

Lepton flavor model with modular A_4 symmetry in large volume limit

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Introduction

SMでは同じ charge を持ち、質量の異なる
3世代のフェルミオンが存在する

3世代のクォークは混合している

クォーク混合行列 (CKM行列)

$$|V_{CKM}| \sim \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

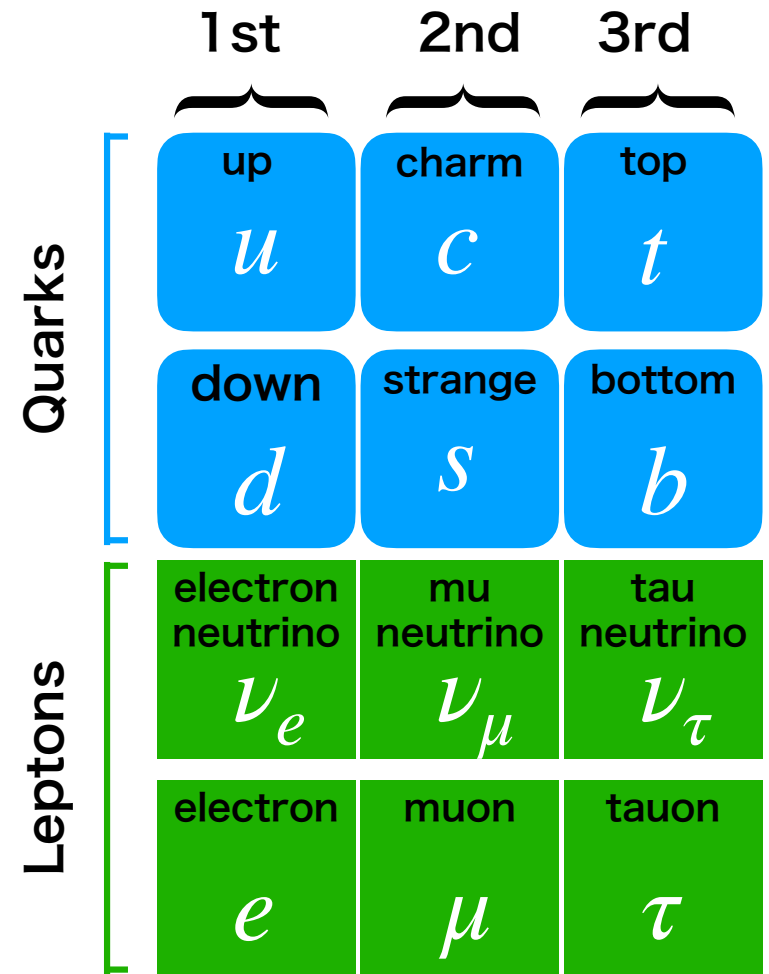
PDG (2020)

3世代のニュートリノ は混合している

ニュートリノ 混合行列 (PMNS行列)

$$|U_{PMNS}| \sim \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.513 \rightarrow 0.579 & 0.143 \rightarrow 0.155 \\ 0.234 \rightarrow 0.500 & 0.471 \rightarrow 0.689 & 0.637 \rightarrow 0.776 \\ 0.271 \rightarrow 0.525 & 0.477 \rightarrow 0.694 & 0.613 \rightarrow 0.756 \end{pmatrix}$$

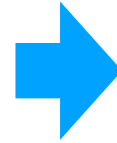
NuFIT 5.0 (2020)



Introduction

標準模型ではYukawa相互作用項がフェルミオンの質量と混合の起源になっている

$$\mathcal{L} = -F^\psi \bar{\psi}_L H \psi_R + h.c.$$



$$M_\psi = F^\psi \langle H \rangle$$

– フレーバー構造の背後にある物理は？

➡ 世代間の対称性によってYukawa couplings を関係付けることができる (フレーバー対称性)

Non-Abelian discrete symmetries ($S_3, A_4, S_4, A_5, \dots$)は典型的なニュートリノ混合パターンをよく説明する

[Altarelli & Feruglio (2010); Ishimori, Kobayashi, Ohki, Okada, Shimizu & Tanimoto (2010); King & Luhn (2013); King, Merle, Morisi, Shimizu & Tanimoto (2014) . . .]

Introduction

- このような非可換離散対称性の起源が余剰次元のコンパクト化に伴う **モジュラー対称性** によって説明できるのではないかと
ということが提案された [Feruglio 1706.08749]

クォークやレプトンの混合角、質量、CP位相を説明できる

T. Kobayashi, K. Tanaka and T. H. Tatsuishi, Phys. Rev. D 98 (2018) no.1, 016004
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P. P. Novichkov, J. T. Penedo, S. T. Petcov and A. V. Titov, JHEP 07 (2019), 165
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M. C. Chen, S. Ramos-Sánchez and M. Ratz, Phys. Lett. B 801 (2020), 135153 , Any more...

