_{*,s} = 80$
- The Gaussian window function $W(k) = \exp(-k^2/2)$
- Including the transfer function of the density perturbations
- The nonlinear relation between the curvature and density perturbations has been neglected.
- We have not adopted the effects of the critical collapse.

Studies by other groups

The effects of non-Gaussianity were studied in

[Franciolini, Iovino, Vaskonen, Veermäe, 2306.17149]
[Wang, Zhao, Li, Zhu, 2307.00572]
[Liu, Chen, Huang, 2307.01102]

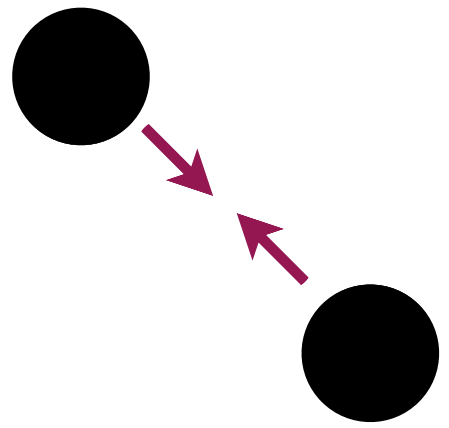
PBH overproduction was reported (except from Wang et al.).

The effects of softening w and/or c_s were studied in

[De Luca, Franciolini, Riotto, 2009.08268]
See also [Franciolini, Racco, Rompineve, 2306.17136], [Abe, Tada, 2307.01653]

GWs from Binary PBH Mergers

Binary formation in the radiation era



[Nakamura, Sasaki, Tanaka, Thorne, 1997]
[Sasaki, Suyama, Tanaka, Yokoyama, 1603.08338]

comoving merger rate

[Sasaki, Suyama, Tanaka, Yokoyama, 1603.08338; 1801.05235]

$$R(z) = \left(\frac{f_{\rm PBH}\Omega_{\rm CDM}\rho_{\rm c}}{M}\right)\frac{\mathrm{d}P_t}{\mathrm{d}t}$$

$$\frac{\mathrm{d}P_t}{\mathrm{d}t} = \begin{cases} \frac{3}{58}\left[-\left(\frac{t}{t_0}\right)^{\frac{3}{8}} + \left(\frac{t}{t_0}\right)^{\frac{3}{37}}\right]\frac{1}{t} & \text{for } t < t_c \\ \frac{3}{58}\left(\frac{t}{t_0}\right)^{\frac{3}{8}}\left[-1 + \left(\frac{t}{t_c}\right)^{-\frac{29}{56}}\left(\frac{4\pi}{3}f_{\rm PBH}\right)^{-\frac{29}{8}}\right]\frac{1}{t} & \text{for } t \geq t_c, \end{cases}$$

$$t_0 = (3/170)\left\{\bar{x}^4/\left[(GM)^3(4\pi f_{\rm PBH}/3)^4\right]\right\}$$

$$t_c = t_0(4\pi f_{\rm PBH}/3)^{37/3}$$

$$\bar{x} = \left[3M/(4\pi\rho_{\rm PBH,eq})\right]^{1/3}$$

$$\Omega_{\rm GW}^{\rm merger}(f) = \frac{f}{3H_0^2}\int_0^{\frac{f_{\rm cut}}{f}-1}\mathrm{d}z\,\frac{R(z)}{(1+z)H(z)}\frac{\mathrm{d}E_{\rm GW}}{\mathrm{d}f_{\rm s}}$$

Energy spectrum at the source frame

[Ajith et al., 0710.2335] [Ajith et al., 0909.2867]

$$\frac{\mathrm{d}E}{\mathrm{d}f_{\rm s}} = \frac{(G\pi)^{2/3}M_c^{5/3}}{3}\begin{cases} f_{\rm s}^{-1/3} & \text{for } f_{\rm s} < f_1 \\ w_1 f_{\rm s}^{2/3} & \text{for } f_1 \leq f_{\rm s} < f_2 \\ w_2\frac{\sigma^4 f_{\rm s}^2}{\left(\sigma^2 + 4(f_{\rm s}-f_2)^2\right)^2} & \text{for } f_2 \leq f_{\rm s} \leq f_3 \\ 0 & \text{for } f_{\rm s} > f_3 \end{cases}$$

