Scale Invariant Extension of the SM

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階層性の問題

なぜ、極端に異なったスケールが存在しているの？

$$\mathrm{EW\ scale} \simeq 10^2\ \mathrm{GeV}\ ,\ M_{\mathrm{PL}} \simeq 10^{18}\ \mathrm{GeV}\ ,\ \Lambda \simeq (10^{-61} M_{\mathrm{PL}})^2$$

関連した問題: Naturalness

パラメータを微調整しないで、これらの極端に異なったスケールを実現できるのか？

難しい話は

この問題を少し違った側面から眺めてみる。

Electroweak scale,
Cosmological constant,
and
Planck scale M_{PL}?

これらのスケールの起源は何なの？

最初から次元のあるパラメータを含む理論から出発すれば、そのパラメータの起源は説明できない。

↓

古典的レベルで次元のある
パラメータを含んでいない理論から出発する。

スケール不変性に基づく標準模型
とEinsteinの重力理論の拡張

とは言っても、スケール不変性にはアノマリーがあり、**hard** に破れている。

その結果

Callan, ’70; Symanzik,’70

1. Running of couplings
2. Change of scaling dimension

しかし…. 3. が今回最も重要

3. アノマリーは質量　(mass gap)をダイレクトに生成できない。

質量を生成するためには、スケール不変性を自発的に破る必要がある。

(Massless theories exit. Loewenstein+Zimmermann,'76.
„Physics BSM may be described by a massless QFT" , JK, at MPI, 2017)

3. がNaturalness問題を和らげる🔑になっている。

The SM does not, by itself, has a fine tuning problem (Bardeen,'95), if there is no large intermediate scale between the SM and Planck scales.

アノマリーの、Higgsの質量への寄与は

logarithmicであり，　quadraticではない！！！

スケール不変性に基づく標準模型
とEinsteinの重力理論を拡張する

動機

M1. 我々の時空は4次元。

その心は:

スケール不変なゲージ理論は4次元だけに存在する。

（C-S in d=3理論のような例外もありますが）

スケールの起源の問題は4次元特有の問題。

M2．標準模型には μ_H^2 ，

Einsteinの重力理論には $M_{\rm Pl}$ ，

それぞれ1コの有次元のパラメータしか含んでいない。

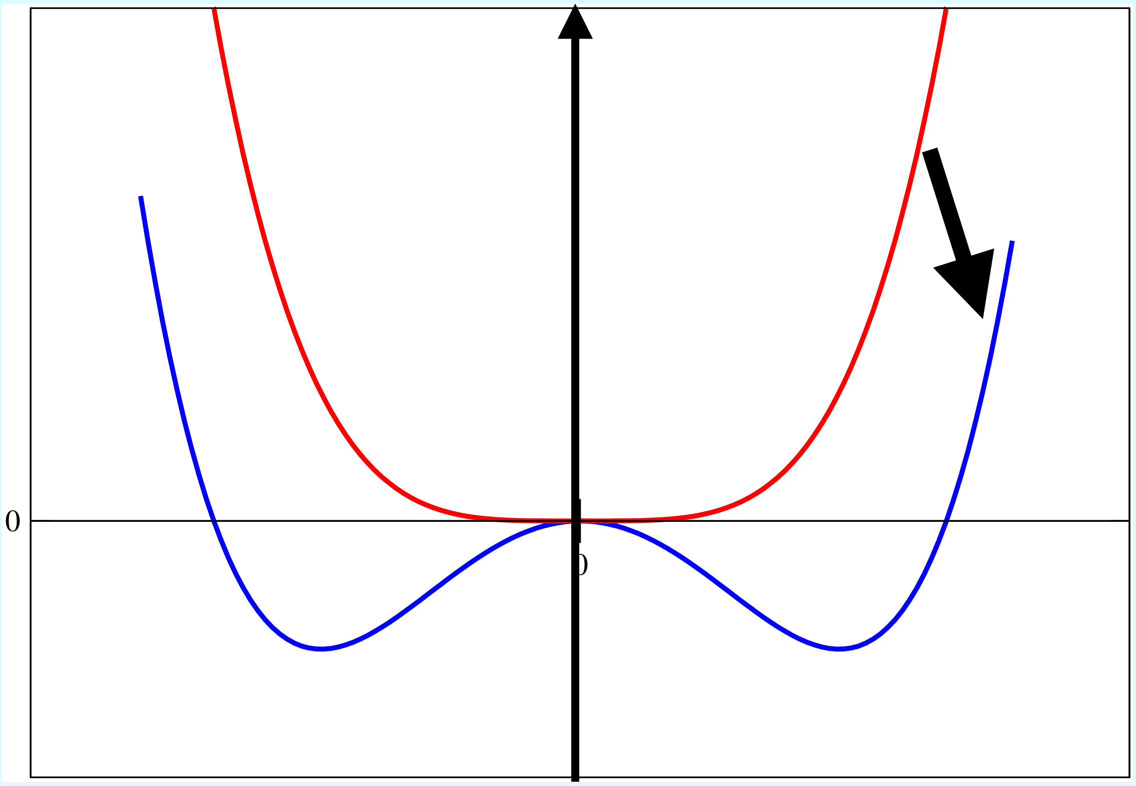
もし沢山独立はパラメータあれば、.....

* Coleman+Weinberg, ‘73

$\mu_H^2 = 0$ 標準模型：もし、m_{top} を無視できれば

scale anomaly=>

Higgs potential



残念ながら：$m_H \sim 10\ \mathrm{GeV}$

* 「 $M_{\rm Pl}$ を生成する。」は昔から。

Induced gravity

with scalars

- *Fujii '74
- *Minkowski, '77
- *Englert, Gunzig, Truffin+Windey,'75
- *Chundnovsky,'78
- *Fradkin+Vilkovisky,'78
- *Zee,'79
- *Smolin,'79
- *Terazawa,'81
- *Nieh, '82

……………

without scalars

- *Akama, Chikashige+ Matsuki,'78
- *Adler,'80
- *Zee,'81

……………

M3. QCDというお手本がある。

その心は:

ハドロンの質量（~98%）の起源はスケール不変性の自発的対称性の破れよるもの。

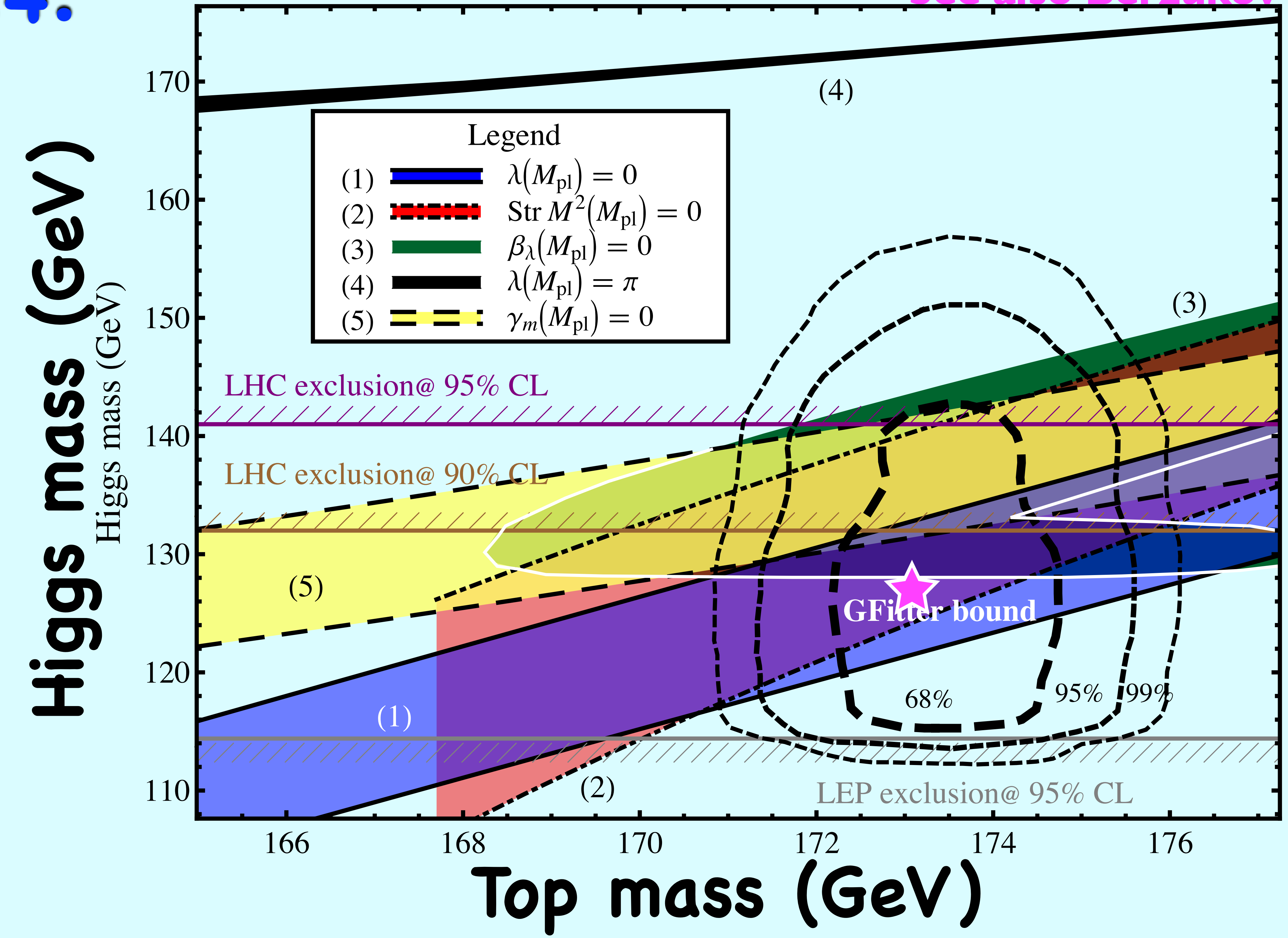
（ちまたでは、

dynamical chiral symmetry breaking と呼ばれている。Nambu, '60; with

Jona-Lasinio, '61 ）

M4.

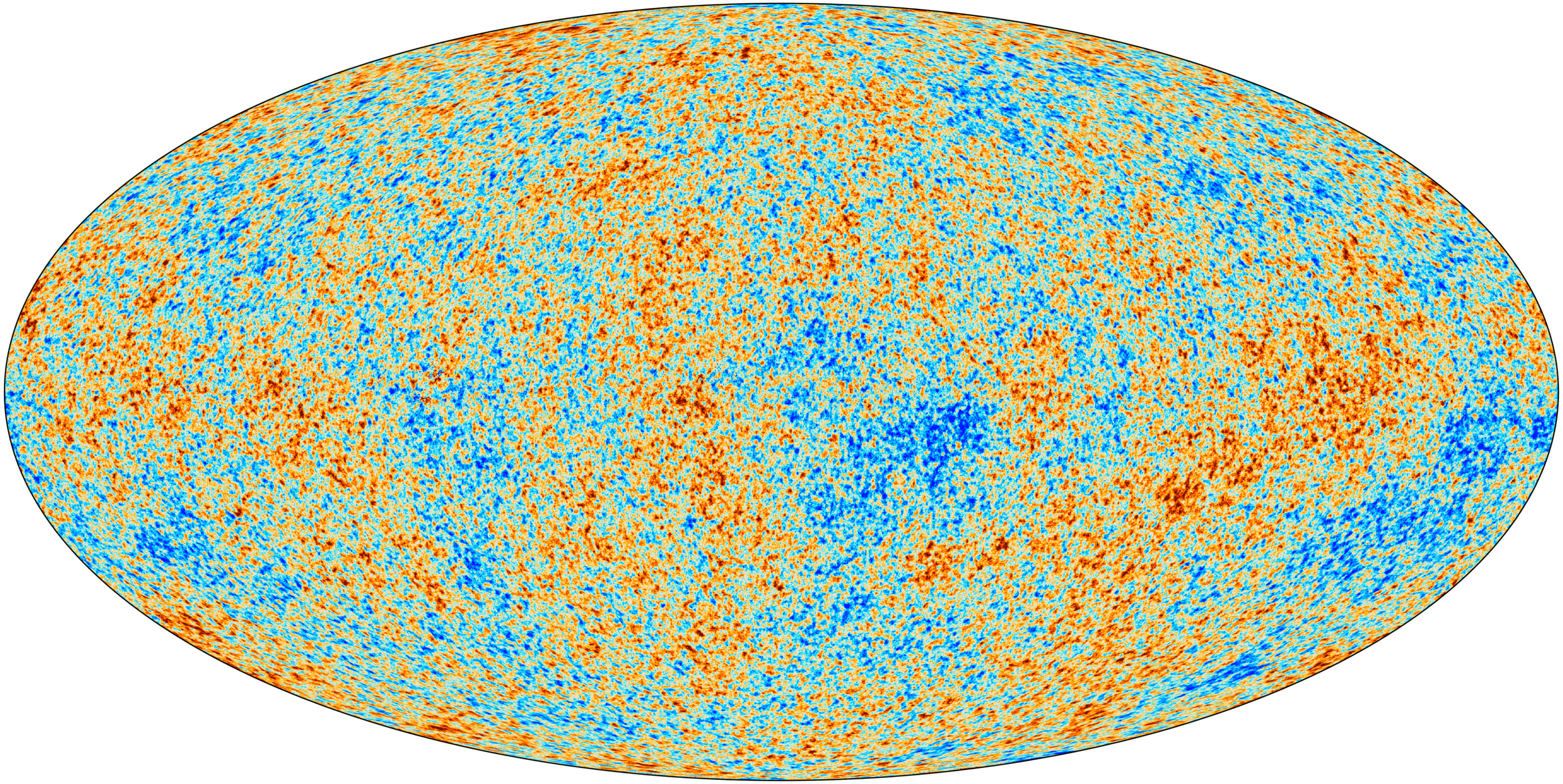
(Holthausen+Lee+Lindner, '12.
See also Berzukov et al, '12)



Bardeen,'95:

The SM does not, by itself, has a fine tuning problem, if there is no large intermediate scale between the SM and Planck scales.

M5.

