Reflection symmetries and universal four-zero texture

素粒子現象論研究会@大阪市立大, Nov. 26, 2020

梁　正樹 (PD) @ 埼玉大学

Arxiv:2003.11701v3　Chin.Phys.Cに投稿中

概要：

Reflection symmetriesというSMの一般化されたCP 対称性を提案

⇒ Majorana位相を $\alpha_{2,3}/2 = 0$ or $\pi/2$に固定 (+ $\theta_{\rm QFD}^{\rm tree} = 0$)

+ universal four-zero texture

⇒ すべてのfermion 質量とV_{CKM} & U_{MNS}を再現し、

δ_{CP} ≒ 203°, 通常質量階層、m1 ≒ 2.5 or 6.2 meVを予言

秋の学会発表
ここまで

+ 対称性の実現のために2HDM+$U(1)_{PQ}$ × CPでSSBを考えた

⇒ $f_a \sim M_{\rm GUT}\sqrt{m_{u,d}m_{c,s}}/v \sim 10^{12}$ GeV ⇒ $\Omega_a h^2 \sim 0.2$.　axion DM??

Reflection symmetries and universal four-zero texture

and flavored axion

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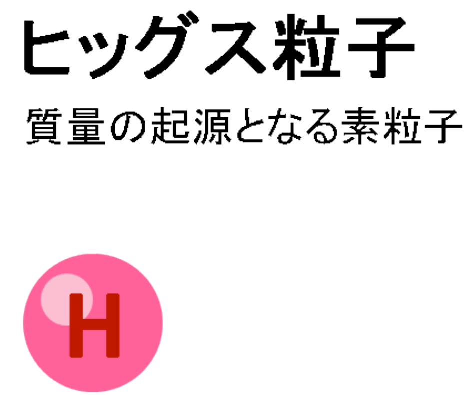
Great success of the SM



from http://www.u-tokyo.ac.jp/ja/utokyo-research/feature-stories/atlas2012/

Unsolved problems

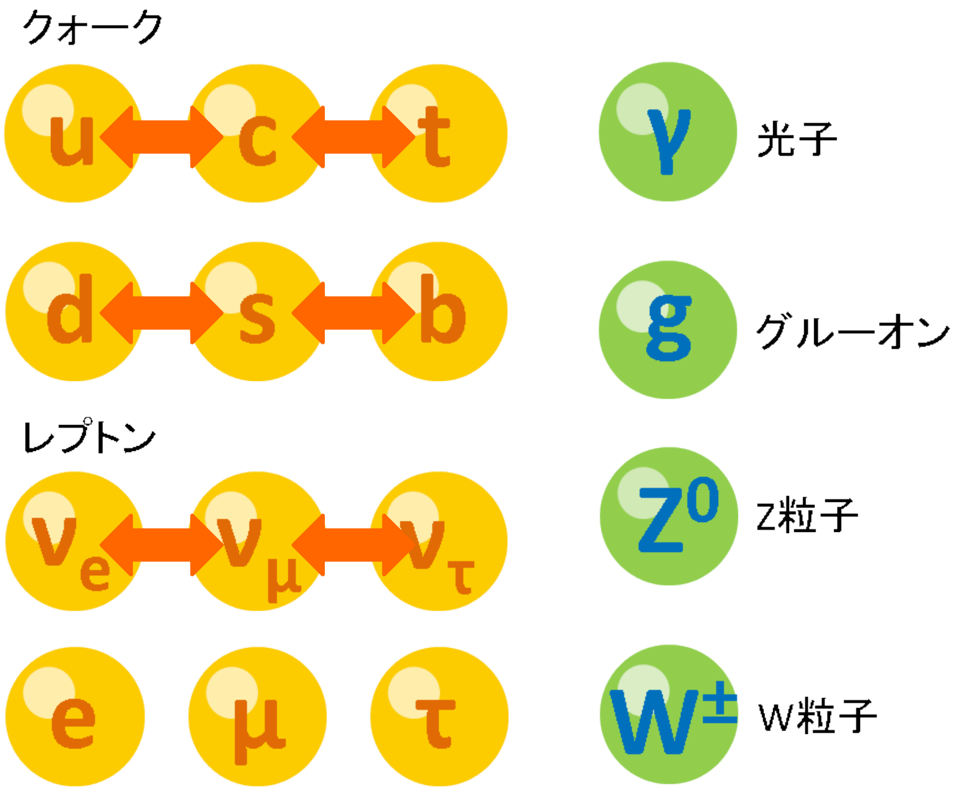
- Neutrino mass
- Flavor puzzle
- Dark matter, etc…

Great success of the SM

物質粒子
物質を形成する素粒子

ゲージ粒子
力を伝える素粒子

ヒッグス粒子
質量の起源となる素粒子



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世代の謎（Flavor puzzle）：概観

- **Flavor symmetry** (top-down)
  - $U(1)_{FN}$ Froggatt & Nielsen, 1979, **1600cited**
  - SU(2), SU(3), …
  - S_3 , Harari, Haut, Weyers, 1978, A_4, S_4, …
  - Family unification Ramond 1979, Wilczek & Zee, 1982

    SO(16), SO(18), E_7, E_8, etc, ⊃ SO(10)×SU(3)
- **Flavor texture** (bottom-up)
  - 6-zero texture Weinberg 1977, Fritzsch 1977, **800cited**
  - n(=2,3,4..)-zero texture
  - Democratic texture Harari, Haut, Weyers, 1978
  - μ-τ symmetry Fukuyama and Nishiura 1997
  - Generalized CP sym. Ecker, Grimus, Konetschny '80, Holthausen et al, '12, Feruglio, et al, '12
  - Etc..

世代構造　⇒　Higgsが何かのヒント？

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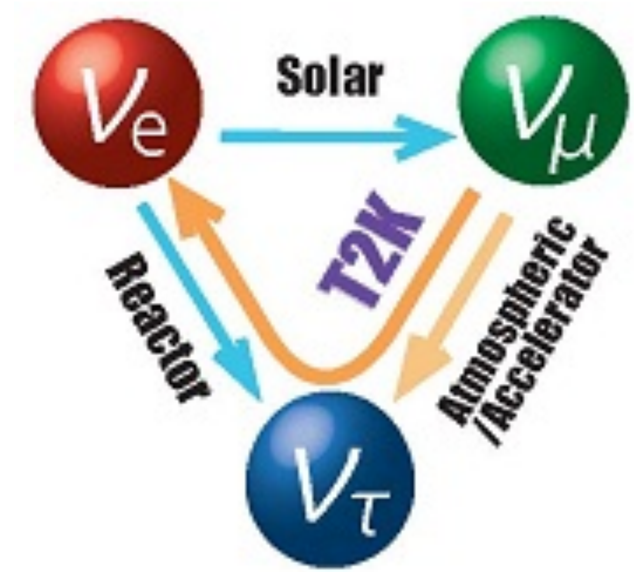
Neutrino oscillation