Introduction

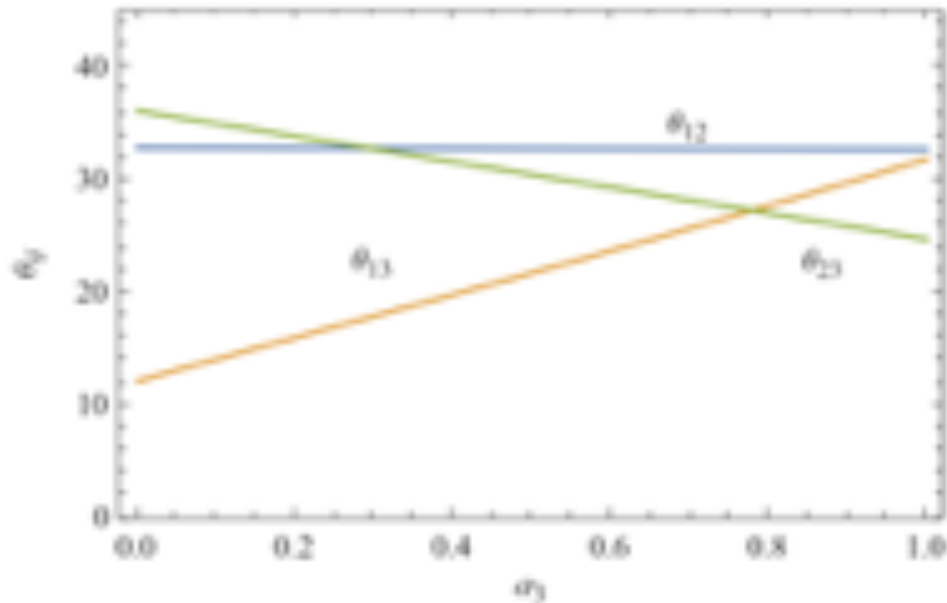
Modular変換の元で不変な理論：

– Superpotential は τ と少ないパラメータを含む

➡ Non-trivialな質量や混合角を予言

– Kähler potential

minimalでない項からadditionalな混合が生じる可能性



➡ Kähler potentialから生じる additional mixing が予言を変える可能性がある

[M. Chen, S. Ramons-Sanchez, M. Ratz, Phys.Lett.B 801 (2020) 135153]

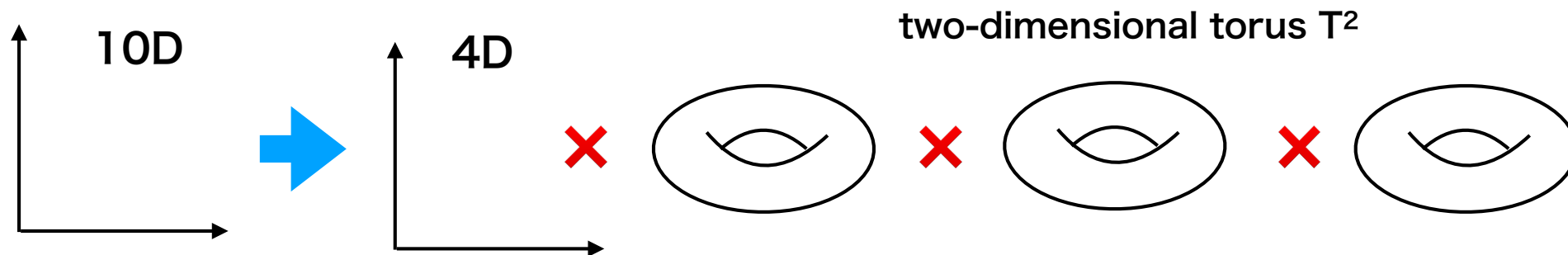
- 今回の発表
 - Lepton セクターにmodular A_4 対称性を持つ模型を考える
 - 特に $\langle \text{Im } \tau \rangle \gg 1$ (Large volume limit) を考えると additionalな混合が生じないことをみる
 - 検討した模型が予言するneutrinoの性質について紹介する

Modular Symmetry

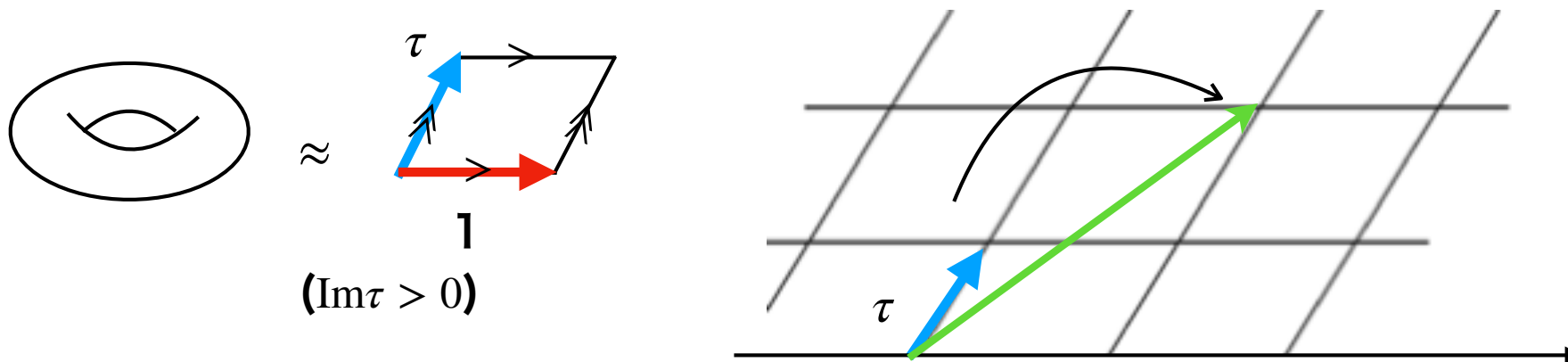
トーラスコンパクト化

背景にsuperstring理論を考える。

10次元から4次元の理論を構成する際に、余分な6次元はコンパクト化する必要がある。



2次元トーラスはモジュラス τ で特徴付けられる。



複素数 τ を格子上の別の点に移す変換 = モジュラー変換

Modular群

- モジュラー群はモジュラス τ に作用する変換群 γ として定義される.

$$\tau \rightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2, \mathbb{Z})$$

$$PSL(2, \mathbb{Z}) = SL(2, \mathbb{Z}) / \{I, -I\} \quad SL(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