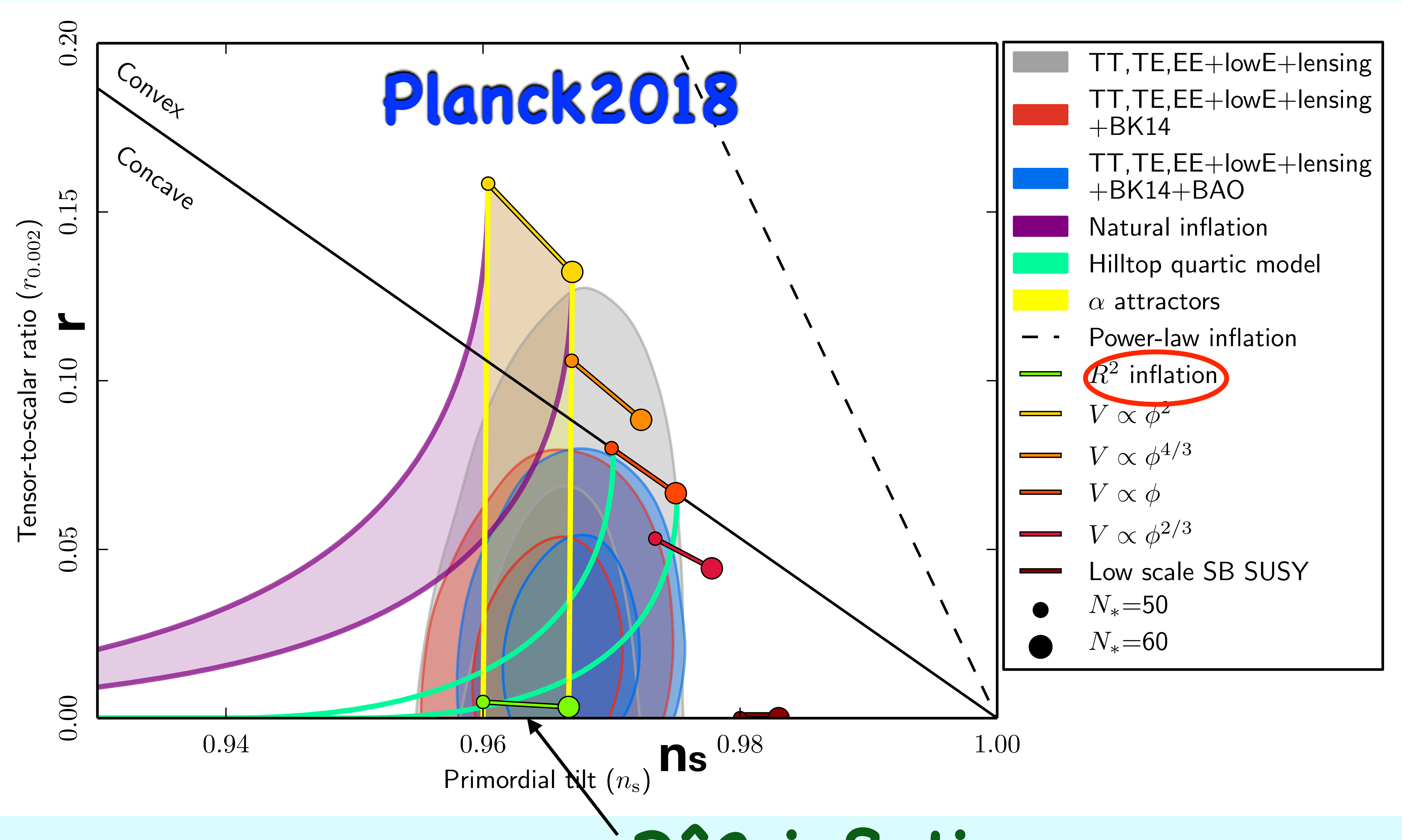


http://sci.esa.int/planck/60506-the-cosmic-microwave-background-temperature-and-polarization/

その心は

Inflation

Planck Collaboration: Constraints on Inflation

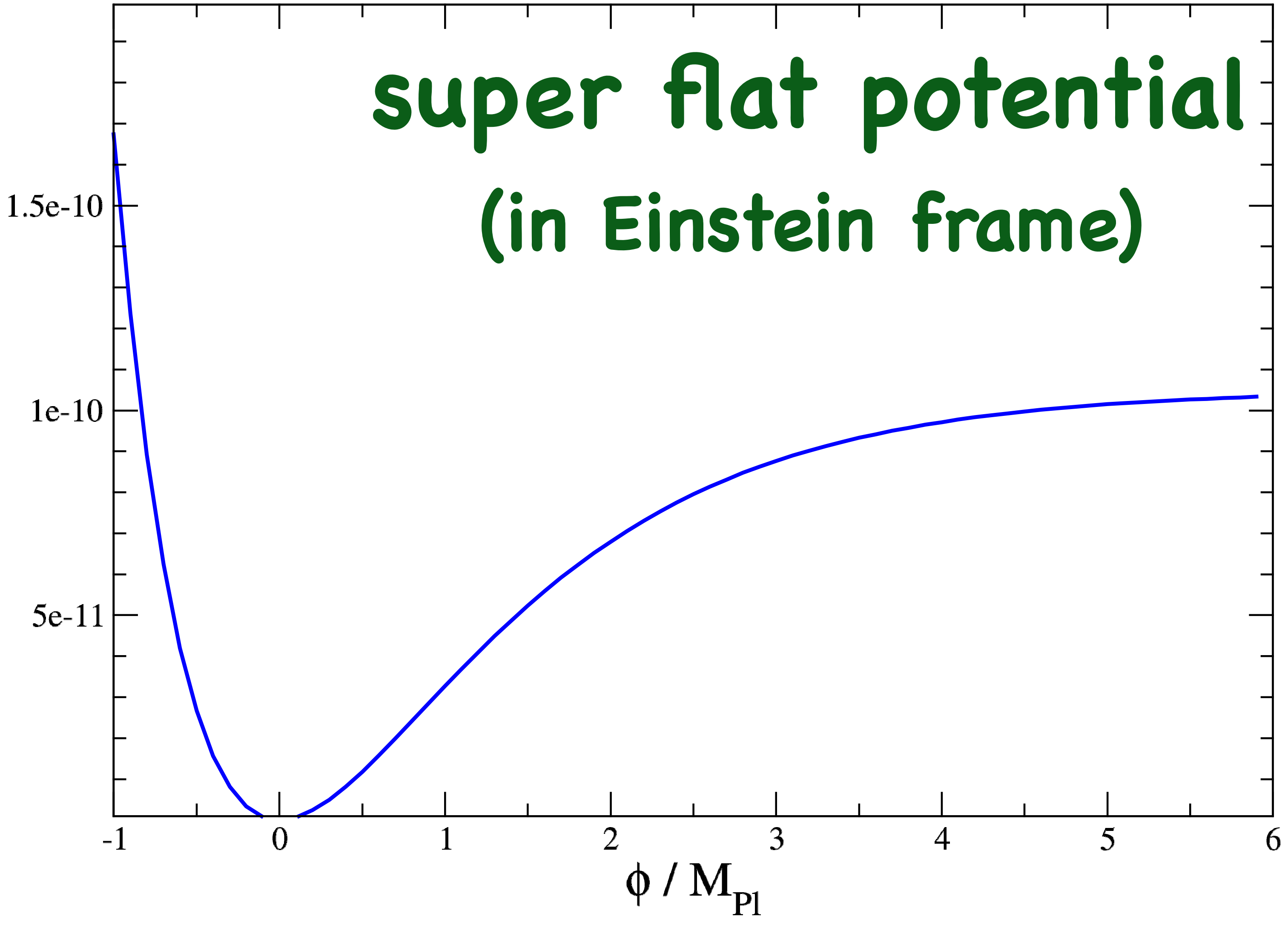


R^2 inflation

scale invariant

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_{\rm Pl}^2}{2} R + \left\{ \begin{array}{c} \gamma R^2 \quad (\gamma \sim 10^9) \\ \beta |H|^2 R - \lambda_H |H|^4 \quad (\beta \sim 10^4) \end{array} \right.$$

$$\text{for} \left\{ \begin{array}{c} R^2 \text{ inflation, Starobinsky, '80; Mukhanov+Chibisov,'81} \\ \text{Higgs inflation, Bezrukov and Shaposhnikov, '08.} \end{array} \right.$$



super flat potential

(in Einstein frame)

M6.　その他

⋮

Conformal gravity,

't Hooft, Mannheim, .. K. Aoki...

⋮

Asymptotically safe gravity,

Wetterich.. Yamada, Hamada……

⋮

Two scenarios for scalegenesis

* **Nonperturbative scenario**

* **Perturbative scenario**

Nonperturbative scenario

.

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*Weinberg,‘76;79
*Susskind,’79

..

*Hur,Jung,Ko,Lee,’07
*Hur,Ko,’11
*Heikinheimo et al,’13
*JK,Holthausen,Lim,Lindner,’13
*JK,Lim,Lindner,’14
*Ametani, Aoki,Goto, JK,`15
*Hatanaka, Jung,Ko,`16
* Haba, Ishida,Kitazawa,
+Yamaguchi,’16
*

*JK, Yamada,’15
*
*

....

Perturbative scenario

*Coleman -Weinberg, ’73
*Gildener -Weinberg, ’76

*Hempfling,’96

.

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*Meissner,Nicolai,’07,
*Chang,Ng,Wu,’07
*Foot,Kobakhidze,Volkas,’07
*Espinosa,Quiros,’07
*Iso,Okada,Orisaka,’09
*Holthausen,Lindner, Schmidt,’09
*A-Nunneley,Pilaftsis,’10

....

*Ishiwata,`11
*

*

*Hamada, Kawai,Oda,Yagyu,’20

....

Strong Dynamics in Yang-Mills theory

$\langle\bar{\psi}\psi\rangle \neq 0$ **breaks chiral symmetry at the same time scale invariance and generate a robust energy scale.**

$\langle S^\dagger S\rangle \neq 0$ **breaks scale invariance and generate a robust energy scale.**

How to deal with this non-perturbative effect ?

- **Direct approach**: Lattice gauge theory
- **Effective theory approach**:

Massless QCD

$$\mathcal{L}_{QCD} = -\frac{1}{2}\mathrm{tr}F^2 + i\bar{\psi}_i\gamma^\mu D_\mu\psi_i$$

At low energy:

$$\langle\bar{\psi}_i\psi_j\rangle = \langle\sum_{c=1}^{N_c}\bar{\psi}_i^c\psi_j^c\rangle \propto \delta_{ij}$$

****Effective theory for chiral condensate (order parameter): Nambu-Jona-Lasinio (NJL) theory**

The NJL model

$$\mathcal{L}_{\mathrm{NJL}} = i\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i + 2G\,\Phi^\dagger\Phi + \ldots = i\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i + G\left[(\bar{\psi}\lambda^a\psi)^2 - (\bar{\psi}\gamma_5\lambda^a\psi)^2\right] + \ldots$$

(4-fermi)

$$\Phi_{ij} = \bar{\psi}_i(1-\gamma_5)\psi_j = \frac{1}{2}\sum_{a=0}^{N_f^2-1}\lambda^a_{ji}\,\bar{\psi}\lambda^a(1-\gamma_5)\psi$$

+ 6-fermi

The relevant global symmetry

$(N_f = 3, N_c = 3)$

★ **At the classical level**

$$SU(3)_L \times SU(3)_R \times U(1)_V \times \left\{ \begin{array}{c} U(1)_A \\ Z_6 \end{array} \right.$$

QCD

NJL (4+6 fermi)

★ **At the quantum level**

$$U(1)_A \to Z_6$$

Chiral anomaly in QCD

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

Dynamical chiral symmetry breaking

★ **Finally**

$$SU(3)_V \times U(1)_V \times Z_6$$

NJL in the mean field approximation

(For a review: Hatsuda+Kunihiro, PR.'94)

1. Go from the conventional vacuum $|0\rangle$ to the "BCS" vacuum $|\text{"BCS"}\rangle = |\sigma, \pi, K, \cdots\rangle$, where $\sigma, \pi, K, \cdots$ are identified with $\langle\text{"BCS"}|\ \mathrm{Tr}\ \overline{\psi}\lambda^a(1, \gamma_5, \cdots)\psi\ |\text{"BCS"}\rangle$.

2. Express $\mathcal{L}_{NJL} = \mathcal{L}_0 + \mathcal{L}_I$, where
   (a) $\mathcal{L}_I$ is normal-ordered with respect to $|\text{"BCS"}\rangle$, i.e. $\langle\text{"BCS"}|\ \mathcal{L}_I |\ \text{"BCS"}\rangle = 0$.
   (b) $\mathcal{L}_0$ is at most quadratic in fermions, where the ferimon bilinears are NOT normal-ordered.

3. Compute diagrams with external mean fields (mesons) to predict the meson properties by integrating out the fermions. But at the lowest order of the approximation, $\mathcal{L}_I$ does not contribute.

Mean field Lagrangian (SU(3)_V)

Mean fields

$$\sigma\delta_{ij} = -4G\langle\bar{\psi}_i\psi_j\rangle_{\mathrm{BCS}} \quad , \quad \phi_a = -2iG\langle\bar{\psi}\gamma_5\lambda^a\psi\rangle_{\mathrm{BCS}}$$

Chiral condensate **NG bosons**

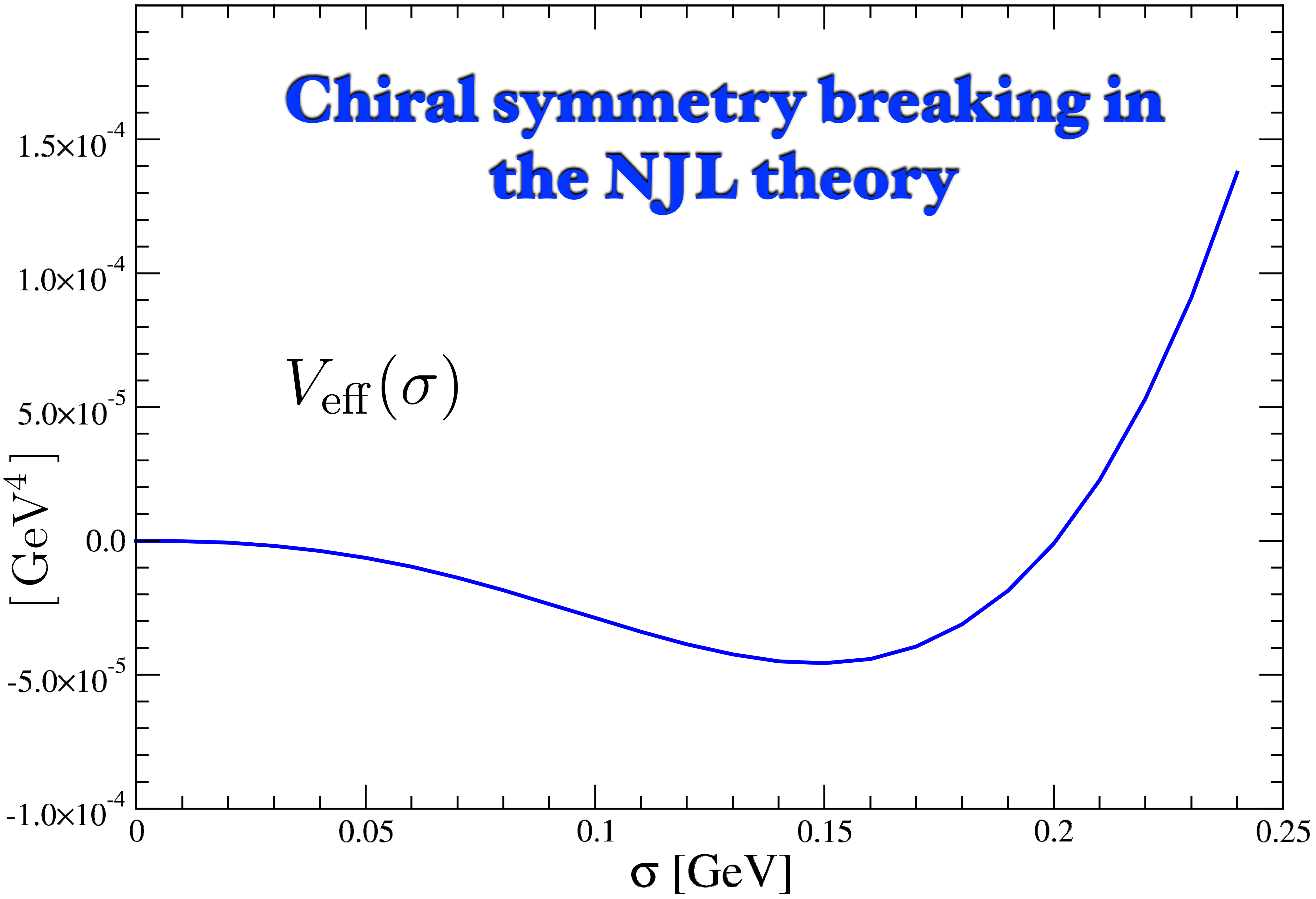
$$\begin{aligned}\mathcal{L}_0 \quad = \quad & \mathrm{Tr}\ \bar{\psi}(i\partial\!\!\!/ - M)\psi - i\mathrm{Tr}\ \bar{\psi}\gamma_5\phi\psi - \frac{1}{8G}\left(3\sigma^2 + 2\sum_{a=1}^{8}\phi_a\phi_a\right) \\ & + \frac{G_D}{8G^2}\left(-\mathrm{Tr}\ \bar{\psi}\phi^2\psi + \sum_{a=1}^{8}\phi_a\phi_a\mathrm{Tr}\ \bar{\psi}\psi + i\sigma\mathrm{Tr}\ \bar{\psi}\gamma_5\phi\psi + \frac{\sigma^3}{2G} + \frac{\sigma}{2G}\sum_{a=1}^{8}(\phi_a)^2\right)\end{aligned}$$

$$\boxed{M = \sigma - (G_D/8G^2)\sigma^2}$$

(G for 4 fermi and GD for 6 fermi)