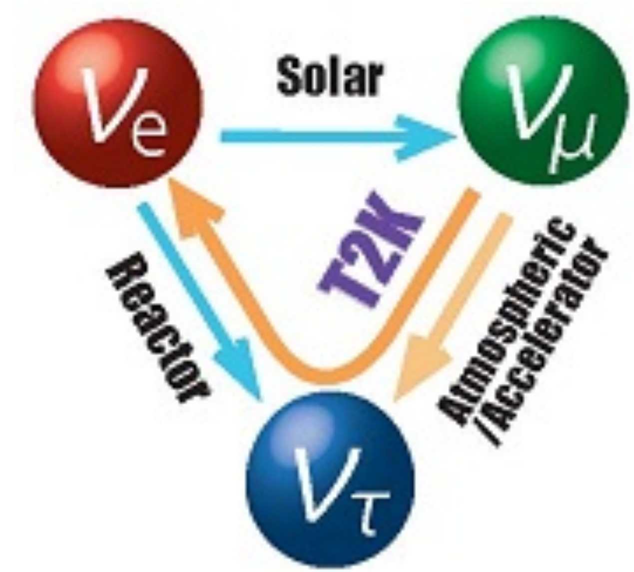


MNS matrix $U_{\mathrm{MNS}} = U_e U_\nu^\dagger.$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric reactor solar Majorana phase

Neutrino oscillation



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From PDG 2020 — atmospheric — reactor — solar — Majorana phase

$$\theta_{23} \simeq 49.2 \pm 1^\circ, \quad \theta_{13} = 8.6 \pm 0.1^\circ, \quad \theta_{12} = 33.4 \pm 0.8^\circ \quad (1\sigma)$$

Neutrino oscillation

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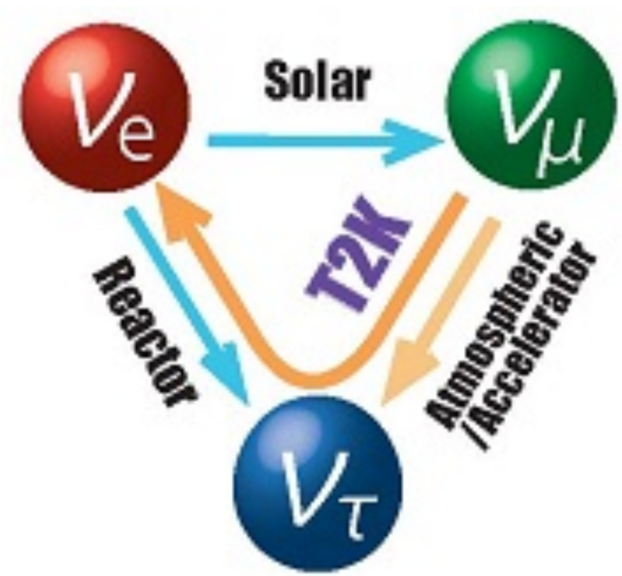

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atmospheric reactor solar Majorana phase

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sin θ_{23} ≒ 1/√2, bi-maximal?

sin θ_{12} ≒ 1/√3, tri-maximal?

μ-τ symmetry

Fukuyama and Nishiura 97,
C. S. Lam 01, E. Ma and M. Raidal 01

bi-maximal 混合を保証するZ_2 symmetry

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad T m_f T = m_f, \qquad m_f = \begin{pmatrix} a & b & b \\ c & d & e \\ c & e & d \end{pmatrix}$$

2-3世代の45°回転 (最大混合) で変換すると、 $a \sim e \in \mathbb{C}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} a & b & b \\ c & d & e \\ c & e & d \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} a & \sqrt{2}b \\ \sqrt{2}c & d+e \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & d-e \end{pmatrix}$$

残りは1-2回転だけで対角化

もう一方の質量行列が対角的な基底で $\Rightarrow \theta_{23} = \pm 45^\circ$

Note: TBMと$K_4 = Z_2 \times Z_2$ 対称性

C.S. Lam, PRD 74, 113004, (2006), PLB 656, 193-198, (2007), PRL 101, 121602 (2008).

２つの離散対称性が厳密なTri-Bi-Maximal 混合を予言

・ μ-τ symmetry

2-3 flavorの交換

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad T m_f T = m_f,$$

・ magic symmetry

どんな行や列の和も等しい（魔法陣）

$$S = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \qquad S m_f S = m_f,$$

C. S. Lam, Phys. Lett. B 640 (2006) 260, P. F. Harrison and W. G. Scott, Phys. Lett. B 594, 324 (2004)
R. Friedberg and T. D. Lee, HEPNP 30, 591 (2006).

この$K_4 = Z_2 \times Z_2$ 対称性（$\triangleleft A_4$）を課すとTBMで対角化

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \underbrace{\begin{pmatrix} c+d-b & b & b \\ b & c & d \\ b & d & c \end{pmatrix}}_{\text{3 parameters}} \boxed{\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}} = \begin{pmatrix} c+d-2b & 0 & 0 \\ 0 & c+d+b & 0 \\ 0 & 0 & c-d \end{pmatrix}$$

それぞれのZ_2対称性が固有ベクトルを固定

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線形代数の試験問題を
無限に生成可能

C. S. Lam, Phys. Lett. B 640 (　W. G. Scott, Phys. Lett. B 594, 324 (2004)
R. Friedberg and T. D. Lee, HE

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3 parameters

それぞれのZ_2 対称性が固有ベクトルを固定

μ-τ reflection symmetry

Harrison & Scott, ‘02
Grimus & Lavoura ‘02

= μ-τ symmetry + 複素共役

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T m_f^* T = m_f, \qquad m_f = \begin{pmatrix} a = a^* & b & b^* \\ c & d & e \\ c^* & e^* & d^* \end{pmatrix}$$

$$a \sim e \in \mathbb{C}$$

23 最大混合で回すと、

$$U_{BM} m_f U_{BM}^\dagger = \begin{pmatrix} a & \sqrt{2}\,\mathrm{Re}\,b & -\sqrt{2}i\,\mathrm{Im}\,b \\ \sqrt{2}\,\mathrm{Re}\,c & \mathrm{Re}\,(d+e) & i\,\mathrm{Im}\,(e-d) \\ -\sqrt{2}i\,\mathrm{Im}\,c & i\,\mathrm{Im}\,(-e-d) & \mathrm{Re}\,(d-e) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix} m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

m_0 は一般的な実行列 (9 parameters).

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23 最大混合で回すと、

pure imaginal

real

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$$\delta_{CP} = \pm 90^\circ,$$

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Harrison & Scott, '02
Grimus & Lavoura '02

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実行列 m_0 は直交行列で対角化

$$R_L\, m_0\, R_R^T = \begin{pmatrix} \pm m_1 & 0 & 0 \\ 0 & \pm m_2 & 0 \\ 0 & 0 & \pm m_3 \end{pmatrix}.$$

$\Rightarrow\ \alpha_{2,3}/2 = 0 \text{ or } \pm\pi/2$

負の固有値 ⇒

最大マヨラナ位相

μ-τ reflection は一般化されたCP対称性のひとつ

a generalized CP (GCP) symmetry

Ecker, Grimus, Konetschny '80,
Holthausen et al, '12, Feruglio, et al, '12

$$\psi_i' = U_{ij} C \psi_j^*(t, -\boldsymbol{x})\,, \quad m_\psi' = U_L m_\psi^* U_R^\dagger\,,$$

μ-τ (reflection) symmetry

Fukuyama and Nishiura 97,
C. S. Lam 01, E. Ma and M. Raidal 01

- μ-τ symmetry $Tm_fT = m_f,$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad m_f = \begin{pmatrix} a & b & b \\ c & d & e \\ c & e & d \end{pmatrix}$$