I : unit matrix

- モジュラー群の生成子 S, T

$$S : \tau \rightarrow -\frac{1}{\tau} \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad T : \tau \rightarrow \tau + 1 \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

- Finite quotient subgroup

$$\Gamma_N = \{S, T \mid S^2 = \mathbf{1}, (ST)^3 = \mathbf{1}, T^N = \mathbf{1}\}$$

$$\Gamma_2 \cong S_3, \Gamma_3 \cong A_4, \Gamma_4 \cong S_4, \Gamma_5 \cong A_5$$

Modular Symmetry in SUGRA

- Supergravityを考える

$$G = K + \ln |W|^2$$

$$\begin{cases} K : \text{Kähler potential} \\ W : \text{superpotential} \end{cases}$$

モジュラス τ のKähler potential

$$K = -\log(-i\tau + i\bar{\tau})$$

$$K \rightarrow K' = K + \log(c\tau + d) + \log(c\bar{\tau} + d)$$

$$W \rightarrow W' = \frac{W}{c\tau + d}$$

$$\leftarrow \tau \rightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

G がmodular変換に対して不変であるためには
superpotential W がmodular weight -1を持つ

Modular transformation

chiral superfieldが受ける変換 S. Ferrara, D. Lust, A. Shapere, S. Theisen, Phys. Lett. B225,4(1989)

$$\phi^{(I)} \rightarrow (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

$$\gamma \in \Gamma_N$$

$-k_I, k$: modular weight

modular form of modular weight k

$\phi^{(I)}, f(\tau)$: representation of Γ_N

$\rho(\gamma), \rho^{(I)}(\gamma)$: unitary repre-

sentation matrix of Γ_N

$$f(\tau) \rightarrow (c\tau + d)^k \rho(\gamma) f(\tau)$$

Γ_N の変換のもとで不変なsuperpotential

$$W = \sum_n f(\tau) \phi^{(I_1)} \phi^{(I_2)} \dots \phi^{(I_n)}$$

$$k - (k_{I_1} + k_{I_2} + \dots + k_{I_n}) = -1$$

$$\rho \times \rho^{(I_1)} \times \rho^{(I_2)} \times \dots \times \rho^{(I_n)} \ni 1 \quad \text{of } \Gamma_N$$

Lepton Flavor Model with Modular A_4 Symmetries ($\text{Im } \tau \sim 1$)

modular form

Dedekind Eta function を使うと weight $k = 2$ の modular form が作れる

Dedekind Eta function : $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad q \equiv e^{i2\pi\tau}$

$$\eta(-1/\tau) = \sqrt{-i\tau} \eta(\tau)$$

$$\eta(\tau + 1) = e^{\pi i/12} \eta(\tau)$$

weight $k = 2$ の modular form (A_4 triplet) F. Feruglio, [arXiv : 1706.08749]

$$Y_1(\tau) = \frac{i}{2\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)} \right)$$

$$Y_2(\tau) = \frac{-i}{\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2 \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right)$$

$$Y_3(\tau) = \frac{-i}{\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2 \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right)$$

$$Y^{A_4} = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \omega = e^{2\pi i/3}$$

$$Y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \dots$$

$$Y_2(\tau) = -6q^{1/3} (1 + 7q + 8q^2 + \dots)$$

$$Y_3(\tau) = -18q^{2/3} (1 + 2q + 5q^2 + \dots)$$

Benchmark model with modular A_4 symmetry

Modular A_4 のseesaw模型を考える

[T. Kobayashi, N. Omoto, Y. Shimizu, K. Takagi, M. Tanimoto

and T. H. Tatsuishi, JHEP **1811** (2018) 196]

	L	e, μ, τ	ν	H_u	H_d
$SU(2)$	2	1	1	2	2
A_4	3	$1, 1'', 1'$	3	1	1
$-k_I$	-1	-1	-1	0	0

$$Y^{A_4}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \begin{array}{l} \text{modular weight } k = 2 \text{ の} \\ A_4 \text{ triplet modular form} \end{array}$$

superpotential

$$q \equiv e^{i2\pi\tau}$$

$$W_e = \alpha e H_d (LY^{A_4}(\tau))_1 + \beta \mu H_d (LY^{A_4}(\tau))_{1'} + \gamma \tau H_d (LY^{A_4}(\tau))_{1''}$$

$$W_D = g_1 \underbrace{(NH_u (LY^{A_4}(\tau))_{3s})_1}_{\text{symmetric}} + g_2 \underbrace{(NH_u (LY^{A_4}(\tau))_{3a})_1}_{\text{anti-symmetric}}$$

$$W_N = \Lambda (NNY^{A_4}(\tau))_1$$

Λ : 右巻きニュートリノの質量スケール

Mass matrices for leptons

$$M_E = v_d \text{diag}[\alpha, \beta, \gamma] \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

$$M_D = v_u \begin{pmatrix} 2g_1 Y_1(\tau) & (-g_1 + g_2) Y_3(\tau) & (-g_1 - g_2) Y_2(\tau) \\ (-g_1 - g_2) Y_3(\tau) & 2g_1 Y_2(\tau) & (-g_1 + g_2) Y_1(\tau) \\ (-g_1 + g_2) Y_2(\tau) & (-g_1 - g_2) Y_1(\tau) & 2g_1 Y_3(\tau) \end{pmatrix}_{RL}$$

$$M_N = \Lambda \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}_{RR} \quad M_\nu = -M_D^T M_N^{-1} M_D$$