Integrate out ψ to get the effective potential:

$\Lambda = 0.93\ ,\ (2G)^{-1/2} = 0.361\ ,\ (-G_D)^{-1/5} = 0.406$ **GeV**



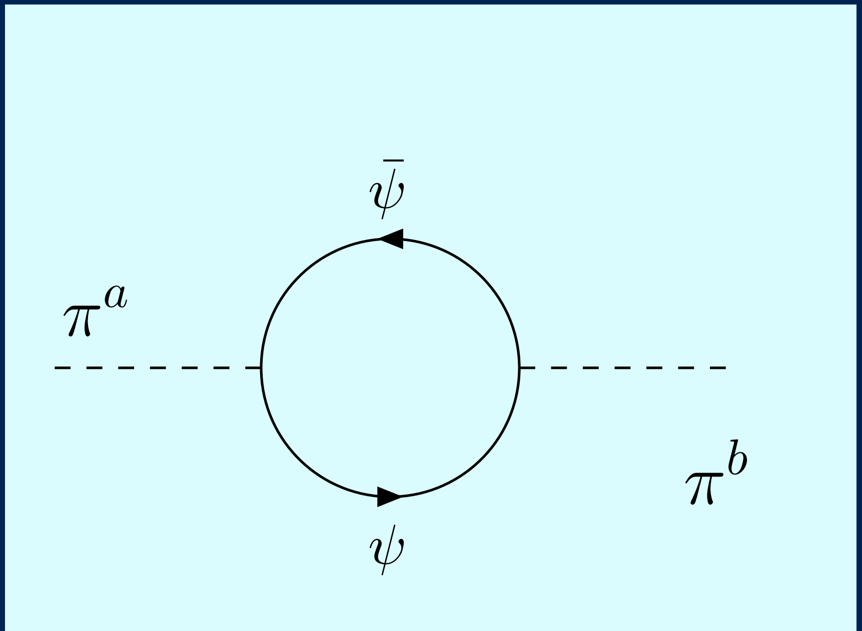
DM candidate

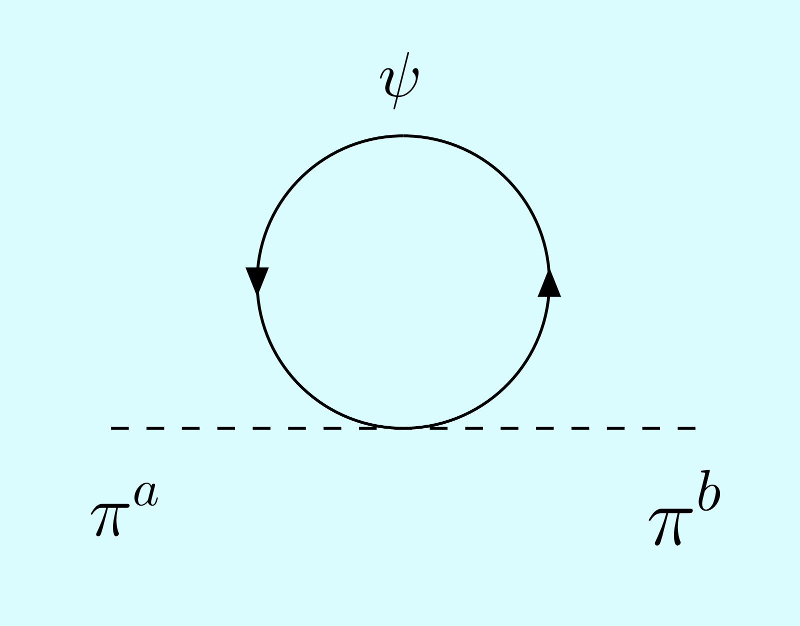
$$\langle \bar{\psi}_i (1-\gamma_5)\psi_j \rangle_{\rm BCS} = -\frac{1}{4G}\left[\, \delta_{ij}\hat{\sigma} + \lambda^a(\sigma'^a + i\pi^a)\,\right]$$

Excitations

with $\langle \sigma'^a \rangle = 0 \;,\; \langle \pi^a \rangle = 0$

The kinetic term for sigma and pion is generated and their masses can be computed :





$$\Lambda = 0.93 \;,\; (2G)^{-1/2} = 0.361 \;,\; (-G_D)^{-1/5} = 0.406 \;,\; m_u = 0.006 \;,\; m_s = 0.163$$

in GeV ***NJL QCD with 6 fermi***

	Exp.	**NJL**
$m_{\pi^0}(m_{\pi^\pm})$	0.135(0.140)	0.136
f_π	0.092	0.093
$m_{K^0}(m_{K^\pm})$	0.498(0.494)	0.499
f_K	0.110	0.105
m_η	0.548	0.460
$m_{\eta'}$	0.958	0.960

in GeV

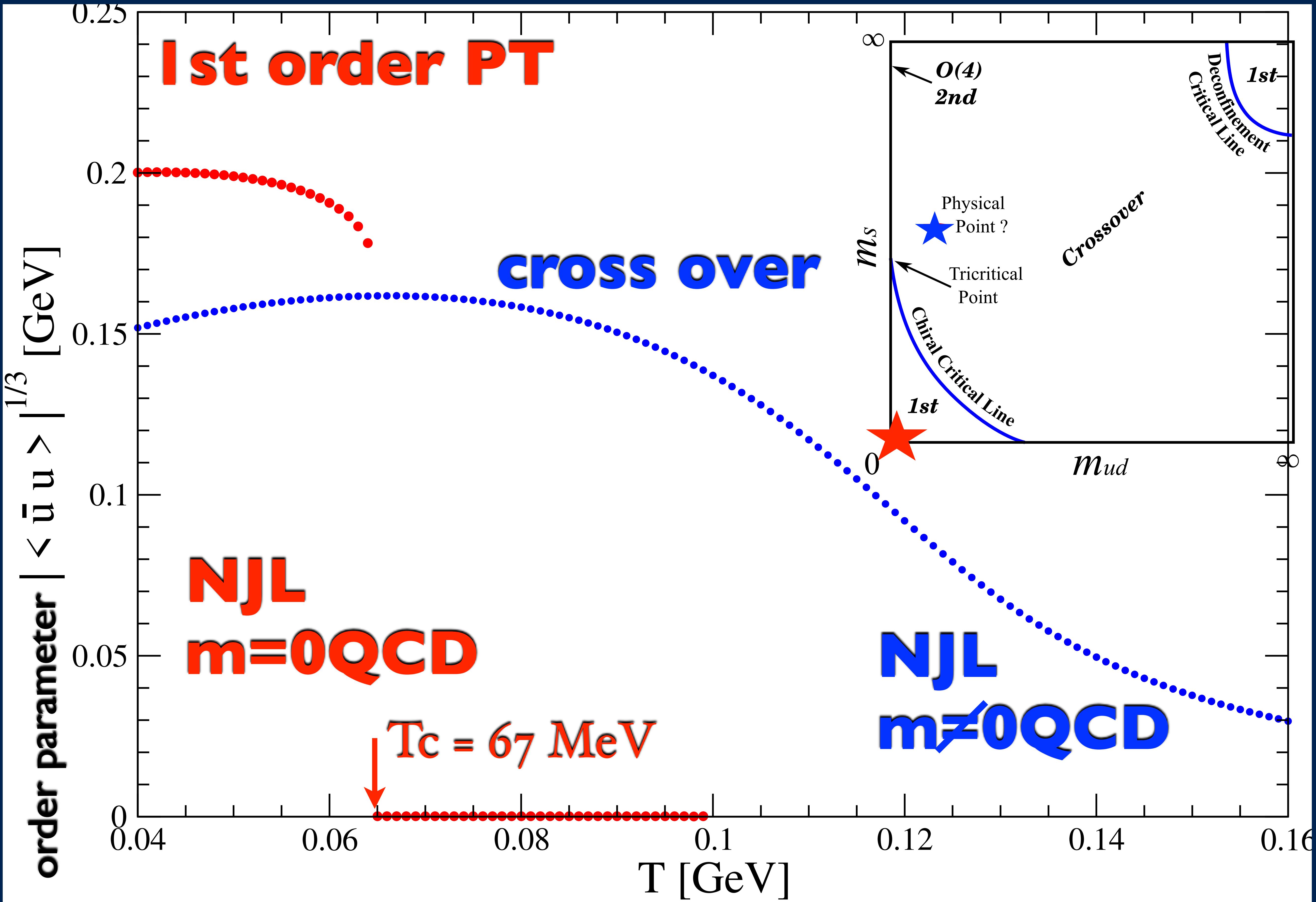
***Goldberger-Treiman relation:**

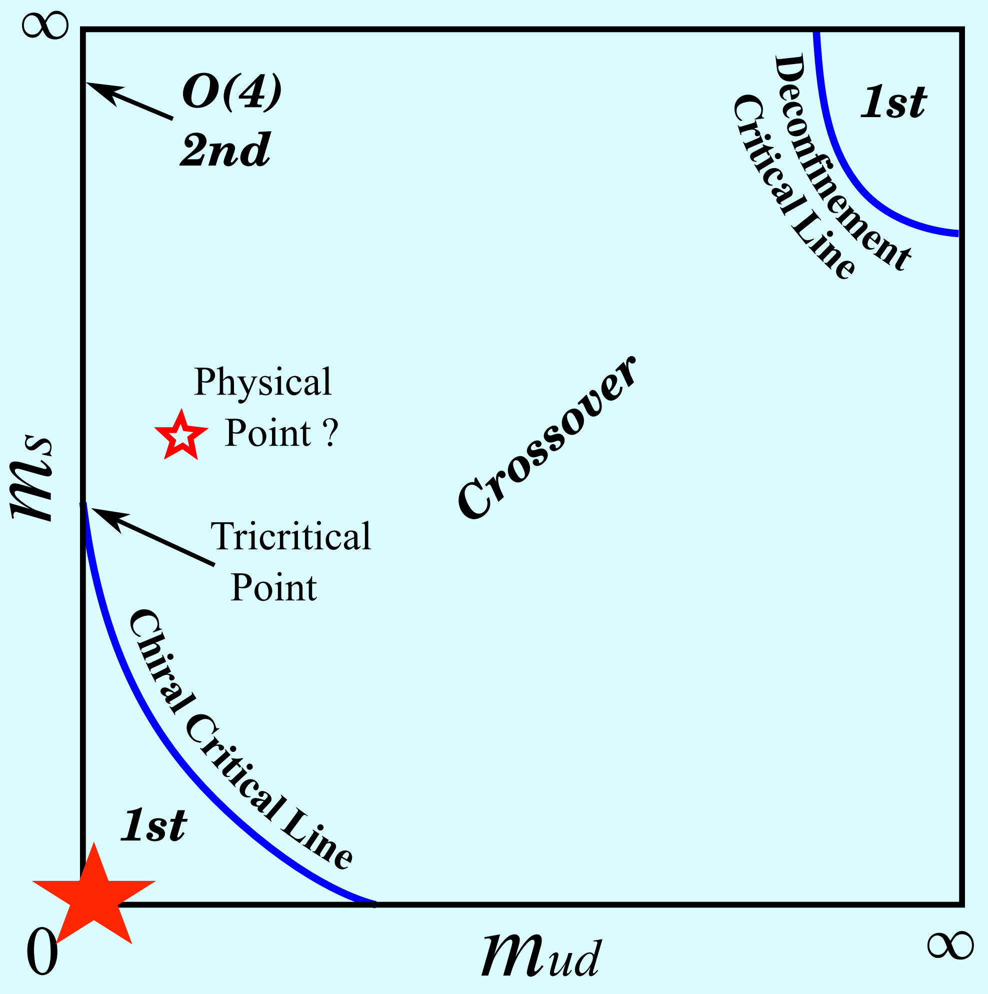
$$f_\pi G_{\pi qq} = 0.98 \times M$$

***Gell-Mann-Oakes-Renner relation:**

$$f_\pi^2 m_\pi^2 \;=\; -1.00 \times \frac{1}{2}(m_u + m_d) < \bar{u}u + \bar{d}d >$$

See also:Hatsuda+Kunihiro, '94


1st order PT
cross over
NJL
m=0QCD
Tc = 67 MeV
NJL
m≠0QCD
order parameter |< ū u >|^1/3 [GeV]
T [GeV]
0.25
0.2
0.15
0.1
0.05
0
0.04
0.06
0.08
0.1
0.12
0.14
0.16
∞
O(4)
2nd
1st
Deconfinement
Critical Line
Physical
Point ?
Crossover
Tricritical
Point
Chiral Critical Line
1st
m_s
0
m_ud
∞



	Tc
NJL	~ 70 MeV
PolyakovNJL (Fukushima, '04)	~ 120 MeV
Lattice QCD (X-Y. Jin et al, '17)	~ 134 MeV

1st order PT can produce a Gravitational Wave (GW) background, which could be observed today.