$a \sim e \in \mathbb{C}$

(in the diagonal basis)

$\Rightarrow \theta_{23} = \pm 45^\circ$

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- Reflection symmetry $Rm_f^*R = m_f,$

M. Yang, '20, submitting to Chin.Phys.C

$$R = \text{diag}\,(-1, 1, 1), \qquad m_f = \begin{pmatrix} A & iB & iC \\ iD & F & G \\ iE & H & K \end{pmatrix}$$

$A \sim K \in \mathbb{R}$

$\delta \sim \pm 90^\circ,$

$\alpha_{2,3}/2 = 0 \text{ or } \pm\pi/2$

動機：Universal texture

Koide, Nishiura, Matsuda, Kikuchi, Fukuyama, '02

すべてのfermionに23対称性と0 textureを課す

$$T\hat{m}_f T = \hat{m}_f, \quad (m_f)_{11} = 0,$$

$$\hat{m}_f = \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix}$$

$A_f \sim C_f \in \mathbb{R}$

質量と混合がだいたい上手くいくが、

$$\Rightarrow \quad \sin\theta_{13} \simeq 0.05$$

2020現在: $\sin\theta_{13} \simeq 0.15$

課題：θ_{13}を大きくしたい

CP位相も含めて統一的textureにしたい

⇒ μ-τ reflection symmetryを用いる

統一的なμ-τ reflection

M. Yang, ’20, PLB 806

- 同じμ-τ reflectionを課すと、相殺してδ_{CKM} = 0になる
- ⇒ 別々のμ-τ reflection symmetriesが必要

$$\boxed{T_u(m_u^{BM})^* T_u = m_u^{BM}, \quad T_d(m_d^{BM})^* T_d = m_d^{BM},}$$

$$T_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

また、仮定として $m_f^\dagger = m_f, \;\; (m_f)_{11} = 0, \;\; (m_f)_{12} = (m_f)_{13}$

→ 質量固有値とCKM混合はOK

Four-zero textureとの関係

階層的基底でfour-zero textureになる

$$m_f^{BM} \equiv U_{BM}^\dagger m_f U_{BM},$$

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

Fritzsch & Xing, 95

$$A \sim K \in \mathbb{R}$$

（元々のUniversal texture では$B_f = 0$）

近似的に 12 × 23 混合のみで対角化 ⇒ CKM行列の良い記述

$$V_{\mathrm{CKM}} = O_u^\dagger \,\mathrm{diag}\,(-i,1,1)\, O_d$$

$$= \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\delta_{\mathrm{FZ}}} & 0 & 0 \\ 0 & c_q & s_q \\ 0 & -s_q & c_q \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mu - \tau$ reflection symmetries $\Rightarrow \delta_{\mathrm{FZ}} = 90^\circ$ 　観測 $\delta_{\mathrm{FZ}} = 87.9^\circ$

Four-zero textureとの関係

階層的基底でfour-zero textureになる

Hermite性を課したとき、
(mf)$_{11}$=0で１つ, (mf)$_{13}$=(mf)$_{31}$=0で１つ

$$m_f^{BM} \equiv U_{BM}^\dagger m_f U_{BM},$$

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

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近似的に 12 × 23 混合のみで対角化 ⇒ CKM行列の良い記述

$$V_{\mathrm{CKM}} = O_u^\dagger \,\mathrm{diag}\,(-i,1,1)\, O_d$$

$$= \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\delta_{\mathrm{FZ}}} & 0 & 0 \\ 0 & c_q & s_q \\ 0 & -s_q & c_q \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mu - \tau$ reflection symmetries $\Rightarrow \delta_{\mathrm{FZ}} = 90^\circ$ 　　観測 $\delta_{\mathrm{FZ}} = 87.9^\circ$

Four-zero textureとの関係

階層的基底でfour-zero textureになる

Hermite性を課したとき、
(mf)$_{11}$=0で１つ, (mf)$_{13}$=(mf)$_{31}$=0で１つ

$$m_f^{BM} \equiv U_{BM}^\dagger m_f U_{BM},$$

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

Fritzsch & Xing, 95

$$A \sim K \in \mathbb{R}$$

（元々のUniversal textureではB_f = 0）

近似的に 12 × 23 混合のみで対角化 ⇒ CKM行列の良い記述

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Maximal CPV！

$\mu - \tau$ reflection symmetries $\Rightarrow \delta_{\rm FZ} = 90^\circ$　　観測　$\delta_{\rm FZ} = 87.9^\circ$

Reflection symmetries

M. Yang, '20, Chin. Phys. Cに投稿中

この階層的基底において、μ-τ reflectionは

対角的なgeneralized CP 対称性になる $\psi_i' = U_{ij} C \psi_j^*(t, -\boldsymbol{x})\,,$

Ecker, Grimus, Konetschny '80, Holthausen et al, '12, Feruglio, et al, '12

$$-U_{BM}^* T_u U_{BM}^\dagger = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv R, \quad U_{BM}^* T_d U_{BM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1_3.$$

$$R m_u^* R = m_u, \qquad m_d^* = m_d.$$

$$m_u = \begin{pmatrix} a_u & \boxed{ib_u \quad ic_u} \\ \boxed{\begin{matrix} -ib_u \\ -ic_u \end{matrix}} & \begin{matrix} d_u & e_u \\ e_u & f_u \end{matrix} \end{pmatrix}, \quad m_d = \begin{pmatrix} a_d & b_d & c_d \\ b_d & d_d & e_d \\ c_d & e_d & f_d \end{pmatrix},$$

pure imaginal

片方の第１世代のみにCP位相を集中する

Reflection symmetries

M. Yang, ’20, Chin. Phys. Cに投稿中

この階層的基底において、μ-τ reflectionは

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片方の第1世代のみに
CP位相を集中する

pure imaginal

μ-τ reflection symmetriesからμ-τ symmetry を引いたもの

→ **reflection symmetries**

Universal four-zero texture

すべてのfermionに同じ対称性とtextureを課す

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv R, \qquad R m_{u,\nu}^* R = m_{u,\nu}, \quad m_{d,e}^* = m_{d,e}.$$

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} 0 & iC_\nu & 0 \\ iC_\nu & \tilde{B}_\nu & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & C_e & 0 \\ C_e & \tilde{B}_e & B_e \\ 0 & B_e & A_e \end{pmatrix},$$

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$$\boxed{\begin{aligned} m_u &= \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix} \\ m_\nu &= \begin{pmatrix} 0 & iC_\nu & 0 \\ iC_\nu & \tilde{B}_\nu & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & C_e & 0 \\ C_e & \tilde{B}_e & B_e \\ 0 & B_e & A_e \end{pmatrix}, \end{aligned}}$$

Concise and Elegant !!

Universal four-zero texture

すべてのfermionに同じ対称性とtextureを課す

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Concise and Elegant !!