$$\text{Re } \tau = 0.530457$$

$$\text{Im } \tau = 1.28698$$

$$|g| = 1.20925$$

$$\phi_g = -1.61095 \quad (\text{rad})$$

$$\hat{\beta} = \beta/\alpha = 16.2773$$

$$\hat{\beta} = \gamma/\alpha = 0.00493867$$

NH

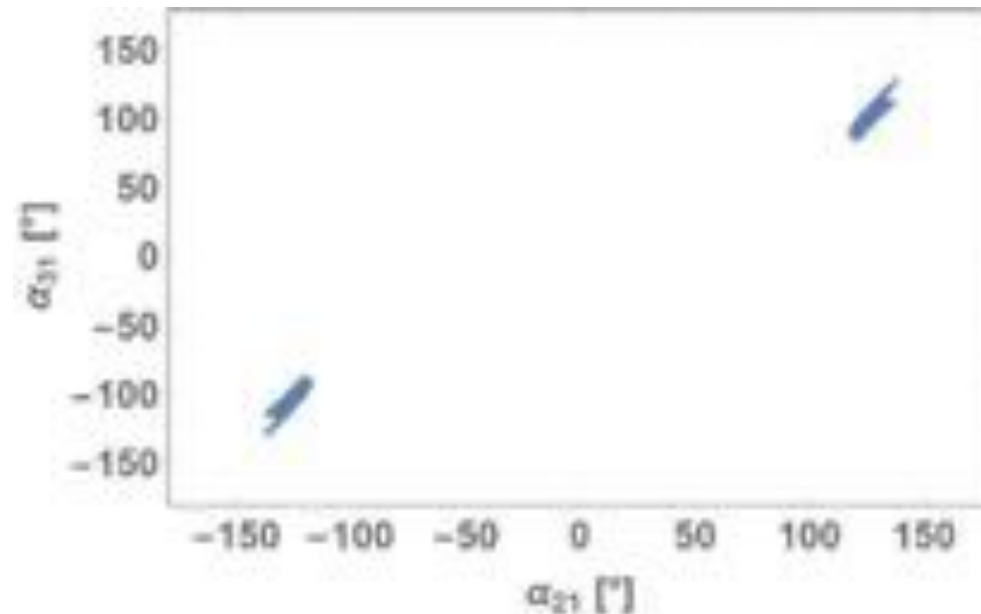
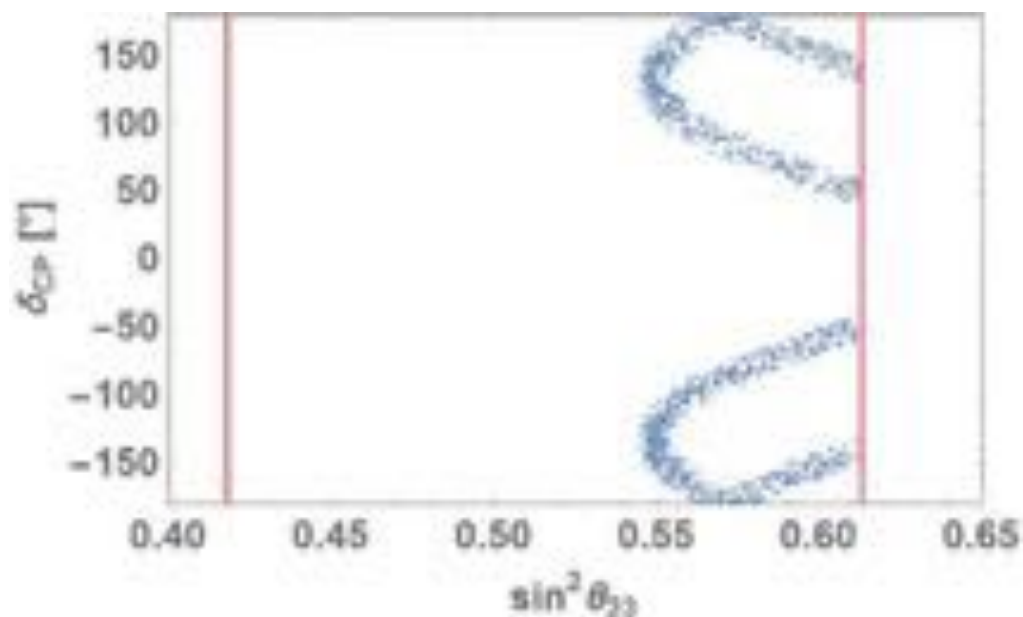
$$\theta_{12} \sim 32.82^\circ$$

$$\theta_{13} \sim 8.539^\circ$$

$$\theta_{23} \sim 49.35^\circ$$



CPV phases



$\text{Im}[\tau]$	$\text{Re}[\tau]$	g	ϕ_g	α/γ	β/γ
0.66–0.73	$\pm(0.25–0.31), \pm(0.46–0.54),$	1.20–1.22	$\pm(87–88)^\circ$	202–203	3286–3306
1.17–1.32	$\pm(0.66–0.75), \pm(1.25–1.31),$ $\pm(1.46–1.50)$		$\pm(92–93)^\circ$		

現実的な混合角を再現するには $\text{Im} \tau \sim O(1)$ が必要

Kähler potential for matters

Kinetic term:

- Typical Kähler potential

$$K^{matter} = (-i\tau + i\bar{\tau})^{-k_I} |\hat{\phi}^{(I)}|^2$$

モジュラー変換で不変

Kähler metric

$$\frac{\partial^2 K^{matter}}{\partial \hat{\phi}_i^{(I)} \partial \hat{\phi}_j^{*(I)}} \propto \delta_{ij}$$