Witten,'84

$$\lambda[\mathrm{m}] \text{ of GW} \sim \text{the size of the Universe at } T = T_{QCD} \sim 30 \text{ km}$$

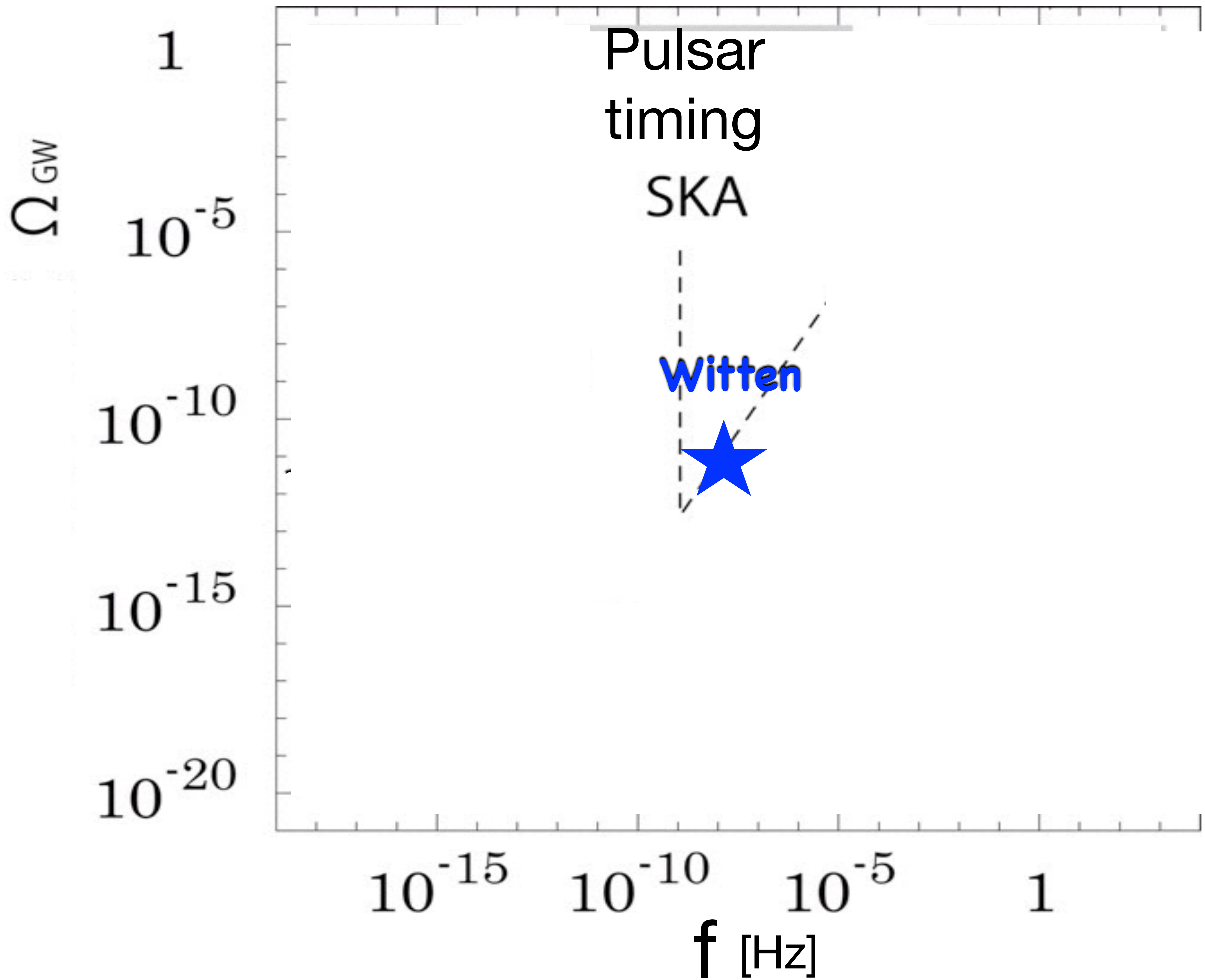
$$\text{As } T_{QCD} \to T_{CMB}, \lambda \text{ is red shifted to } \frac{T_{QCD}}{T_{CMB}}\lambda \sim 1.3 \times 10^{13} \text{ km}$$

$\nu \sim 10^{-8}$ Hz

$1.5 \cdot 10^8$ km

to the Sun

Gravitational Wave (GW) spectrum



GW spectrum for the cosmological chiral phase transition in the massless QCD

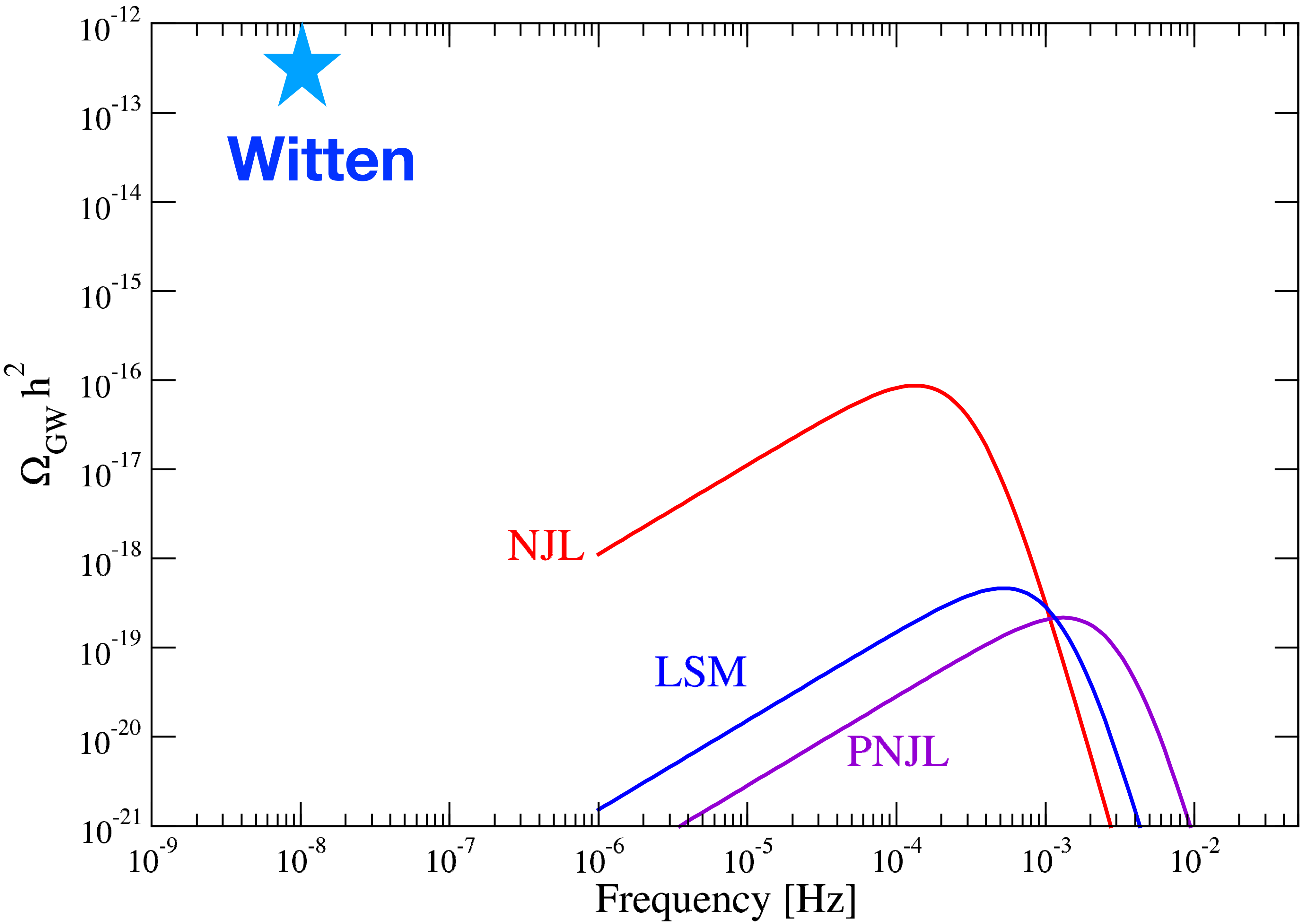
No first principle calculation in QCD!!!

GW spectrum for the massless QCD

Helmboldt, JK + van der Woude,'19.

See also: Tsumura, Yamada+Yamaguchi,'17;

Bai,Long+Lu,'19



A.J. Helmboldt,, JK and S. van der Woude, PRD, `19

**Effective theory for $\langle S^\dagger S \rangle \neq 0$

(approximate order parameter)

JK and Yamada,'15

SU(Nc) gauge theory with U(Nf)

(Osterwalder+Seiler,'78;Fradkin+Shenker,'79;....)

$$\mathcal{L}_H = -\frac{1}{2}\mathrm{tr}F^2 + ([D_\mu S_i]^\dagger D^\mu S_i) - \hat{\lambda}_S(S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S(S_i^\dagger S_j)(S_j^\dagger S_i)$$

$$(i, j = 1, \ldots, N_f)$$

with S in the fundamental representation of SU(Nc)

(The color indices are suppressed.)

The guiding principle:
The global symmetry

★ **At the classical level:**

$U(N_f)$ flavor symmetry and
scale invariance

★ **At the quantum level:**

$U(N_f)$ flavor symmetry and
(anomalous) scale invariance,
which is dynamically broken
by $\langle S_i^\dagger S_j \rangle \neq 0$ with $\langle S_i \rangle = 0$.

$$\mathcal{L}_H \;=\; -\frac{1}{2}\mathrm{tr}F^2 + ([D_\mu S_i]^\dagger D^\mu S_i) - \hat{\lambda}_S(S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S(S_i^\dagger S_j)(S_j^\dagger S_i)$$

U(Nf)+classi. Scale Invariance

at low energy

UNIQUE !

$$\mathcal{L}_{\mathrm{eff}} \;=\; ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S(S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S(S_i^\dagger S_j)(S_j^\dagger S_i)$$

It remains to show:
Scale invariance is dynamically broken.

Earlier discussions in 70s and later in a different context:

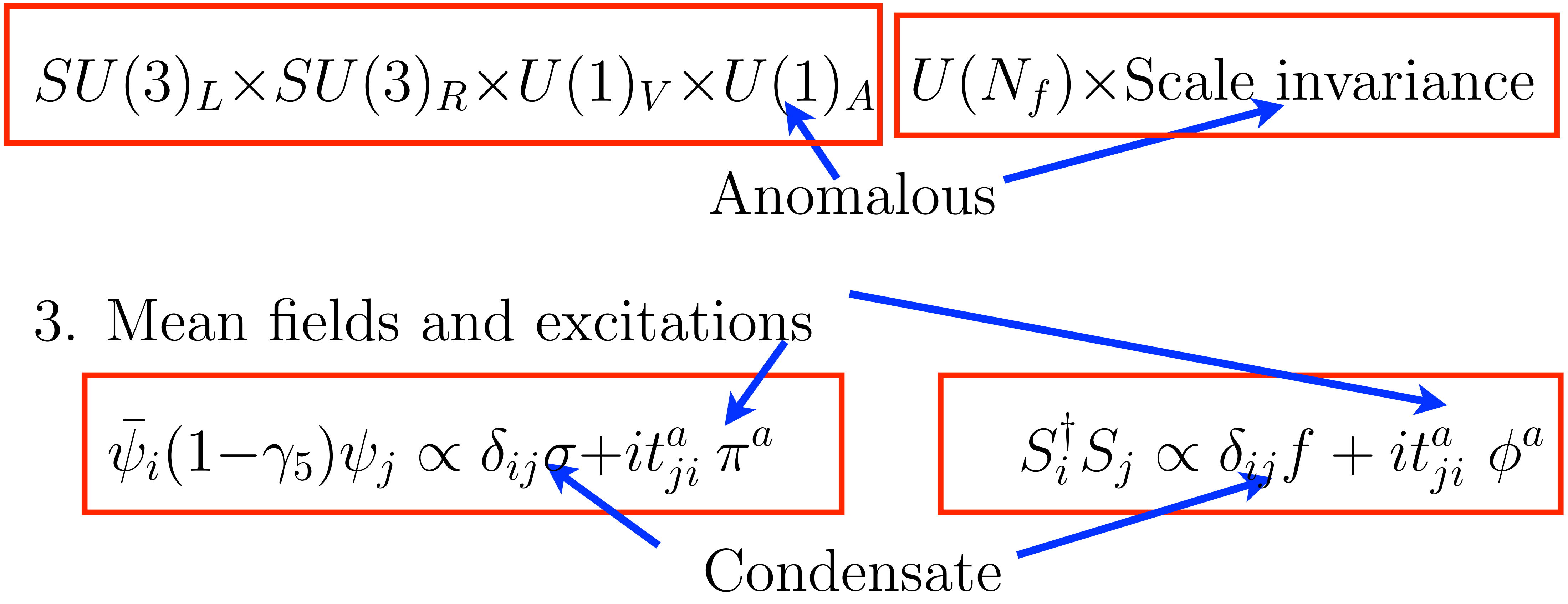
Coleman, Jackiw+Schnitzer,'74; Kobayashi+Kugo, '75; Bardeen+Moshe, '83;…………