レプトンは4 × 2 parameter ← 3 lepton mass, 2 neutrino mass, 3 mixing

⇒ m1, δCP, α2,3を決定

$$U_{\rm MNS} = V_e^T \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_\nu P_M,$$

four-zero textureの正当性（またあとで）

- YukawaのHermite性

  ← $SU(2)_L \times SU(2)_R$ modelでのパリティ対称性

$$\mathcal{L} \ni \bar{\psi}_{Li} Y_{fij} \Phi \psi_{Rj} + \bar{\psi}_{Ri} Y^\dagger_{fij} \Phi^* \psi_{Lj},$$

$$P\ : \psi_L \leftrightarrow \psi_R,\ \Phi^* \leftrightarrow \Phi \quad \Rightarrow \quad Y_f = Y_f^\dagger$$

- $(m_f)_{11} = 0$

  しばしば離散対称性で実現

  第１世代の質量を禁止するカイラル対称性を示唆

- $(m_f)_{12} \neq 0$, $(m_f)_{13} = 0$,

  やや非自明だが、LR modelで正当化可能（あとで）

Calculation of U_{MNS}

νは階層が弱い→対角化は13混合を無視できない

$$V_{\rm CKM} \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & 0 \\ -\sqrt{\frac{m_u}{m_c}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & c_q & s_q \\ 0 & -s_q & c_q \end{pmatrix} \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & 0 \\ \sqrt{\frac{m_d}{m_s}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{\rm MNS} = UP, \quad P \equiv {\rm diag}(1, e^{i\alpha_2/2}, e^{i\alpha_3/2}).$$

$$U \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_e}{m_\mu}} & 0 \\ -\sqrt{\frac{m_e}{m_\mu}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \boxed{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Ue3はO.K. (an input parameter)

$$|U_{e3}| = |s_{13}^{PDG}| \simeq |is_{13} - \sqrt{\frac{m_e}{m_\mu}} c_{13} s_{23}|.$$

Calculation of U_{MNS}

Jarlskog invariant → Dirac phase

$$J = -\mathrm{Im}\left[U_{\mu 3}U_{\tau 2}U^*_{\mu 2}U^*_{\tau 3}\right] \tag{36}$$

$$\simeq \sqrt{m_e/m_\mu}\, c_{13}c_{23}[-c_{12}s_{12}s^2_{23} + s_{13}c_{23}s_{23}(c^2_{12} - s^2_{12}) + s^2_{13}c_{12}s_{12}c^2_{23}] = -0.0130, \tag{37}$$

$$\sin\delta_{CP} \simeq \sqrt{\frac{m_e}{m_\mu}}\frac{c_{13}s_{23}}{s_{13}}$$

$$\boxed{\delta_{CP} \simeq 203^\circ}$$

This value is rather close to the best fit for the normal hierarchy and in the 1σ region $\delta_{CP}/^\circ = 217^{+40}_{-28}$ [40]. This

[40] Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni, and Schwetz, JHEP 01, 106 (2019), 1811.05487.

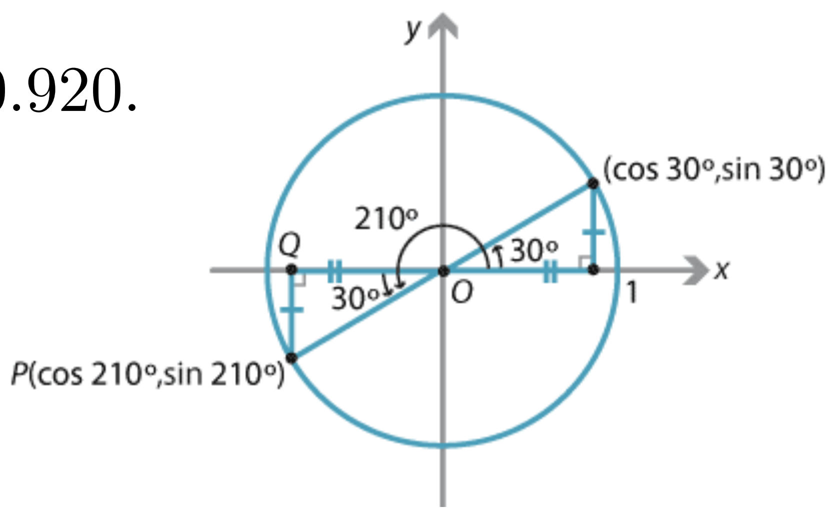
保険用ページ

δCP は第３象限に存在

$$\cos\delta_{CP} = \frac{|U_{22}^{PDG}|^2 - (s_{12}^{PDG}s_{13}^{PDG}s_{23}^{PDG})^2 - (c_{12}^{PDG}c_{23}^{PDG})^2}{-2s_{12}^{PDG}s_{13}^{PDG}s_{23}^{PDG}c_{12}^{PDG}c_{23}^{PDG}} \tag{40}$$

$$= \frac{|U_{22}|^2(1-|U_{13}|^2)^2 - |U_{13}|^2|U_{12}|^2|U_{23}|^2 - |U_{11}|^2|U_{33}|^2}{-2|U_{13}||U_{12}||U_{23}||U_{11}||U_{33}|} \tag{41}$$

$$= -0.920. \tag{42}$$



物理的予言

ニュートリノ質量行列を再構成できる

$$m_\nu = V_e U_{\rm MNS} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\rm MNS}^T V_e^T.$$

(m_e が4-zero texture の基底で)

0 textureから予言が発生

$$(m_\nu)_{11} = (m_\nu)_{13} = 0 \ \Rightarrow$$

$$m_1 = \frac{-e^{i\alpha_2} m_2 s_{12}^2 - e^{i\alpha_3} m_3 t_{13}^2}{c_{12}^2}, \ \simeq 2.5\,[\mathrm{meV}], \quad (\alpha_2, \alpha_3) = (\pi, 0).$$
$$\simeq -6.2\,[\mathrm{meV}], \quad (\alpha_2, \alpha_3) = (0, 0).$$

Double beta decayのeffective mass

$$|m_{ee}| = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| \simeq 1.24\,[\mathrm{meV}]\,(\alpha_2, \alpha_3) = (\pi, 0)$$
$$\simeq 0.17\,[\mathrm{meV}]\,(\alpha_2, \alpha_3) = (0, 0).$$

(逆階層では解なし、textureと矛盾)

再構成された質量行列の数値

(摂動が悪いかも、なので2〜5%くらい誤差あるかも)

Z.-z. Xing & Z.-h. Zhao, NPB 897, '15

$$Y_u \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002i & 0 \\ -0.0002i & \boxed{0.10} & \boxed{0.31} \\ 0 & \boxed{0.31} & \boxed{1} \end{pmatrix} \qquad Y_d \simeq \frac{0.9 m_b \sqrt{2}}{v} \begin{pmatrix} 0 & 0.005 & 0 \\ 0.005 & \boxed{0.13} & \boxed{0.31} \\ 0 & \boxed{0.31} & \boxed{1} \end{pmatrix}$$

O(mt) O(mb)

$m_1 \simeq 2.5[\mathrm{meV}], \quad (\alpha_2, \alpha_3) = (\pi, 0).$ の解

$$m_{\nu 0} \simeq \begin{pmatrix} 0 & -8.86i & 0 \\ -8.86i & 29.3 & 26.4 \\ 0 & 26.4 & 14.6 \end{pmatrix} [\mathrm{meV}] \qquad m_e \simeq \begin{pmatrix} 0 & -7.058 & 0 \\ -7.058 & 107.873 & 96.12 \\ 0 & 96.12 & \boxed{1740} \end{pmatrix} [\mathrm{MeV}]$$

O(mτ)