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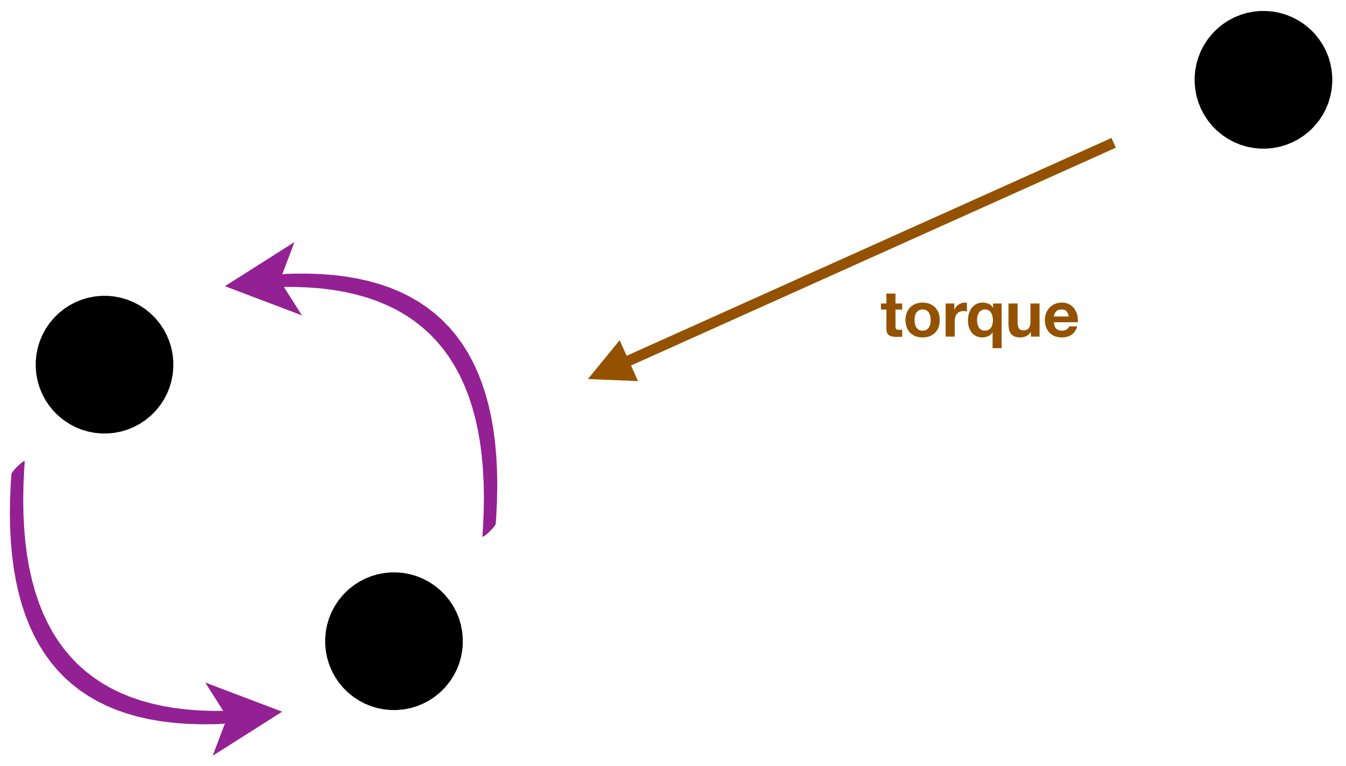