- Typicalな項以外も書くことができる

$$K^{matter} \rightarrow \alpha_0 (-i\tau + i\bar{\tau})^{-k_I} \left(\bar{\hat{\phi}}^{(I)} \hat{\phi}^{(I)} \right)_1 + \sum_i \alpha_i (-i\tau + i\bar{\tau})^{-k_I+k} \left(\bar{f}(\bar{\tau}) \bar{\hat{\phi}}^{(I)} f(\tau) \hat{\phi}^{(I)} \right)_1 + \dots$$

対角的

一般に非対角的

kinetic normalizationから新たな混合が生じ得る

Kähler potential

Kähler potential for L

$$\begin{aligned}
 K &= \alpha_0(-i\tau + i\bar{\tau})^{-1}(\bar{L}L)_1 + \sum_i^7 \alpha_i(-i\tau + i\bar{\tau}) \left(\overline{Y^{A_4}(\bar{\tau})} \bar{L} Y^{A_4}(\tau) L \right)_1 \\
 &= \frac{\alpha_0}{2\text{Im}\tau} \bar{L}_i \left\{ \delta_{ij} + \underbrace{(2\text{Im}\tau)^2}_{\sim 6.625} \sum_I^7 k_I \underbrace{\Delta K_{I,ij}}_{\sim O(1)} \right\} L_j \quad k_i \equiv \alpha_i/\alpha_0 \\
 &\quad \text{Im } \tau = 1.28698
 \end{aligned}$$

→ k_i は小さくなければ第二項は無視できない

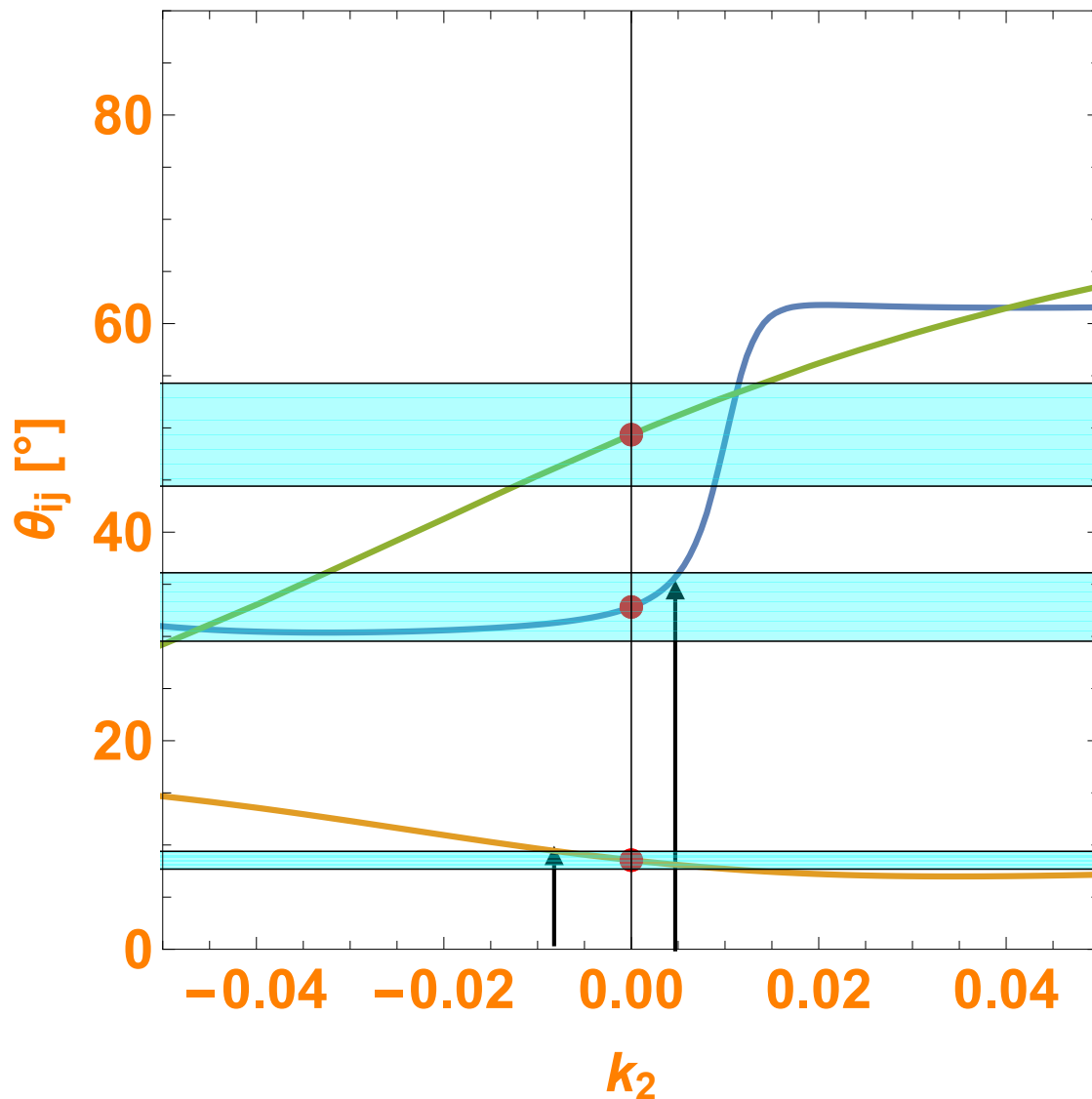
$$\Delta K_1 = \begin{pmatrix} |Y_1|^2 & Y_1^* Y_3 & Y_1^* Y_2 \\ (1,2)^* & |Y_3|^2 & Y_3^* Y_2 \\ (1,3)^* & (2,3)^* & |Y_2|^2 \end{pmatrix} \quad \Delta K_2 = \begin{pmatrix} |Y_2|^2 & Y_2^* Y_1 & Y_2^* Y_3 \\ (1,2)^* & |Y_1|^2 & Y_1^* Y_3 \\ (1,3)^* & (2,3)^* & |Y_3|^2 \end{pmatrix} \quad \Delta K_3 = \begin{pmatrix} |Y_3|^2 & Y_3^* Y_2 & Y_3^* Y_1 \\ (1,2)^* & |Y_2|^2 & Y_2^* Y_1 \\ (1,3)^* & (2,3)^* & |Y_1|^2 \end{pmatrix}$$