（もう一方の解は$(m_e)_{22}$〜m_τでやや不自然）

素朴には d-e 統一に失敗...？？

(エルミート性か$(m_d)_{13} = 0$を放棄 ...？)

シーソー機構

Minkowski ‘77, Yanagida ‘79,
Mohapatra & Senjanovic ‘80

Four-zero textureとreflection symmetriesはSeesaw不変

⇒ Y_νにこれを課せばM_Rも従う

Nishiura, Matsuda, and Fukuyama ’99,

$$Y_\nu = Y_u \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002i & 0 \\ -0.0002i & 0.10 & 0.31 \\ 0 & 0.31 & 1 \end{pmatrix}$$

とすると

$$M_R = \frac{v^2}{2} Y_\nu m_{\nu 0}^{-1} Y_\nu^T$$
$$= \begin{pmatrix} 0 & -1.08\, i \times 10^8 & 0 \\ -1.08\, i \times 10^8 & 1.26 \times 10^{14} & 4.07 \times 10^{14} \\ 0 & 4.07 \times 10^{14} & 1.32 \times 10^{15} \end{pmatrix} [\mathrm{GeV}].$$

$$\Rightarrow \; R M_R^* R = M_R. \qquad (M_{R1}\, , M_{R2}\, , M_{R3})$$
$$= (2.86 \times 10^6\, , 3.73 \times 10^9\, , 1.44 \times 10^{15})\, [\mathrm{GeV}],$$

くりこみに対する安定性

Four-zero textureとreflection sym.は量子補正をほとんど受けない

Fritzsch & Xing, PLB 413, ’97
Xing & Zhao, NPB 897, ‘15

$$16\pi^2 \frac{dY_u}{dt} = [\alpha_u + C_u^u (Y_u Y_u^\dagger) + C_u^d (Y_d Y_d^\dagger)] Y_u,$$

$$16\pi^2 \frac{dY_d}{dt} = [\alpha_d + C_d^u (Y_u Y_u^\dagger) + C_d^d (Y_d Y_d^\dagger)] Y_d,$$

$$Y_u Y_u^\dagger Y_d = \begin{pmatrix} 1.17\times 10^{-9} i & 2.34\times 10^{-12} + 2.56\times 10^{-7} i & 7.99\times 10^{-7} i \\ 6.22\times 10^{-6} & 0.00140 - 1.17\times 10^{-9} i & 0.00438 \\ 2.00\times 10^{-5} & 0.00450 - 3.63\times 10^{-9} i & 0.0141 \end{pmatrix}$$

$$\simeq \begin{pmatrix} iC_u \tilde{B}_u C_d & iC_u (B_u B_d + \tilde{B}_u \tilde{B}_d) & iC_u (B_u A_d + \tilde{B}_u B_d) \\ (B_u B_u + \tilde{B}_u \tilde{B}_u) C_d & O(B_u A_u B_d) - i\tilde{B}_u C_u C_d & O(B_u A_u A_d) \\ (A_u B_u + B_u \tilde{B}_u) C_d & O(A_u A_u B_d) - iB_u C_u C_d & O(A_u A_u A_d) \end{pmatrix}.$$

第１世代 $C_{u,d} \sim \sqrt{m_{u,d} m_{c,s}}$ が小さい ⇒ the textureとthe sym.は独立に安定

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$$\simeq \begin{pmatrix} iC_u\tilde{B}_u C_d & iC_u(B_u B_d + \tilde{B}_u \tilde{B}_d) & iC_u(B_u A_d + \tilde{B}_u B_d) \\ (B_u B_u + \tilde{B}_u \tilde{B}_u) C_d & O(B_u A_u B_d) - i\tilde{B}_u C_u C_d & O(B_u A_u A_d) \\ (A_u B_u + B_u \tilde{B}_u) C_d & O(A_u A_u B_d) - iB_u C_u C_d & O(A_u A_u A_d) \end{pmatrix}.$$

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第１世代 $C_{u,d} \sim \sqrt{m_{u,d} m_{c,s}}$ が小さい ⇒ the textureとthe sym.は独立に安定

⇒ 高エネルギー(GUT scale)のflavorやCPを受け継いでいる

UV theory?

G-J Ding, S. F. King, A. J. Stuart
JHEP 1013) 006, ArXiv 1307.4212.

これらは別々に破れたresidual symmetries

UV theory with the only GCP sym.

Spontaneous CPV

$\langle \theta_u \rangle = iV_u$

$\langle \theta_d \rangle = V_d$

$$R m^*_{u,\nu} R = m_{u,\nu}, \quad m^*_{d,e} = m_{d,e}.$$

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

flavored CPV (もしくはSCPV)が、第１世代のみ特別扱い

第１世代の質量を禁止するカイラル対称性の破れに付随？？

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$\langle\theta_d\rangle = V_d$

$$Rm^*_{u,\nu}R = m_{u,\nu}, \quad m^*_{d,e} = m_{d,e}.$$

$$m_u = \left(\begin{array}{c:cc} 0 & iC_u & 0 \\ \hdashline -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{array}\right) \quad m_d = \left(\begin{array}{c:cc} 0 & C_d & 0 \\ \hdashline C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{array}\right)$$

flavored CPV (もしくはSCPV)が、第１世代のみ特別扱い

第１世代の質量を禁止するカイラル対称性の破れに付随？？

⇒ $U(1)_{PQ}$ symmetry?

Realization of reflection symmetries

Model : 2HDM + 2 flavons $\theta_{u,d}$ + Z_2^{NFC} × $U(1)_{PQ}$ × GCP

- Z_2^{NFC} : FCNCを禁止 + ポテンシャルを制限
- $U(1)_{PQ}$: 第１世代の質量を禁止 + $\bar{\theta}$ = 0
- GCP : 湯川の複素位相を制御

類似の模型がK. Kang and M. Shin, Phys. Rev. D 33, 2688 (1986).

	$SU(2)_L$	$U(1)_Y$	$Z_2^{\rm NFC}$	$U(1)_{\rm PQ}$	CP
q_{Li}	$\mathbf{2}$	$1/6$	1	$-1,0,0$	1
u_{Ri}	$\mathbf{1}$	$2/3$	1	$1,0,0$	1
d_{Ri}	$\mathbf{1}$	$-1/3$	-1	$1,0,0$	1
l_{Li}	$\mathbf{2}$	$-1/2$	1	$-1,0,0$	1
ν_{Ri}	$\mathbf{1}$	0	1	$1,0,0$	1
e_{Ri}	$\mathbf{1}$	-1	-1	$1,0,0$	1
H_u	$\mathbf{2}$	$-1/2$	1	0	1
H_d	$\mathbf{2}$	$1/2$	-1	0	1
θ_u	$\mathbf{1}$	1	1	-1	$+i$
θ_d	$\mathbf{1}$	1	-1	-1	$-i$

$\theta_{u,d}$と第１世代のみ
$U(1)_{PQ}$ chargeを持つ

$\theta_{u,d}$のみ
GCP chargeを持つ

$\theta_{u,d}$のvevで
$U(1)_{PQ}$ とGCPが破れる

1. flavored U(1)$_{PQ}$

F. Wilczek, Phys.Rev.Lett. 49, 1549 (1982),
A. Davidson and K. C. Wali, Phys. Rev. Lett. 48, 11 (1982),
Y. Ahn, Phys.Rev.D 91, 056005 (2015).