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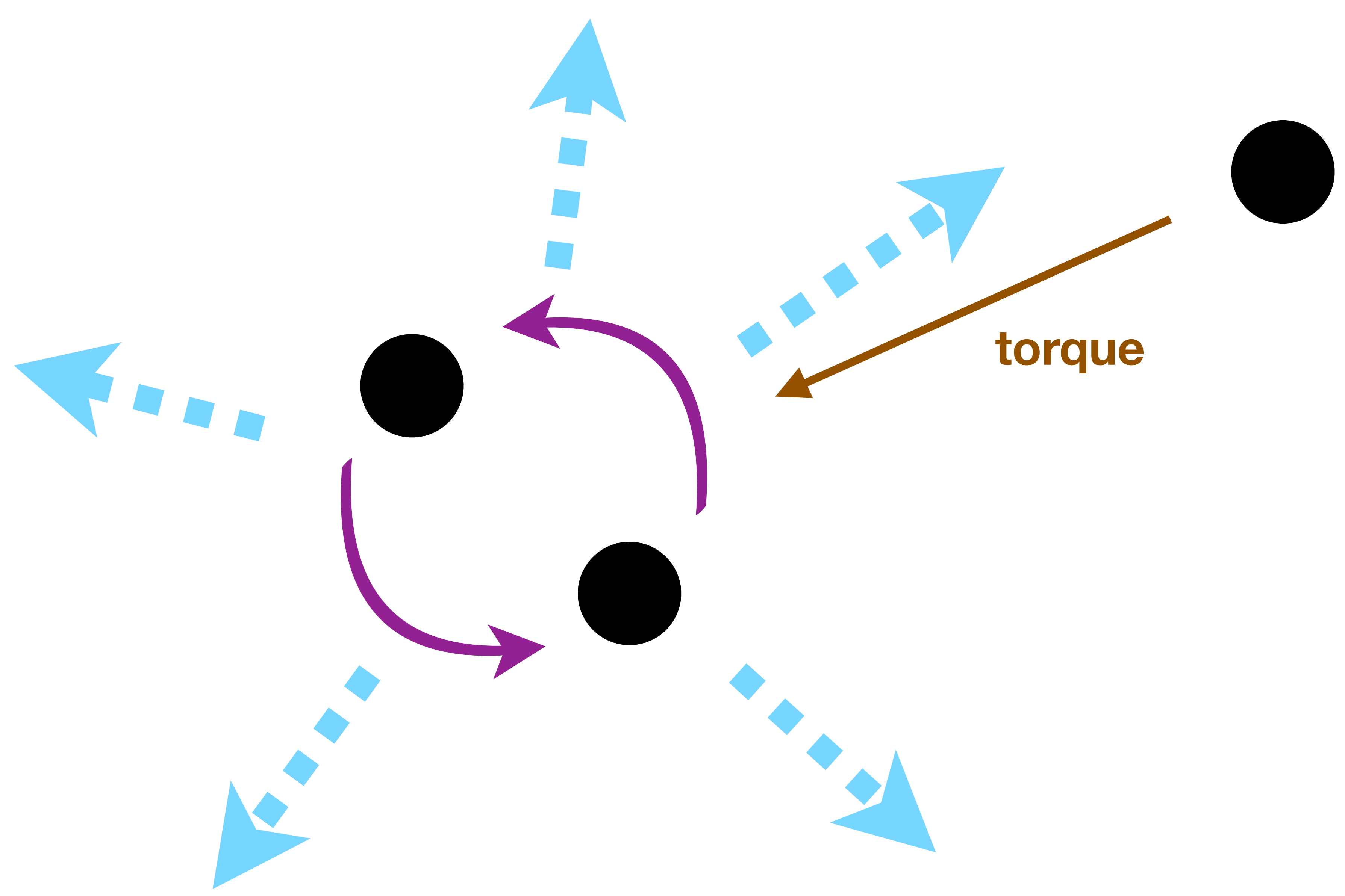

Binary Black Holes loose energy by emitting Gravitational Waves.

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