$$\Delta K_4 = \frac{1}{9} \begin{pmatrix} 4|Y_1|^2 + |Y_2|^2 + |Y_3|^2 & -2Y_1^* Y_3 + Y_2^* Y_1 - 2Y_3^* Y_2 & -2Y_1^* Y_2 - 2Y_2^* Y_3 + Y_3^* Y_1 \\ (1,2)^* & |Y_1|^2 + 4|Y_2|^2 + |Y_3|^2 & Y_3^* Y_2 - 2Y_1^* Y_3 - 2Y_2^* Y_1 \\ (1,3)^* & (2,3)^* & |Y_1|^2 + |Y_2|^2 + 4|Y_3|^2 \end{pmatrix} \quad \Delta K_5 = \frac{1}{4} \begin{pmatrix} |Y_2|^2 + |Y_3|^2 & -Y_2^* Y_1 & -Y_3^* Y_1 \\ (1,2)^* & |Y_1|^2 + |Y_3|^2 & -Y_3^* Y_2 \\ (1,3)^* & (2,3)^* & |Y_1|^2 + |Y_2|^2 \end{pmatrix}$$

$$\Delta K_6 = \frac{1}{3} \begin{pmatrix} -|Y_2|^2 + |Y_3|^2 & 2Y_1^* Y_3 + Y_2^* Y_1 & -2Y_1^* Y_2 - Y_3^* Y_1 \\ (1,2)^* & |Y_1|^2 - |Y_3|^2 & 2Y_2^* Y_1 + Y_3^* Y_2 \\ (1,3)^* & (2,3)^* & -|Y_1|^2 + |Y_2|^2 \end{pmatrix} \quad \Delta K_7 = \frac{i}{3} \begin{pmatrix} 0 & -Y_1^* Y_3 - Y_2^* Y_1 - Y_3^* Y_2 & Y_1^* Y_2 + Y_2^* Y_3 + Y_3^* Y_1 \\ -(1,2)^* & 0 & -Y_1^* Y_3 - Y_2^* Y_1 - Y_3^* Y_2 \\ -(1,3)^* & -(2,3)^* & 0 \end{pmatrix}$$

Effect on mixing angle

Contribution from $\left(\overline{Y^{A_4}(\bar{\tau})L}\right)_{\bar{1}'} (Y^{A_4}(\tau)L)_{1'}$



— θ_{12}

— θ_{13}

— θ_{23}

● $\Delta K = 0$: typical Kähler

$k_2 \sim 0.005$ でも θ_{ij} は
10%くらいずれる

Our Model

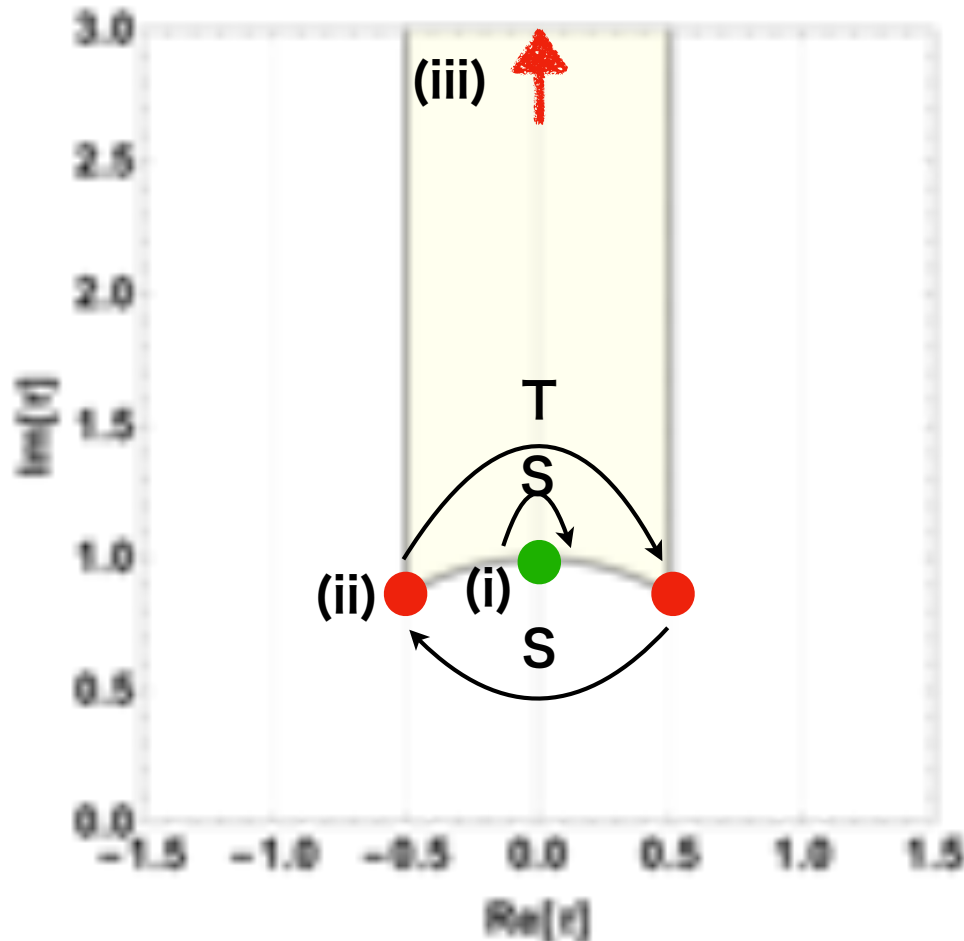
Residual symmetry

- $\langle \tau \rangle$ が特定の値をとるとき部分的な対称性が残る (residual symmetry)

$$\Gamma_N = \{S, T \mid S^2 = 1, (ST)^3 = 1, T^N = 1\}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov and A. V. Titov, JHEP **04** (2019) 005