NJL

Our approach

1. Integrating out the gauge fields.

2. Global symmetries



$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$

$U(N_f) \times$ Scale invariance

Anomalous

3. Mean fields and excitations

$\bar{\psi}_i(1-\gamma_5)\psi_j \propto \delta_{ij}\sigma + i t^a_{ji}\, \pi^a$

$S^\dagger_i S_j \propto \delta_{ij} f + i t^a_{ji}\, \phi^a$

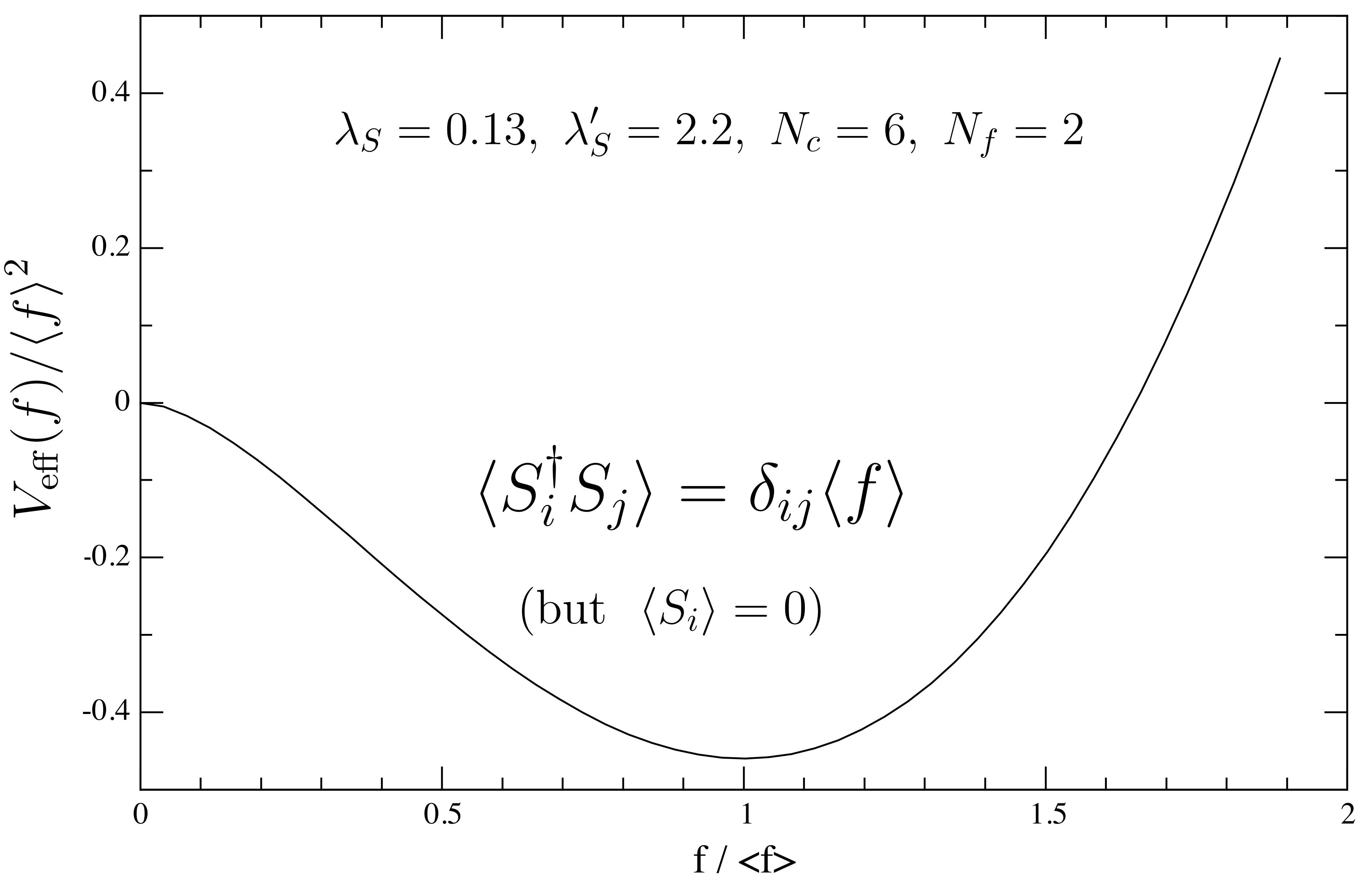
Condensate

4. Effective potential from

integrating out ψ

integrating out δS around $\bar{S}$

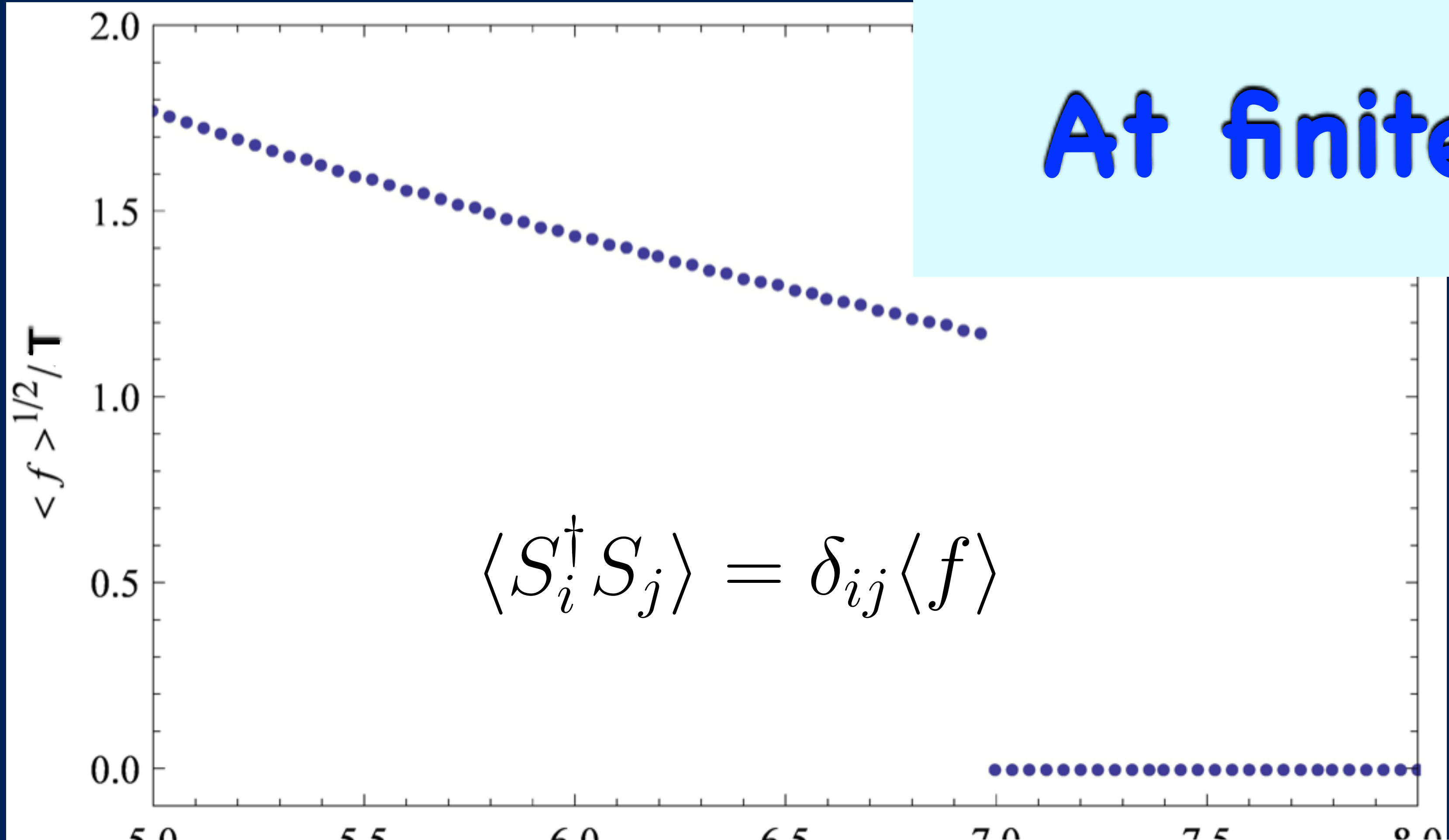
Spontaneous scale symmetry breaking



in an effective theory using the mean field approximation.

JK and Yamada,'15

At finite temperature



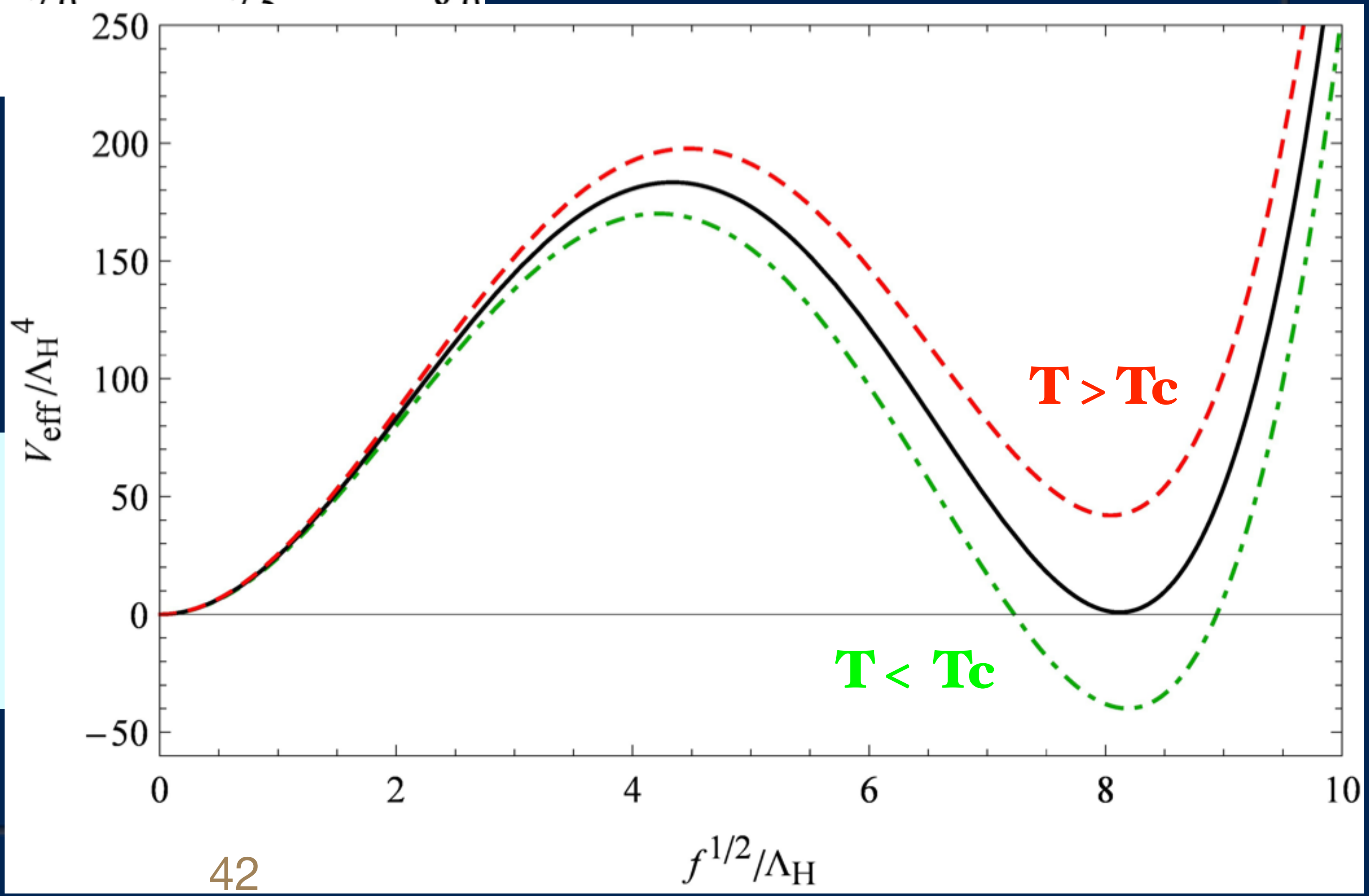
$$\langle S_i^\dagger S_j \rangle = \delta_{ij} \langle f \rangle$$

JK and Yamada,'15

Scale Phase Transition

is 1st order.

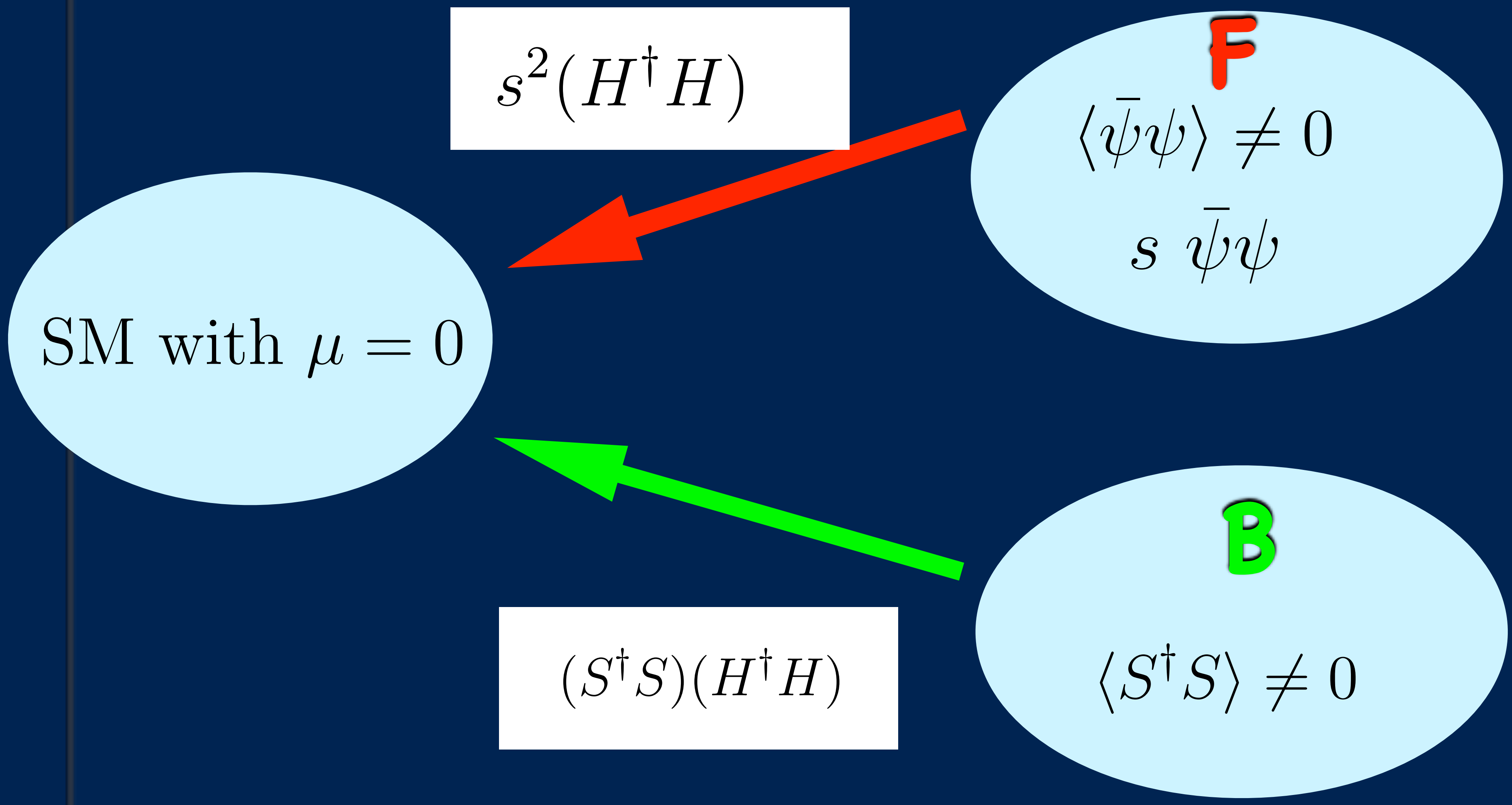
$$N_f = 1,\ N_c = 6,\ \lambda_S + \lambda'_S = 2.083$$



Applications to
extending the SM and Einstein GR
based on scale invariance

Realistic models (Fermionic and Bosonic <>)

QCD-like hidden sector



$s^2(H^\dagger H)$

F

$\langle\bar\psi\psi\rangle \neq 0$

$s\ \bar\psi\psi$

SM with $\mu = 0$

B

$\langle S^\dagger S\rangle \neq 0$

$(S^\dagger S)(H^\dagger H)$

Hur+Ko, '11;
Heikinheimo et al,'13;
Holthausen,JK,Lim+Lindner,'13
JK,Lim+Lindner,'14;
Ametani, M.Aoki,Goto+JK,'15;
M.Aoki,Goto+JK,`17

JK,Lim+Lindner,'14;
JK + Yamada,'15,'16,;
JK, Soesanto+Yamada,'17.