θ$_{u,d}$と第１世代のみU(1)$_{PQ}$ chargeを持つ

$$q_{1L} \to e^{-i\alpha} q_{1L}, \quad u_{1R} \to e^{i\alpha} u_{1R}, \quad d_{1R} \to e^{i\alpha} d_{1R},$$
$$l_{1L} \to e^{-i\alpha} l_{1L}, \quad \nu_{1R} \to e^{i\alpha} \nu_{1R}, \quad e_{1R} \to e^{i\alpha} e_{1R}.$$

湯川行列の電荷

$$\left(\begin{array}{c|cc} e^{2i\alpha} & e^{i\alpha} & e^{i\alpha} \\ \hline e^{i\alpha} & 1 & 1 \\ e^{i\alpha} & 1 & 1 \end{array}\right)$$

最も一般的な
湯川相互作用

$$-\mathcal{L} \ni \bar{q}_L(\tilde{Y}_u^0 + \frac{\theta_u}{\Lambda}\tilde{Y}_u^1 + \frac{\theta_u^2}{\Lambda^2}\tilde{Y}_u^2 + \frac{\theta_d^2}{\Lambda^2}\tilde{Y}_u'^2)u_R H_u$$
$$+ \bar{q}_L(\tilde{Y}_d^0 + \frac{\theta_d}{\Lambda}\tilde{Y}_d^1 + \frac{\theta_u\theta_d}{\Lambda^2}\tilde{Y}_d^2)d_R H_d + h.c.\,,$$

$$\tilde{Y}_{u,d}^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ 0 & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix}, \quad \tilde{Y}_{u,d}^1 = \begin{pmatrix} 0 & \tilde{e}_{u,d} & \tilde{f}_{u,d} \\ \tilde{g}_{u,d} & 0 & 0 \\ \tilde{h}_{u,d} & 0 & 0 \end{pmatrix}$$

flavored axion (flaxion or axiflavon) の一種

Y. Ema, K. Hamaguchi, T. Moroi, and K. Nakayama, JHEP 01, 096 (2017),
L. Calibbi, F. Goertz, D. Redigolo, R. Ziegler, and J. Zupan, Phys. Rev. D 95, 095009 (2017).

2. Generalized CP

	Z_2^{NFC}	$U(1)_{\mathrm{PQ}}$	CP
H_u	1	0	1
H_d	-1	0	1
θ_u	1	-1	$+i$
θ_d	-1	-1	$-i$

$\theta_{u,d}$のみGCP chargeを持つ

$$\theta_u^* = +i\theta_u, \quad \theta_d^* = -i\theta_d, \quad \phi^* = \phi \quad \text{for other fields}$$

ポテンシャルはZ_2と$U(1)_{PQ}$で実(GCP inv.)なものしか許されない

$$V = V^1(H_u, H_d) + V^2(H_{u,d}, \theta_{u,d}) + V^3(\theta_u, \theta_d).$$

（４次項の具体例：$|\theta_u|^2|\theta_d^2|$ or $\theta_u^*\theta_d\theta_u^*\theta_d$ ）

θu,dの真空期待値（実で位相なし）でGCPと$U(1)_{PQ}$が破れる

⇒ この基底において、CP位相は湯川の第１世代のみに集中

$$\tilde{Y}^0_{u,d} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ 0 & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix}, \quad \tilde{Y}^1_{u,d} = \begin{pmatrix} 0 & \tilde{e}_{u,d} & \tilde{f}_{u,d} \\ \tilde{g}_{u,d} & 0 & 0 \\ \tilde{h}_{u,d} & 0 & 0 \end{pmatrix}$$

$$(\tilde{Y}^0_{u,d})^* = \tilde{Y}^0_{u,d}, \quad \tilde{Y}^1_u = e^{i\pi/4}|\tilde{Y}^1_u|, \quad \tilde{Y}^1_d = e^{-i\pi/4}|\tilde{Y}^1_d|.$$

結果

$$Y_{u,d} = (\tilde{Y}^0_{u,d} + \frac{\langle\theta_{u,d}\rangle}{\Lambda}\tilde{Y}^1_{u,d}) = \begin{pmatrix} O(\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2}) & \tilde{e}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{f}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}\,e^{i\varphi_{u,d}} \\ \tilde{g}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ \tilde{h}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix},$$

$$\varphi_u = +\pi/4, \quad \varphi_d = -\pi/4. \quad \xrightarrow[\text{位相変換}]{\text{q,u,dの}} \quad \varphi_u = \pi/2, \quad \varphi_d = 0.$$

quark質量行列のbest fitから、

$$\frac{\langle\theta_u\rangle}{\Lambda}|\tilde{Y}^1_u| \simeq \frac{\sqrt{2m_u\,m_c}}{v\,\sin\beta} \simeq \frac{3\times10^{-4}}{\sin\beta}, \quad \frac{\langle\theta_d\rangle}{\Lambda}|\tilde{Y}^1_d| \simeq \frac{\sqrt{2m_d\,m_s}}{v\,\cos\beta} \simeq \frac{1\times10^{-4}}{\cos\beta},$$

$$\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2} \simeq \frac{10^{-8}(\times\tan^2\beta)}{|\tilde{Y}^1_{u,d}|^2} \lesssim (y_u, y_d) \simeq (\frac{m_u}{v\sin\beta}, \frac{m_d}{v\cos\beta}) \simeq (10^{-5}, 10^{-5}\tan\beta).$$

11成分は十分小さい → reflection symmetriesと$(m_f)_{11} = 0$をみたす

（さらに、（別のCPの基底で）$\tilde{Y}^1_u = \tilde{Y}^1_d$であれば$(m_f)_{13} = 0$もみたす）

結果

$$Y_{u,d} = (\tilde{Y}^0_{u,d} + \frac{\langle\theta_{u,d}\rangle}{\Lambda}\tilde{Y}^1_{u,d}) = \begin{pmatrix} O(\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2}) & \tilde{e}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{f}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}\,e^{i\varphi_{u,d}} \\ \tilde{g}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ \tilde{h}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix},$$

$$\varphi_u = +\pi/4, \quad \varphi_d = -\pi/4. \xrightarrow[\text{位相変換}]{\text{q,u,dの}} \varphi_u = \pi/2, \quad \varphi_d = 0.$$

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$$\frac{\langle\theta_u\rangle}{\Lambda}|\tilde{Y}^1_u| \simeq \frac{\sqrt{2m_u\,m_c}}{v\,\sin\beta} \simeq \frac{3\times 10^{-4}}{\sin\beta}, \quad \frac{\langle\theta_d\rangle}{\Lambda}|\tilde{Y}^1_d| \simeq \frac{\sqrt{2m_d\,m_s}}{v\,\cos\beta} \simeq \frac{1\times 10^{-4}}{\cos\beta},$$

The strong CP problem

$$10^{-10} \gtrsim \bar{\theta} = \theta_{\rm QCD} + \theta_{\rm QFD}(\equiv \mathrm{Arg\,Det}[m_u m_d]),$$