P. P. Novichkov, S. T. Petcov and M. Tanimoto, Phys. Lett. B **793** (2019)



(i) $\langle \tau \rangle = i$

S-trans. $\langle \tau \rangle \rightarrow \langle -1/\tau \rangle = i = \langle \tau \rangle$

→ Z_2 invariant

(ii) $\langle \tau \rangle = \omega = -1/2 + \sqrt{3}/2 i$

ST-trans. $\langle \tau \rangle \rightarrow \langle -1/(\tau+1) \rangle$
 $= -1/(-\omega^*) = \omega = \langle \tau \rangle$

→ Z_3 invariant

(iii) $\langle \tau \rangle = i \infty$

T-trans. $\langle \tau \rangle \rightarrow \langle \tau+1 \rangle \sim i \infty = \langle \tau \rangle$

→ Z_N invariant

Kähler potential also has Z_2 (, Z_3 or Z_N) invariance.

The case of A_4 at $\langle \tau \rangle = i \infty$

- $\langle \tau \rangle = i \infty$ の場合を考える

T変換が対角なベースをとる

$$\rho(S) = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad \rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$Y^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3} (1 + 7q + 8q^2 + \dots) \\ -18q^{2/3} (1 + 2q + 5q^2 + \dots) \end{pmatrix} \quad q \equiv e^{i2\pi\tau}$$

$$\longrightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (\tau \rightarrow i \infty)$$

$$Y^{(2)} \otimes Y^{(2)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_3 \otimes \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_3 = (1)_1 \oplus (0)_1' \oplus (0)_1'' \oplus \frac{1}{3} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_3 \oplus \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_3$$

weight $2k$ A_4 triplet modular form は $(1, 0, 0)$, $k = 1, 2, \dots$

singlet modular form は自明な singlet のみ , $k = 2, 3, \dots$

The case of A_4 at $\langle \tau \rangle = i \infty$

$$\hat{\phi}^{(I)} = \begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \hat{\phi}_3 \end{pmatrix} \quad : \text{modular weight } -k_I \text{ の } A_4 \text{ triplet chiral superfield}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_3 \otimes \begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \hat{\phi}_3 \end{pmatrix}_3 = (\hat{\phi}_1)_1 \oplus (\hat{\phi}_2)_{1'} \oplus (\hat{\phi}_3)_{1''} \oplus \frac{1}{3} \begin{pmatrix} 2\hat{\phi}_1 \\ -\hat{\phi}_2 \\ -\hat{\phi}_3 \end{pmatrix}_3 \oplus \frac{1}{2} \begin{pmatrix} 0 \\ \hat{\phi}_2 \\ -\hat{\phi}_3 \end{pmatrix}_3$$

Modular A_4 不変な Kähler potential

$$K_{\hat{\phi}^{(I)}} = (-i\tau + i\bar{\tau})^{-k_I} |\hat{\phi}^{(I)}|^2 + \sum_{a=1}^7 \alpha_a (-i\tau + i\bar{\tau})^{-k_I+2} (\bar{Y}^{(2)} \bar{\hat{\phi}}^{(I)} Y^{(2)} \phi^{(2)})_1 + \dots$$

More higher order terms

$$\langle \tau \rangle \rightarrow i\infty \rightarrow \begin{pmatrix} \bar{\hat{\phi}}_1 & \bar{\hat{\phi}}_2 & \bar{\hat{\phi}}_3 \end{pmatrix} \begin{pmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{pmatrix} \begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \hat{\phi}_3 \end{pmatrix}$$



Z_3 is left

$$K\rho(T) = \rho(T)K$$

K : Kähler metric

Z_3 対称性が残っているので、Kähler metric は対角的となり、混合を生じない

d_i ($i=1,2,3$) are positive value

Model

	\hat{L}	$\hat{E}_1^c, \hat{E}_2^c, \hat{E}_3^c$	\hat{N}^c	\hat{H}_u	\hat{H}_d	\hat{S}	Y^{A_4}
$SU(2)_L$	2	1	1	2	2	1	1
A_4	3	1, 1'', 1'	3	1	1	3	3
M.W.	-1	-1	-1	-1	-1	-1	+2
R	-	-	-	+	+	+	+

T. Asaka, Y. Heo, TY
[arXiv2009.12120]

Superpotential

$$\begin{aligned}
 W = & k (\hat{S} Y^{A_4} \hat{H}_u \hat{H}_d)_1 \\
 & + f_1 (\hat{L} Y^{A_4})_1 \hat{E}_1^c H_d + f_2 (\hat{L} Y^{A_4})_{1'} \hat{E}_2^c H_d + f_3 (\hat{L} Y^{A_4})_{1''} \hat{E}_3^c H_d \\
 & + g_1 ((\hat{L} Y^{A_4})_{3s} \hat{N}^c \hat{H}_u)_1 + g_2 ((\hat{L} Y^{A_4})_{3a})_1 \hat{N}^c \hat{H}_u)_1 \\
 & + h_1 ((\hat{S} Y^{A_4})_1 \hat{N}^c \hat{N}^c)_1 + h_2 ((\hat{S} Y^{A_4})_{1'} \hat{N}^c \hat{N}^c)_1 + h_3 ((\hat{S} Y^{A_4})_{1''} \hat{N}^c \hat{N}^c)_1 \\
 & + h_4 ((\hat{S} Y^{A_4})_{3s} \hat{N}^c \hat{N}^c)_1 + h_5 ((\hat{S} Y^{A_4})_{3a} \hat{N}^c \hat{N}^c)_1.
 \end{aligned}$$