The lightest bound states in the hidden sector are dark matter!!

Model F

uses

the chiral condensate

$$\langle\bar{\psi}\psi\rangle \neq 0$$

Real singlet

$$\mathcal{L}_{\rm H} = -\frac{1}{2}{\rm tr}F^2 + \bar{\psi}_i \left(i\gamma^\mu D_\mu - yS\right)\psi_i \qquad \cancel{\chi}$$

$$-\frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H) - \lambda_H (H^\dagger H)^2 + \mathcal{L}'_{\rm SM}$$

(SM with no mass term)

$$\boxed{\langle\bar{\psi}\psi\rangle \neq 0 \quad \to \langle S\rangle \neq 0}$$

Higgs portal

(Hur+Ko, PRL 106 (2011) 141802;
Heikinheimo et al,Mod.Phys.Lett.A29 (2014)1450077;
Holthusen+JK+Lim+Lindner, JHEP1312 (2013)076...)

Effective theory

$\cancel{U(1)}\ A$

$$\mathcal{L}_{\text{eff}} = \text{Tr}\ \bar{\psi}\left(i\partial\!\!\!/ - \left[\sigma - \frac{G_D}{8G^2}\sigma^2 + \mathbf{y}S\right]\right)\psi + 2G\,\text{Tr}\,\Phi^\dagger\Phi + G_D\,(\det\Phi + h.c.)$$

4-fermi 6-fermi

DM

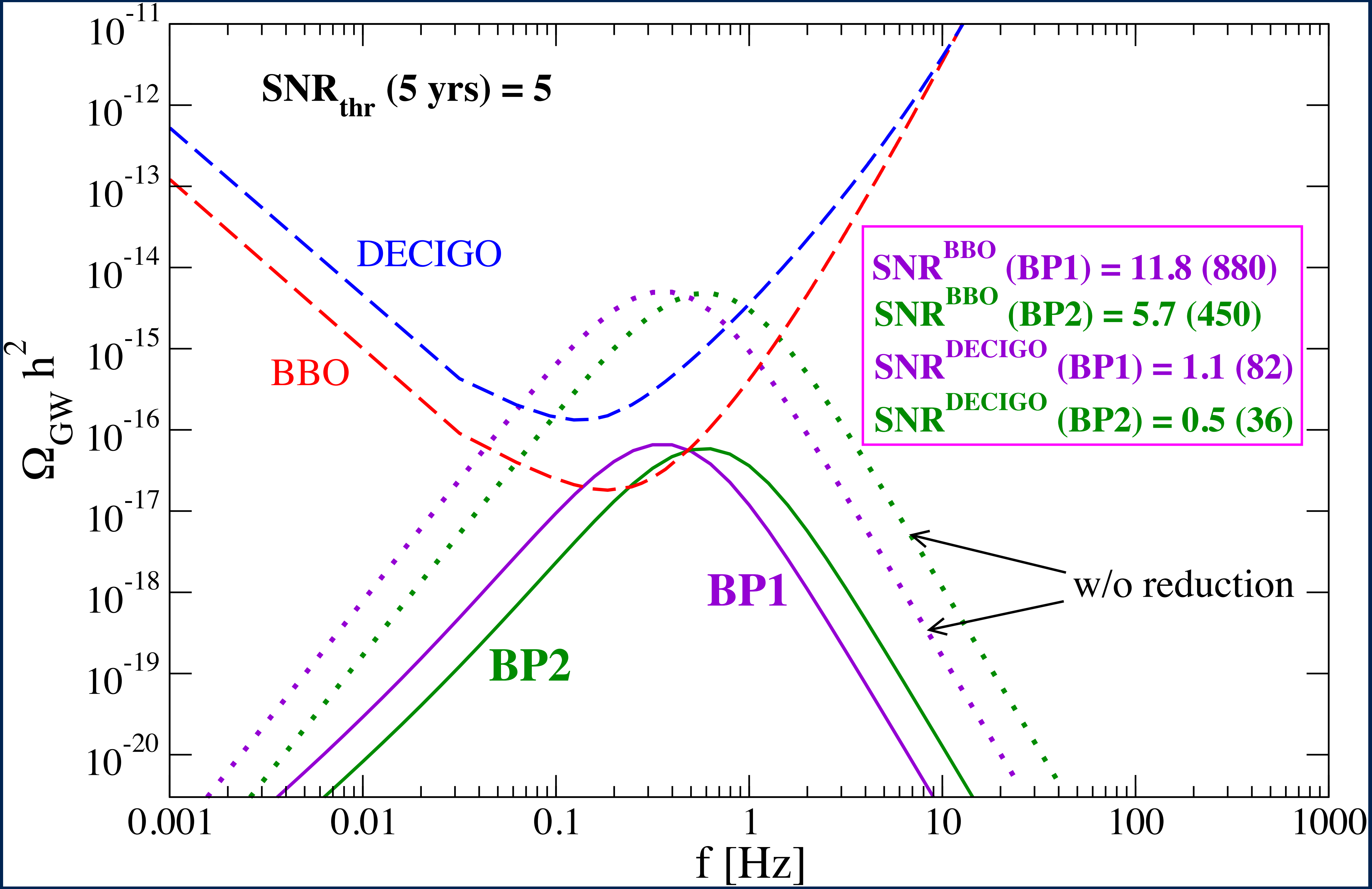
$$\Phi_{ij} = \bar{\psi}_i(1-\gamma_5)\psi_j = -\frac{1}{4G}\lambda^a_{ji}\,(\,\sigma_a + i\pi_a\,)$$

To apply the m=0 QCD for a hidden sector, we scale-up and assume:

$$G^{1/2}\Lambda_H = 1.82\,, \qquad (-G_D)^{1/5}\Lambda_H = 2.29$$

even for $\Lambda_H \gg \Lambda = 0.93$ GeV

✓ Dark Matter
✓ Gravitational Waves

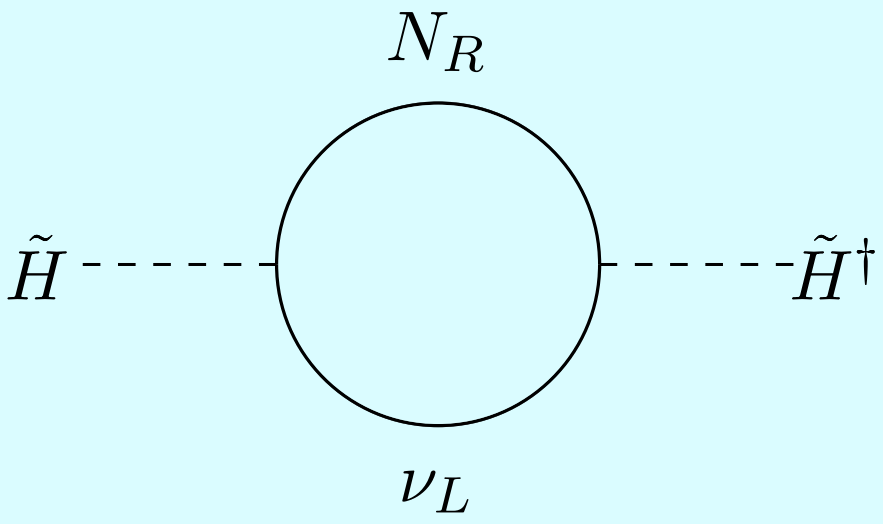


(Aoki, Goto+Kubo, '17; Aoki+Kubo, '20)

Application to the neutrino option

（開き直り）

Brivo+Trott , ,17

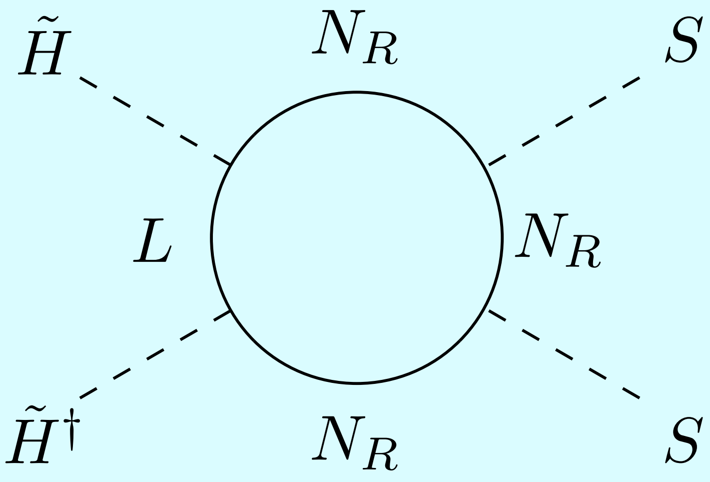


$$|\Delta\mu_H^2| \sim \frac{y_\nu^2 m_N^2}{4\pi^2}$$

~$(100\ \mathrm{GeV})^2$ if mN ~10^7 GeV and seesaw is used.

Brdar, Emonds, Helmboldt+Lindner,'19;
Brdar, Helmboldt+JK,'19;
Brivo+Trott,'19, '20;
Brdar, Helmboldt, Iwamoto+Schmitz ,'19;
Brivo, Moflat, Pascoli+Turner,'20

Scale invariant extension by Brdar, Emonds, Helmboldt+Lindner,`19, using Gildener-Weinberg.



$$|\Delta\mu_H^2| \sim \frac{y_\nu^2 m_N^2}{4\pi^2}$$

$$\frac{y_N}{2} N_R^T C N_R \langle S \rangle \; + \; \frac{\lambda_{HS}}{4} |H|^2 \langle S \rangle^2$$

$$m_N = y_N \langle S \rangle \sim 10^7 \text{ GeV} \qquad \lambda_{HS} \sim 0$$

No dark matter

Use the QCD like hidden sector, instead of the CW.

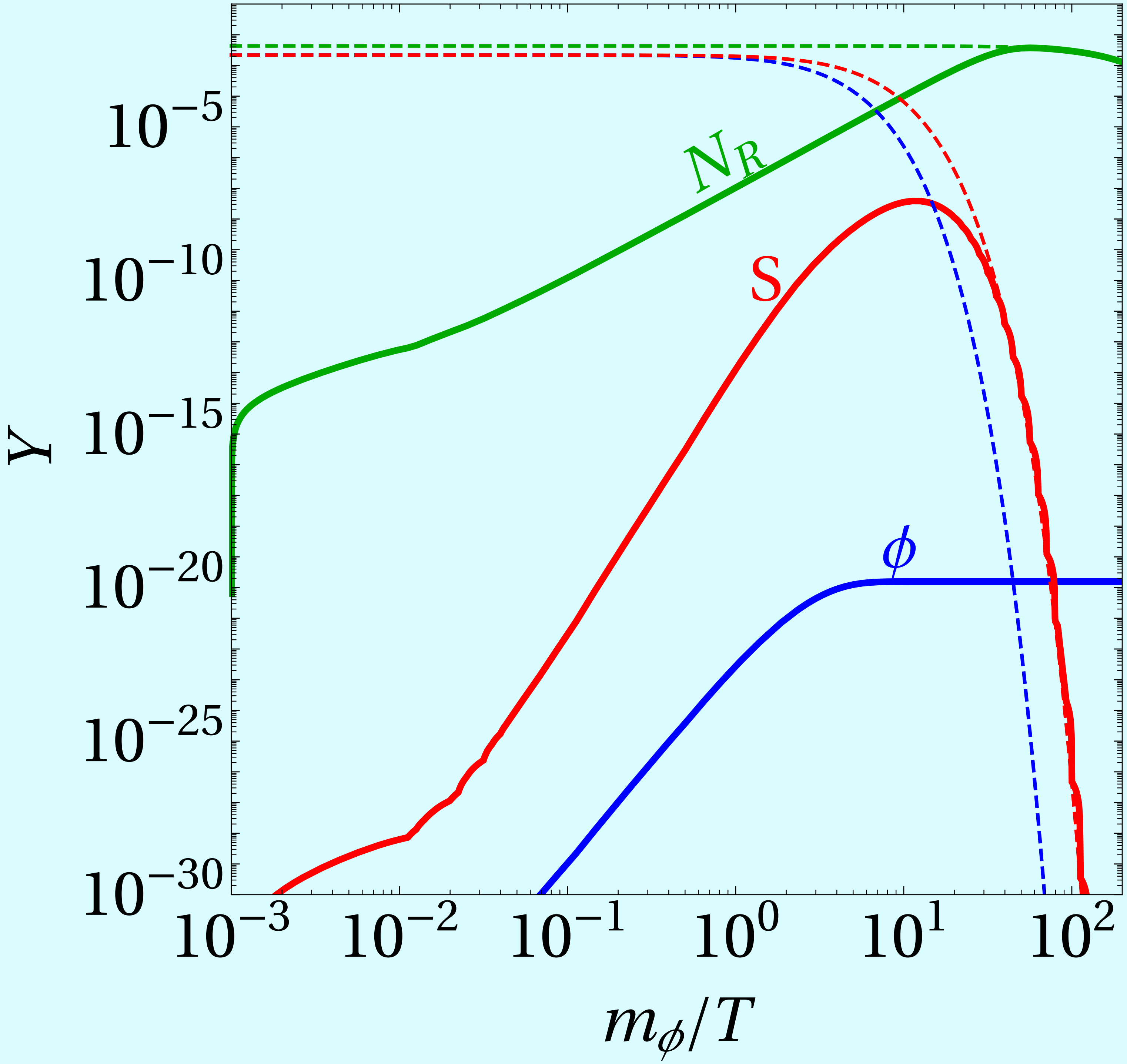
Aoki, Brdar+JK,`20.

Heavy dark matter (10^{9} GeV)

Freeze-in mechanism

Hall, Jedamzik+M-Russell, ,10

$m_{\rm DM} = 3.44 \times 10^9\ \text{GeV}\,,\ m_S = 2.07 \times 10^9\ \text{GeV}\,,\ v_\sigma = 4.17 \times 10^{10}\ \text{GeV}\,.$



Aoki, Brdar+JK,'20.