エルミート性やミラー粒子なしに$\delta_{\rm CKM}$と$\theta^{\rm tree}_{\rm QFD}$= 0が両立

→ PやCPを用いたstrong CPの解にも適用可能？

結果

$$Y_{u,d} = (\tilde{Y}^0_{u,d} + \frac{\langle\theta_{u,d}\rangle}{\Lambda}\tilde{Y}^1_{u,d}) = \begin{pmatrix} O(\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2}) & \tilde{e}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{f}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}\,e^{i\varphi_{u,d}} \\ \tilde{g}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ \tilde{h}\,\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix},$$

$$\varphi_u = +\pi/4, \quad \varphi_d = -\pi/4. \quad \xrightarrow[\text{位相変換}]{\text{q,u,dの}} \quad \varphi_u = \pi/2, \quad \varphi_d = 0.$$

quark質量行列のbest fitから、

$$\frac{\langle\theta_u\rangle}{\Lambda}|\tilde{Y}^1_u| \simeq \frac{\sqrt{2m_u\,m_c}}{v\,\sin\beta} \simeq \frac{3\times 10^{-4}}{\sin\beta}, \quad \frac{\langle\theta_d\rangle}{\Lambda}|\tilde{Y}^1_d| \simeq \frac{\sqrt{2m_d\,m_s}}{v\,\cos\beta} \simeq \frac{1\times 10^{-4}}{\cos\beta},$$

今回はflavored U(1)$_{\mathrm{PQ}}$により $\bar{\theta}$ はdynamicalに消滅、

$$\Lambda_{\mathrm{GUT}} \simeq 10^{16}\ [\mathrm{GeV}], \ \Rightarrow\ \langle\theta_{u,d}\rangle \sim \Lambda_{\mathrm{GUT}}\frac{\sqrt{m_{u,d}\,m_{c,s}}}{v} \sim 10^{12}\,[\mathrm{GeV}].$$

⇒ 現象論的制限と矛盾せず、

Y. Ema, K. Hamaguchi, T. Moroi, and K. Nakayama, JHEP 01, 096 (2017),

$$m_a \simeq 10^{-6}\,[\mathrm{eV}], \text{ the dark matter abundance } \Omega_a h^2 \sim 0.2.$$

まとめ

- Reflection sym.というSMの新しいGCPを提案
  - ⇒ Majorana位相 $\alpha_{2,3}/2 = 0$ or $\pi/2$
  - ⇒ エルミート性やミラー粒子なしにδ_{CKM}と$\theta_{\mathrm{QFD}}^{\mathrm{tree}}$= 0が両立
- + universal four-zero texture
  - ⇒ すべてのfermion 質量とV_{CKM} & U_{MNS}を再現
  - ⇒ δ_{CP} ≒ 203°, 通常質量階層、m1 ≒ 2.5 or 6.2 meVを予言
- この構造はseesaw不変なので、Y_νに課せばM_Rも従う
- 第１世代の軽さから、くりこみに耐えうる
- + 対称性の実現のために2HDM+U(1)$_{PQ}$ × CPでSSBを考えた

⇒ $f_a \sim M_{\mathrm{GUT}}\sqrt{m_{u,d}m_{c,s}}/v \sim 10^{12}$ GeV ⇒ $\Omega_a h^2 \sim 0.2$. axion DM?

$$m_u = \begin{pmatrix} 0 & iC_u & 0 \\ -iC_u & \tilde{B}_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde{B}_d & B_d \\ 0 & B_d & A_d \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} 0 & iC_\nu & 0 \\ iC_\nu & \tilde{B}_\nu & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & C_e & 0 \\ C_e & \tilde{B}_e & B_e \\ 0 & B_e & A_e \end{pmatrix},$$

That's all. Thank you!

Back ups

位相変換後のreflection symmetries

rephasing of quark fields $Q = q, u, d$

$$Q' = P_Q^\dagger Q, \quad P_Q = \mathrm{diag}(e^{i\phi_Q}, 1, 1),$$

$$R_{q,u} \equiv P_{q,u} R P_{q,u} = \begin{pmatrix} -e^{2i\phi_{q,u}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tilde{R}_{q,d} \equiv P_{q,d} 1_3 P_{q,d} = \begin{pmatrix} +e^{2i\phi_{q,d}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_q^\dagger \tilde{m}_u^* R_u = \tilde{m}_u, \quad \tilde{R}_q^\dagger \tilde{m}_d^* \tilde{R}_d = \tilde{m}_d.$$

先ほどの例にするには $\quad \phi_u = 3\pi/4,\ \phi_q = -\phi_d = \pi/4$

$$\begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tilde{m}_u^* \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \tilde{m}_u, \quad \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tilde{m}_d^* \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \tilde{m}_d,$$

$$\varphi_u = +\pi/4, \quad \varphi_d = -\pi/4.$$

Global Fit

I. Esteban, M.C.Gonzalez-Garcia, M. Maltoni, T. Schwetz, A. Zhou,
JHEP 09 (2020) 178 • e-Print: 2007.14792

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.155 \\ 0.234 \to 0.500 & 0.471 \to 0.689 & 0.637 \to 0.776 \\ 0.271 \to 0.525 & 0.477 \to 0.694 & 0.613 \to 0.756 \end{pmatrix}$$

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
	$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
	$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
	$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
	$\delta_{\rm CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

Fritzsch-Xing parameterization

H.Fritzsch and Z-z. Xing, *Phys.Lett.B* 413 (1997) 396-404, ArXiv 9707215

Another parameterization of the CKM matrix

$$
\begin{aligned}
V &= \begin{pmatrix} c_{\rm u} & s_{\rm u} & 0 \\ -s_{\rm u} & c_{\rm u} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-{\rm i}\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_{\rm d} & -s_{\rm d} & 0 \\ s_{\rm d} & c_{\rm d} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} s_{\rm u}s_{\rm d}c + c_{\rm u}c_{\rm d}e^{-{\rm i}\varphi} & s_{\rm u}c_{\rm d}c - c_{\rm u}s_{\rm d}e^{-{\rm i}\varphi} & s_{\rm u}s \\ c_{\rm u}s_{\rm d}c - s_{\rm u}c_{\rm d}e^{-{\rm i}\varphi} & c_{\rm u}c_{\rm d}c + s_{\rm u}s_{\rm d}e^{-{\rm i}\varphi} & c_{\rm u}s \\ -s_{\rm d}s & -c_{\rm d}s & c \end{pmatrix} ,
\end{aligned}
$$

Since the first-generation has tiny mass, only the heavy quark mixing receive quantum corrections

Fritzsch-Xing parameterization

H.Fritzsch and Z-z. Xing, *Phys.Lett.B* 413 (1997) 396-404, ArXiv 9707215

Another parameterization of the CKM matrix

Almost RGE invariant!

$$V = \begin{pmatrix} \boxed{\begin{matrix} c_{\rm u} & s_{\rm u} \\ -s_{\rm u} & c_{\rm u} \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix} \begin{pmatrix} \boxed{e^{-{\rm i}\varphi}} & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} 0 \\ 0 \end{matrix} & \boxed{\begin{matrix} c & s \\ -s & c \end{matrix}} \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} c_{\rm d} & -s_{\rm d} \\ s_{\rm d} & c_{\rm d} \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_{\rm u} s_{\rm d} c + c_{\rm u} c_{\rm d} e^{-{\rm i}\varphi} & s_{\rm u} c_{\rm d} c - c_{\rm u} s_{\rm d} e^{-{\rm i}\varphi} & s_{\rm u} s \\ c_{\rm u} s_{\rm d} c - s_{\rm u} c_{\rm d} e^{-{\rm i}\varphi} & c_{\rm u} c_{\rm d} c + s_{\rm u} s_{\rm d} e^{-{\rm i}\varphi} & c_{\rm u} s \\ -s_{\rm d} s & -c_{\rm d} s & c \end{pmatrix},$$