Model B
uses
the condensation of scalar bi-linear

$$\langle S^{\dagger} S \rangle \neq 0$$

Couple to the SM

JK+Lim+Lindner, '14
JK+Yamada ,'15

Dynamical generation of $M_{\rm Pl}$ and inflation

JK,Lindner+Schmitz+Yamada, '19

$$S_{\rm C} = \int d^4x\sqrt{-g}\left(-\tilde{\beta}S^\dagger S\ R + \tilde{\gamma}R^2 + \tilde{\kappa}W_{\mu\nu}W^{\mu\nu} - \frac{1}{2}{\rm Tr}\ F^2 + g^{\mu\nu}[D_\mu S]^\dagger D_\nu S - \tilde{\lambda}(S^\dagger S)^2\right)$$

Curvature portal

$$\frac{M_{\rm Pl}}{2} = \hat{\beta}\langle S^\dagger S\rangle$$

Most general form under:

1 General invariance
2 SU(N) local gauge invariance
3 Classical scale invariance

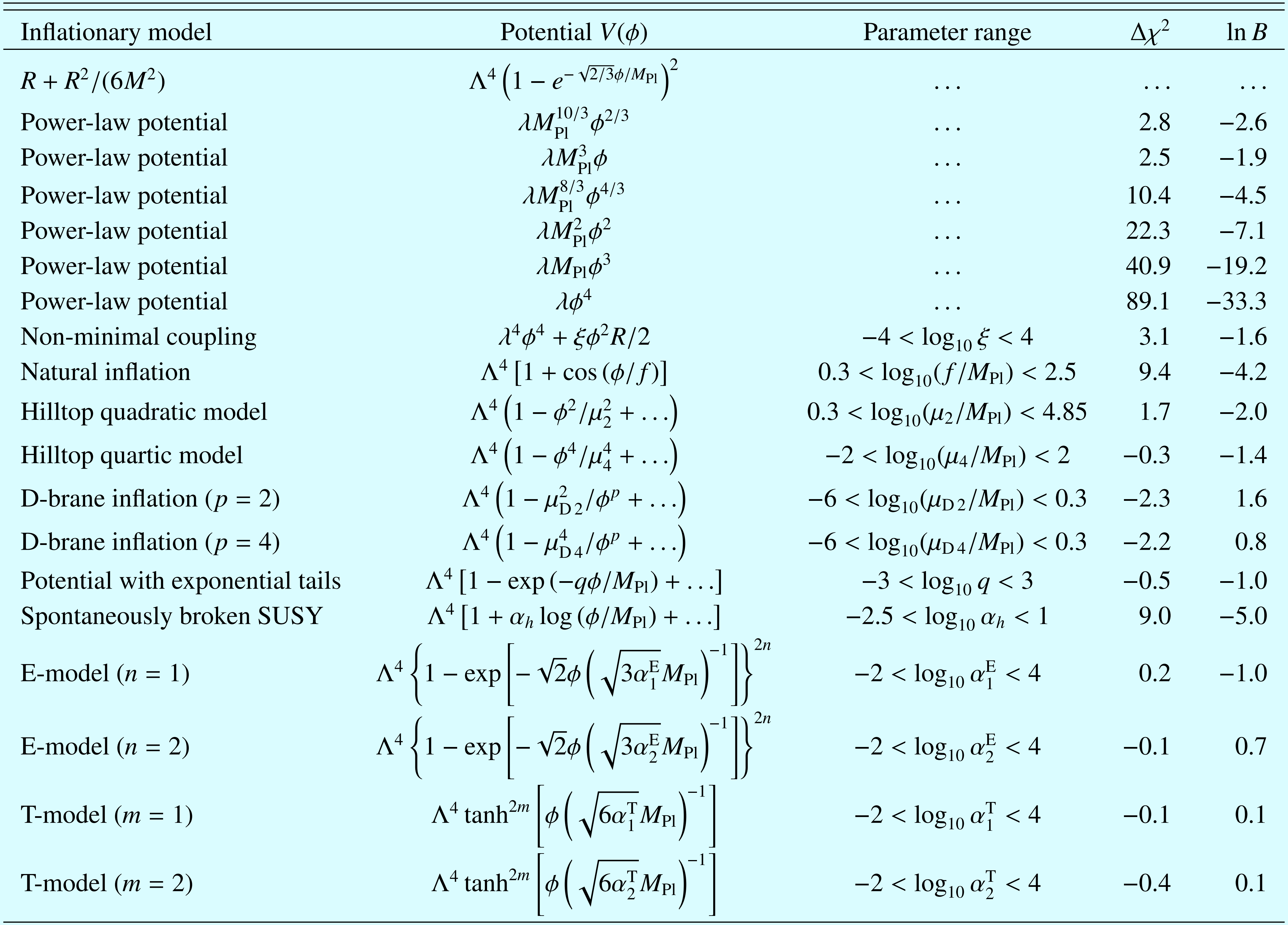
The Planck mass can be generated:

$$\frac{M_{\rm Pl}}{2} = \hat{\beta}\langle S^{\dagger}S\rangle$$

In addition there is a byproduct:

The dilaton $\sigma\,(\chi)$ may play the role of inflaton.

Inflationary models



Inflationary model	Potential $V(\phi)$	Parameter range	$\Delta\chi^2$	$\ln B$
$R + R^2/(6M^2)$	$\Lambda^4\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)^2$	...	...	...
Power-law potential	$\lambda M_{\rm Pl}^{10/3}\phi^{2/3}$	...	2.8	−2.6
Power-law potential	$\lambda M_{\rm Pl}^{3}\phi$	...	2.5	−1.9
Power-law potential	$\lambda M_{\rm Pl}^{8/3}\phi^{4/3}$	...	10.4	−4.5
Power-law potential	$\lambda M_{\rm Pl}^{2}\phi^{2}$	...	22.3	−7.1
Power-law potential	$\lambda M_{\rm Pl}\phi^{3}$	...	40.9	−19.2
Power-law potential	$\lambda\phi^{4}$	...	89.1	−33.3
Non-minimal coupling	$\lambda^4\phi^4 + \xi\phi^2 R/2$	$-4 < \log_{10}\xi < 4$	3.1	−1.6
Natural inflation	$\Lambda^4\left[1 + \cos\left(\phi/f\right)\right]$	$0.3 < \log_{10}(f/M_{\rm Pl}) < 2.5$	9.4	−4.2
Hilltop quadratic model	$\Lambda^4\left(1 - \phi^2/\mu_2^2 + \ldots\right)$	$0.3 < \log_{10}(\mu_2/M_{\rm Pl}) < 4.85$	1.7	−2.0
Hilltop quartic model	$\Lambda^4\left(1 - \phi^4/\mu_4^4 + \ldots\right)$	$-2 < \log_{10}(\mu_4/M_{\rm Pl}) < 2$	−0.3	−1.4
D-brane inflation ($p = 2$)	$\Lambda^4\left(1 - \mu_{\rm D\,2}^2/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{\rm D\,2}/M_{\rm Pl}) < 0.3$	−2.3	1.6
D-brane inflation ($p = 4$)	$\Lambda^4\left(1 - \mu_{\rm D\,4}^4/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{\rm D\,4}/M_{\rm Pl}) < 0.3$	−2.2	0.8
Potential with exponential tails	$\Lambda^4\left[1 - \exp\left(-q\phi/M_{\rm Pl}\right) + \ldots\right]$	$-3 < \log_{10} q < 3$	−0.5	−1.0
Spontaneously broken SUSY	$\Lambda^4\left[1 + \alpha_h \log\left(\phi/M_{\rm Pl}\right) + \ldots\right]$	$-2.5 < \log_{10}\alpha_h < 1$	9.0	−5.0
E-model ($n = 1$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_1^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^{\rm E} < 4$	0.2	−1.0
E-model ($n = 2$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_2^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^{\rm E} < 4$	−0.1	0.7
T-model ($m = 1$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^{\rm T} < 4$	−0.1	0.1
T-model ($m = 2$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^{\rm T} < 4$	−0.4	0.1

(Planck paper, '18)

Inflation

$$S_{\mathrm{C}} = \int d^4x\sqrt{-g}\left(-\tilde{\beta}S^\dagger S\ R + \tilde{\gamma}R^2 + \xcancel{\tilde{\kappa}W_{\mu\nu}}W^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\ F^2 + g^{\mu\nu}[D_\mu S]^\dagger D_\nu S - \tilde{\lambda}(S^\dagger S)^2\right)$$

JK+Lindner+Schmitz+Yamada, '19

Effective field theory description

$$S_{\mathrm{C,eff}} = \int d^4x\sqrt{-g}\left(\gamma R^2 + \kappa \xcancel{W_{\mu\nu\alpha\beta}}W^{\mu\nu\alpha\beta} + g^{\mu\nu}[\partial_\mu S]^\dagger \partial_\nu S - (2f\lambda + \beta R)S^\dagger S + \lambda f^2\right)$$

Integrate out S around the background S to obtain the effective potential of f.

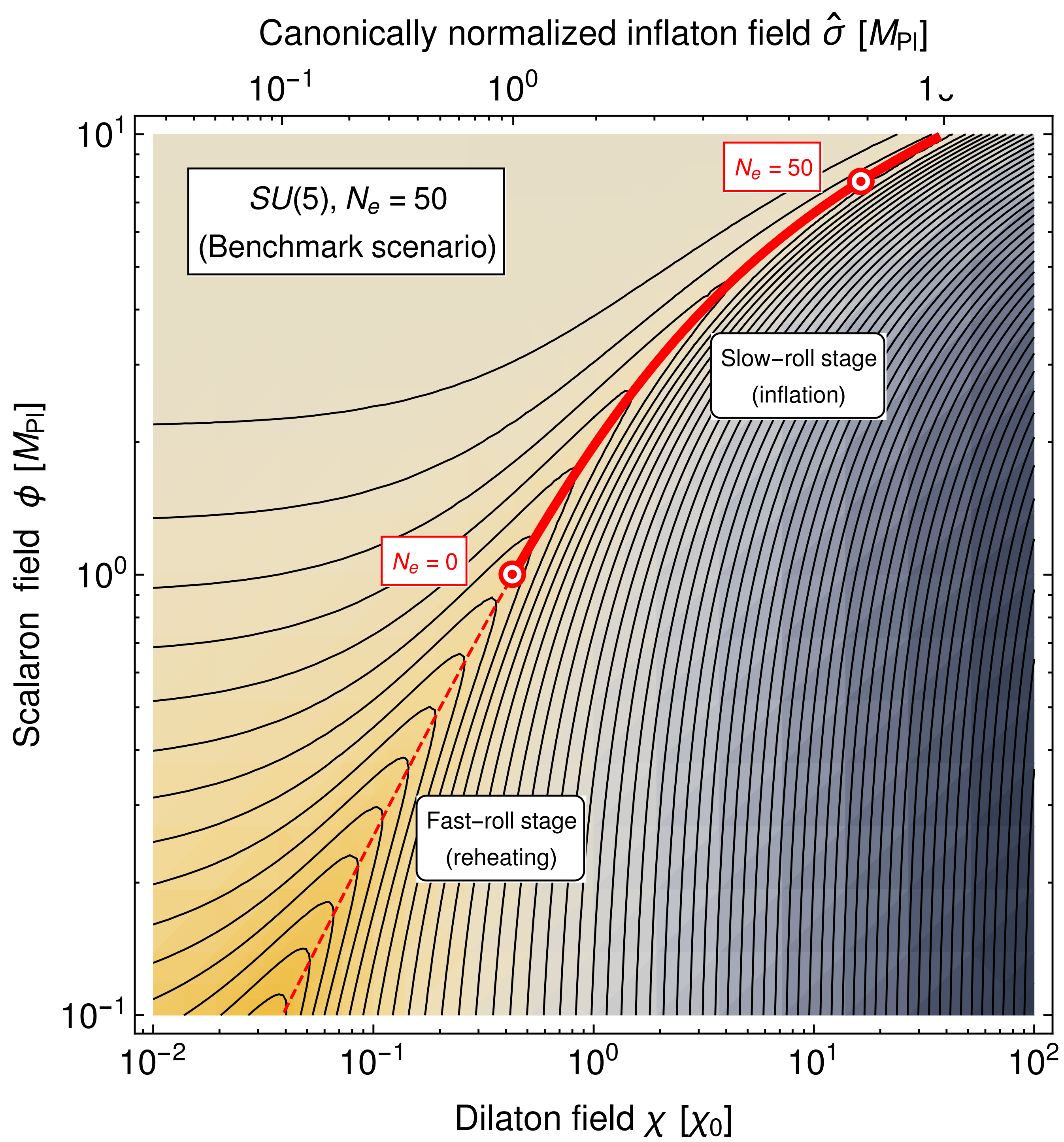
Two-field inflation system

$$\chi \ \text{(dilaton)} \quad \text{and} \quad \phi = \sqrt{\frac{3}{2}}\,M_{\rm Pl}\left(B\left(\chi\right) - \frac{4G(\chi)\psi}{M_{\rm Pl}^2}\right) \ \text{(scalaron)}$$

Deep potential valley along a line

Effective single-field inflation system

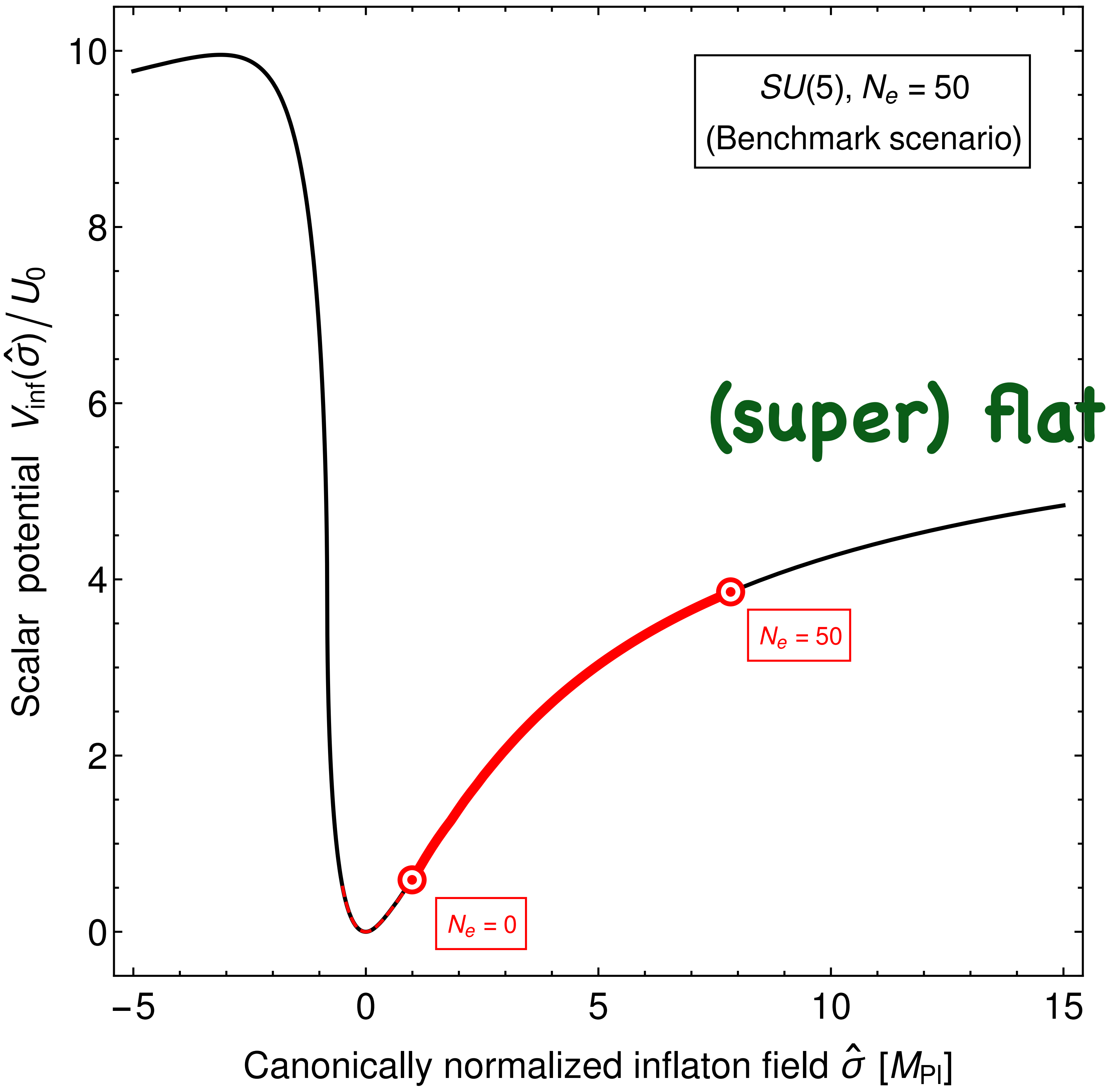
Equipotential lines (Jordan frame)