Since the first-generation has tiny mass, only
the heavy quark mixing receive quantum corrections

おまけ

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_{\rm u} = \begin{pmatrix} \mathbf{0} & +\,{\rm i}x & \mathbf{0} \\ -{\rm i}x & y & ry \\ \mathbf{0} & ry & z \end{pmatrix} ,$$

$$M_{\rm d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & y' & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix} ,$$

$$M_{\rm e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & -\,3y' & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix} ,$$

6 parameters!! @ GUT scale

(r = √2)

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_\mathrm{u} = \begin{pmatrix} \mathbf{0} & +\mathrm{i}x & \mathbf{0} \\ -\mathrm{i}x & y & ry \\ \mathbf{0} & ry & z \end{pmatrix},$$

$$M_\mathrm{d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

$$M_\mathrm{e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{-3y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

Georgi-Jarlskog relation

6 parameters!! @ GUT scale

(r = √2)

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_{\rm u} = \begin{pmatrix} \mathbf{0} & +{\rm i}x & \mathbf{0} \\ -{\rm i}x & y & ry \\ \mathbf{0} & ry & z \end{pmatrix},$$

$$M_{\rm d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

$$M_{\rm e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{-3y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

Solve RGEs

$\Rightarrow$

Georgi-Jarlskog relation

$$m_d = 3m_e \left(1 + \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau} \xi_\tau^3\right) \zeta_{\rm de} ,$$

$$m_s = \frac{m_\mu}{3} \left(1 - \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau} \xi_\tau^3\right) \zeta_{\rm de} ,$$

$$m_b = m_\tau \frac{\xi_t \xi_b^3}{\xi_\tau^3} \zeta_{\rm de} .$$

$$\xi_i = \exp\left[-\frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} f_i^2(\chi)\, {\rm d}\chi\right]$$

6 parameters!! @ GUT scale

(r = √2)

renormalized mass @ mZ scale

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_{\rm u} = \begin{pmatrix} \mathbf{0} & +{\rm i}x & \mathbf{0} \\ -{\rm i}x & y & ry \\ \mathbf{0} & ry & z \end{pmatrix},$$

$$M_{\rm d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

$$M_{\rm e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{-3y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

Georgi-Jarlskog relation

Solve RGEs $\Rightarrow$

$$m_d = 3m_e \left(1 + \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau}\, \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_s = \frac{m_\mu}{3} \left(1 - \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau}\, \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_b = m_\tau \frac{\xi_t \xi_b^3}{\xi_\tau^3} \boxed{\zeta_{\rm de}}.$$

RGE factor $\sim$ 3 from strong ints.

$$\xi_i = \exp\left[-\frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} f_i^2(\chi)\, {\rm d}\chi\right]$$

6 parameters!! @ GUT scale

(r = √2)

renormalized mass @ mZ scale

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_{\rm u} = \begin{pmatrix} \mathbf{0} & +{\rm i}x & \mathbf{0} \\ -{\rm i}x & y & ry \\ \mathbf{0} & ry & z \end{pmatrix},$$

$$M_{\rm d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

$$M_{\rm e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{-3y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

Solve RGEs $\Longrightarrow$

Georgi-Jarlskog relation

$$m_d = 3m_e \left(1 + \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau}\ \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_s = \frac{m_\mu}{3} \left(1 - \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau}\ \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_b = m_\tau \frac{\xi_t \xi_b^3}{\xi_\tau^3} \boxed{\zeta_{\rm de}}.$$

RGE factor $\sim$ 3 from strong ints.

$$\xi_i = \exp\left[-\frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} f_i^2(\chi)\ {\rm d}\chi\right]$$

6 parameters!! @ GUT scale (r = √2)

renormalized mass @ mZ scale

Taking $r^2 = 2$ and $\tan\beta_{\rm susy} = 50$ for example, we obtain $m_d \approx 3.6$ MeV, $m_s \approx 76$ MeV and $m_b \approx 3.2$ GeV, essentially in agreement with the results listed in (2.9).

Note: Predictive GUT

Fritzsch and Xing review, 9912358

universal texture and d-e unification has predictions.

$$M_{\rm u} = \begin{pmatrix} \mathbf{0} & +ix & \mathbf{0} \\ -ix & y & ry \\ \mathbf{0} & ry & z \end{pmatrix}, \quad M_{\rm d} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix}, \quad M_{\rm e} = \begin{pmatrix} \mathbf{0} & x' & \mathbf{0} \\ x' & \boxed{-3y'} & ry' \\ \mathbf{0} & ry' & z' \end{pmatrix},$$

Solve RGEs $\Rightarrow$

Georgi-Jarlskog relation

$$m_d = 3m_e \left(1 + \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau} \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_s = \frac{m_\mu}{3} \left(1 - \frac{4r^2}{9} \cdot \frac{m_\mu}{m_\tau} \xi_\tau^3\right) \boxed{\zeta_{\rm de}},$$

$$m_b = m_\tau \frac{\xi_t \xi_b^3}{\xi_\tau^3} \boxed{\zeta_{\rm de}}.$$

RGE factor ~ 3 from strong ints.

$$\xi_i = \exp\left[-\frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} f_i^2(\chi)\, d\chi\right]$$

6 parameters!! @ GUT scale (r = √2)

renormalized mass @ mZ scale

Taking $r^2 = 2$ and $\tan\beta_{\rm susy} = 50$ for example, we obtain $m_d \approx 3.6$ MeV, $m_s \approx 76$ MeV and $m_b \approx 3.2$ GeV, essentially in agreement with the results listed in (2.9).

Unfortunately, this scheme contradicts recent observation of Vub …

再構成された質量行列の数値

(摂動が悪いかも、なので2〜5%くらい誤差あるかも)

Xing & Zhao, NPB 897, '15

$$Y_u \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002i & 0 \\ -0.0002i & 0.10 & 0.31 \\ 0 & 0.31 & 1 \end{pmatrix} \qquad Y_d \simeq \frac{0.9 m_b \sqrt{2}}{v} \begin{pmatrix} 0 & 0.005 & 0 \\ 0.005 & 0.13 & 0.31 \\ 0 & 0.31 & 1 \end{pmatrix}$$

O(mt) O(mb)

$m_1 \simeq 2.5[\mathrm{meV}], \quad (\alpha_2, \alpha_3) = (\pi, 0).$ の解

$$m_{\nu 0} \simeq \begin{pmatrix} 0 & -8.86i & 0 \\ -8.86i & 29.3 & 26.4 \\ 0 & 26.4 & 14.6 \end{pmatrix} [\mathrm{meV}] \qquad m_e \simeq \begin{pmatrix} 0 & -7.058 & 0 \\ -7.058 & 107.873 & 96.12 \\ 0 & 96.12 & \boxed{1740} \end{pmatrix} [\mathrm{MeV}]$$

O(mτ)

（もう一方の解は$(m_e)_{22}$〜m_τでやや不自然）

素朴には d-e 統一に失敗…？？

(エルミート性か$(m_d)_{13}$ = 0を放棄 …？)