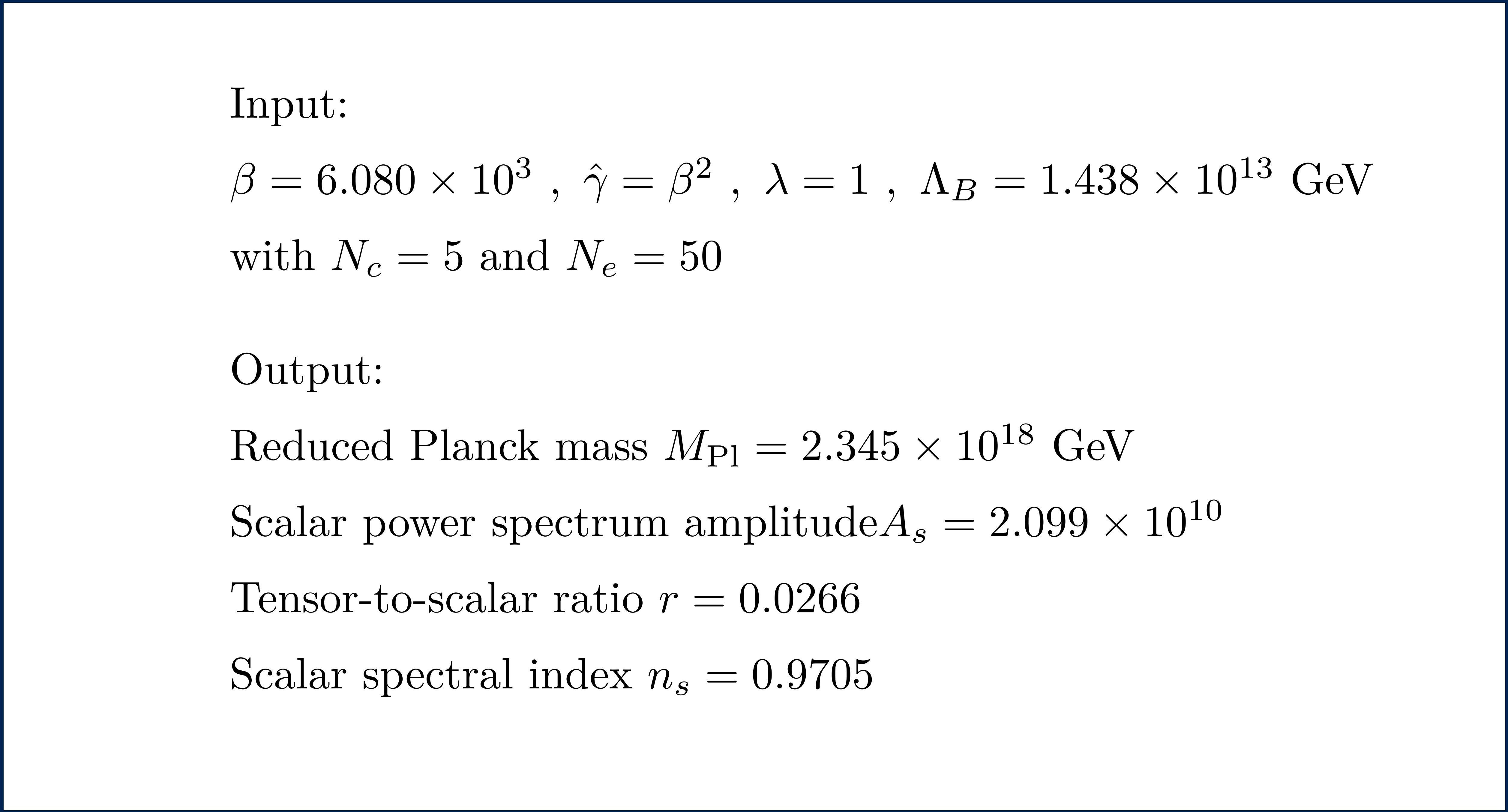
$$\beta = 6.080 \times 10^3 \ , \ \ \hat{\gamma} = \beta^2 \ , \ \ \lambda = 1 \ , \ \ \Lambda_B = 1.438 \times 10^{13} \ \text{GeV}$$

$$S_{\rm C} = \int d^4x \sqrt{-g} \left(-\tilde{\beta} S^\dagger S \ R + \tilde{\gamma} R^2 + \xcancel{\tilde{\kappa} W_{\mu\nu} W^{\mu\nu}} - \frac{1}{2} \text{Tr} \ F^2 + g^{\mu\nu} [D_\mu S]^\dagger D_\nu S - \tilde{\lambda} (S^\dagger S)^2 \right)$$

Potential along the valley (Einstein frame)



Parameters and observables: Benchmark point



Input:

$\beta = 6.080 \times 10^3$, $\hat{\gamma} = \beta^2$, $\lambda = 1$, $\Lambda_B = 1.438 \times 10^{13}$ GeV

with $N_c = 5$ and $N_e = 50$

Output:

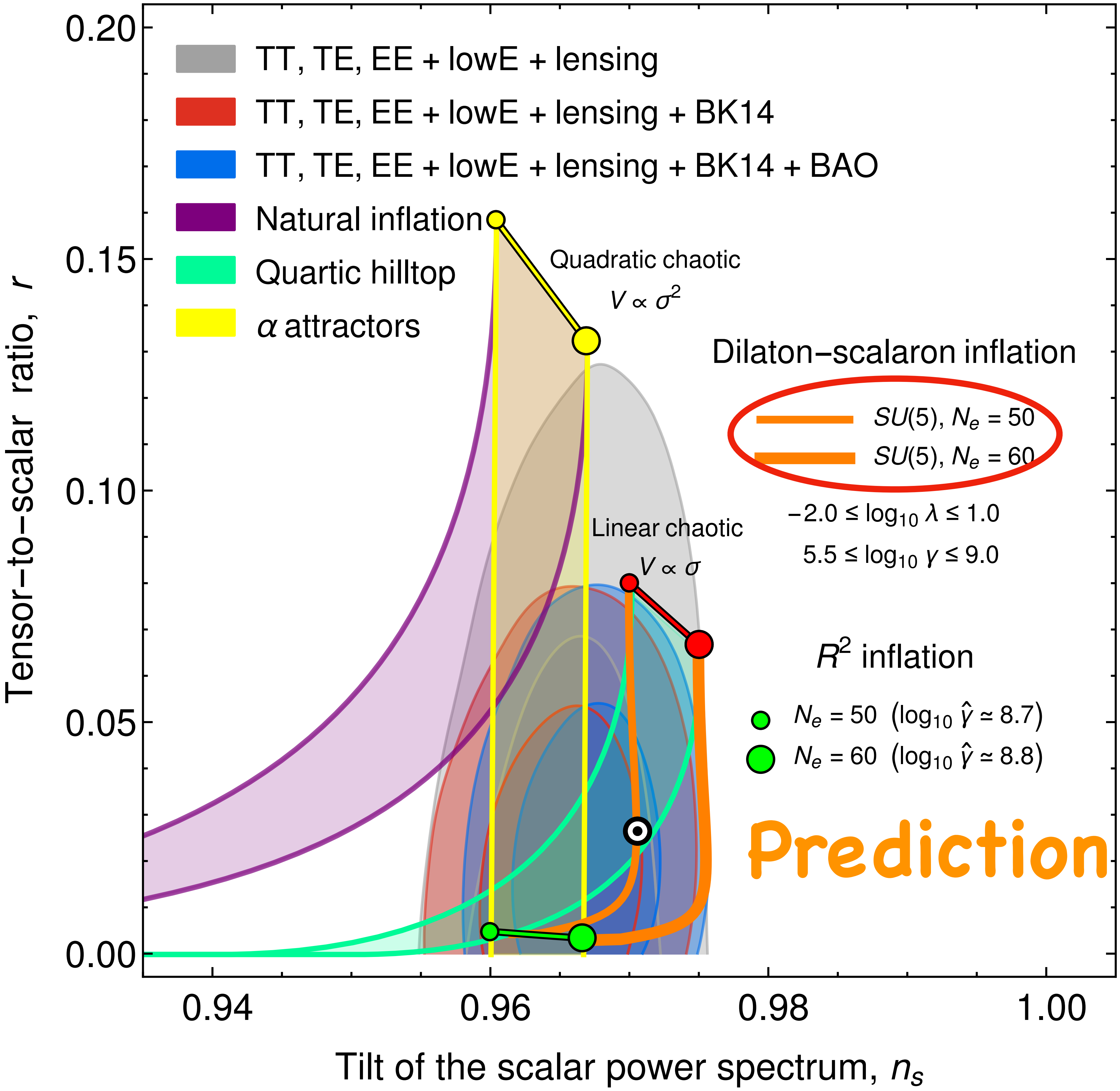
Reduced Planck mass $M_{\rm Pl} = 2.345 \times 10^{18}$ GeV

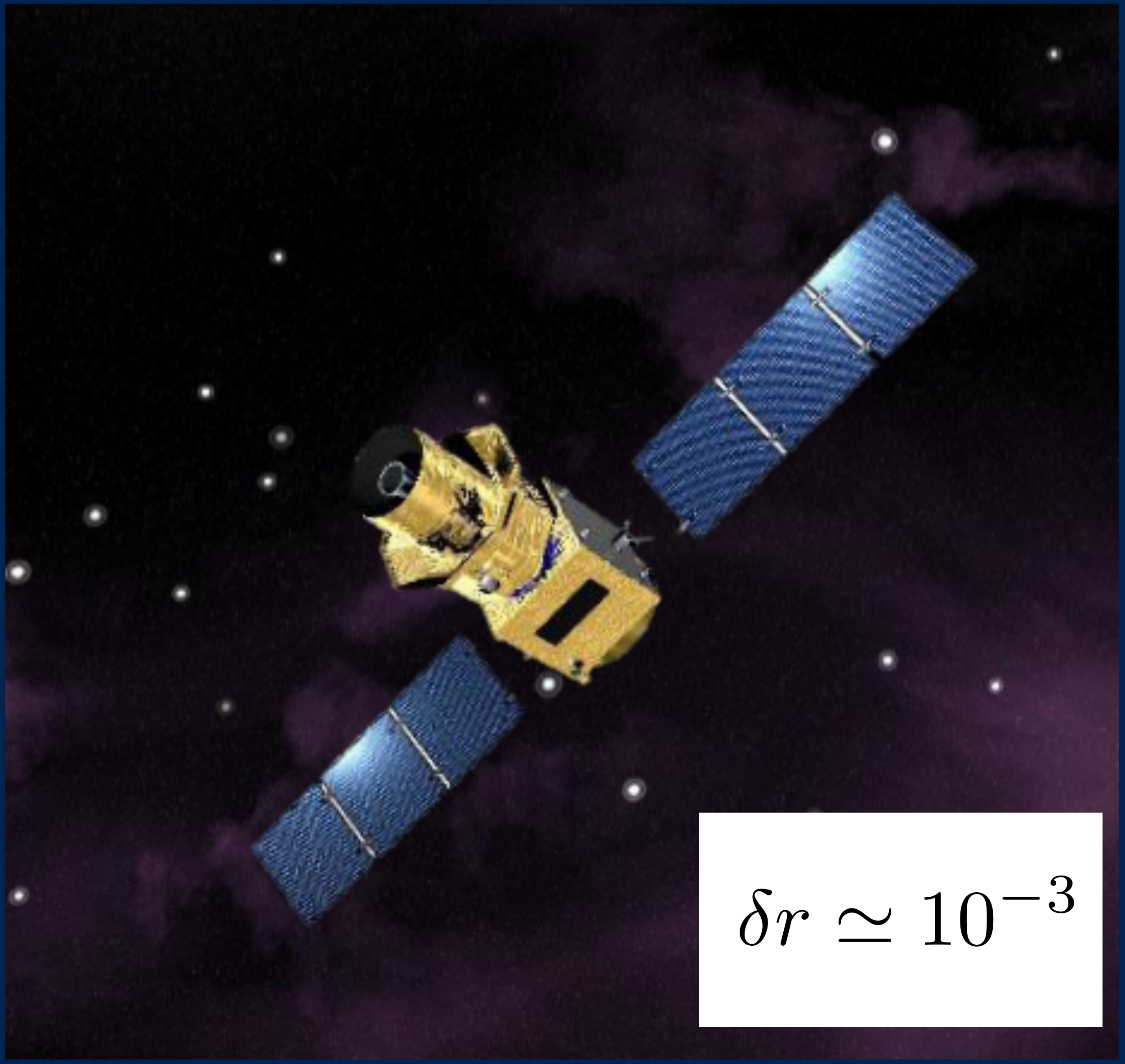
Scalar power spectrum amplitude$A_s = 2.099 \times 10^{10}$

Tensor-to-scalar ratio $r = 0.0266$

Scalar spectral index $n_s = 0.9705$

JK+Lindner+Schmitz+Yamada, ‘19


0.20
0.15
0.10
0.05
0.00
0.94
0.96
0.98
1.00
Tensor–to–scalar ratio, r
Tilt of the scalar power spectrum, n_s
TT, TE, EE + lowE + lensing
TT, TE, EE + lowE + lensing + BK14
TT, TE, EE + lowE + lensing + BK14 + BAO
Natural inflation
Quartic hilltop
α attractors
Quadratic chaotic
$V \propto \sigma^2$
Linear chaotic
$V \propto \sigma$
Dilaton–scalaron inflation
$SU(5)$, $N_e = 50$
$SU(5)$, $N_e = 60$
$-2.0 \leq \log_{10} \lambda \leq 1.0$
$5.5 \leq \log_{10} \gamma \leq 9.0$
R^2 inflation
$N_e = 50$ $(\log_{10} \hat{\gamma} \simeq 8.7)$
$N_e = 60$ $(\log_{10} \hat{\gamma} \simeq 8.8)$
Prediction



$$\delta r \simeq 10^{-3}$$

LiteBIRD (JAXA, KEK,… project) can confirm our prediction!

Conclusion

- Scale invariant extension of the SM may provide a solution to the fine-tuning problem: Quantum corrections are at most logarithmic.

- Good reasons for SI extension of the SM

- But:
  Hierarchy of dimensionless parameters
  is sometimes needed to explain mass hierarchy.

ありがとうございました。