Model

	\hat{L}	$\hat{E}_1^c, \hat{E}_2^c, \hat{E}_3^c$	\hat{N}^c	\hat{H}_u	\hat{H}_d	\hat{S}	Y^{A_4}
$SU(2)_L$	2	1	1	2	2	1	1
A_4	3	1, 1'', 1'	3	1	1	3	3
M.W.	-1	-1	-1	-1	-1	-1	+2
R	-	-	-	+	+	+	+

T. Asaka, Y. Heo, TY
[arXiv2009.12120]

Superpotential

$$\begin{aligned}
 W = & k (\hat{S} Y^{A_4} \hat{H}_u \hat{H}_d)_1 && \leftarrow \mu \text{ term} \\
 & + f_1 (\hat{L} Y^{A_4})_1 \hat{E}_1^c H_d + f_2 (\hat{L} Y^{A_4})_{1'} \hat{E}_2^c H_d + f_3 (\hat{L} Y^{A_4})_{1''} \hat{E}_3^c H_d && \leftarrow \text{Charged lepton mass} \\
 & + g_1 ((\hat{L} Y^{A_4})_{3s} \hat{N}^c \hat{H}_u)_1 + g_2 ((\hat{L} Y^{A_4})_{3a})_1 \hat{N}^c \hat{H}_u)_1 && \leftarrow \text{Dirac mass} \\
 & + h_1 ((\hat{S} Y^{A_4})_1 \hat{N}^c \hat{N}^c)_1 + h_2 ((\hat{S} Y^{A_4})_{1'} \hat{N}^c \hat{N}^c)_1 + h_3 ((\hat{S} Y^{A_4})_{1''} \hat{N}^c \hat{N}^c)_1 \\
 & + h_4 ((\hat{S} Y^{A_4})_{3s} \hat{N}^c \hat{N}^c)_1 + h_5 ((\hat{S} Y^{A_4})_{3a} \hat{N}^c \hat{N}^c)_1. && \left. \vphantom{h_1} \right\} \text{Majorana mass}
 \end{aligned}$$

Mass matrix of leptons

Superpotential W

$$\langle \text{Im}\tau \rangle \rightarrow \infty, \quad Y^{A_4} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



$$\langle \hat{H}_u \rangle \quad \langle \hat{H}_d \rangle \quad \langle \hat{S} \rangle = \begin{pmatrix} S_1 \\ S_2 \\ 0 \end{pmatrix}$$

breaks Z_3 symmetry

P_{ijk} : Permutation matrix

$$M_E = \langle \hat{H}_d \rangle P_{ijk} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} P_{ijk}^T$$

$$(\hat{E}_1^c, \hat{E}_2^c, \hat{E}_3^c)$$



$$(\hat{e}^c, \hat{\mu}^c, \hat{\tau}^c)$$

$$M_D = \langle \hat{H}_u \rangle \begin{pmatrix} 2g_1 & 0 & 0 \\ 0 & 0 & -g_1 + g_2 \\ 0 & -g_1 - g_2 & 0 \end{pmatrix} P_{ijk}^T$$

Here $P_{ijk} = \text{diag}(1, 1, 1)$

$$M_M = \begin{pmatrix} (h_1 + 4h_4) S_1 & 0 & (h_2 + h_4 + h_5) S_2 \\ 0 & (h_2 - 2h_4 - 2h_5) S_2 & (h_1 - 2h_4) S_1 \\ (h_2 + h_4 + h_5) S_2 & (h_1 - 2h_4) S_1 & 0 \end{pmatrix}$$

Mass matrix of leptons

- Neutrino mass matrix
 - Seesaw mechanism

$$M_\nu = -M_D^T M_M^{-1} M_D = \Lambda \begin{pmatrix} 1 & b_2 b_3 & b_3 \\ b_2 b_3 & b_1 b_2 & b_1 \\ b_3 & b_1 & b_3^2 \end{pmatrix}$$

$$\Lambda = - \frac{4g_1^2 (h_1 - 2h_4)^2 \langle \hat{H}_u \rangle^2 S_1^2}{(h_1 - 2h_4)^2 (h_1 + 4h_4) S_1^3 + (h_2 + h_4 + h_5)^2 (h_2 - 2h_4 - 2h_5) S_2^3}$$

$$b_1 = \frac{(g_1^2 - g_2^2) (h_1 + 4h_4)}{4g_1^2 (h_1 - 2h_4)}, \quad b_2 = - \frac{(g_1 + g_2) (h_2 - 2h_4 - 2h_5) S_2}{(g_1 - g_2) (h_1 - 2h_4) S_1},$$

$$b_3 = \frac{(g_1 - g_2) (h_2 + h_4 + h_5) S_2}{2g_1 (h_1 - 2h_4) S_1}$$

Mass matrix of leptons

- Neutrino mass matrix
 - Seesaw mechanism

$$M_\nu = -M_D^T M_M^{-1} M_D = \Lambda \begin{pmatrix} 1 & b_2 b_3 & b_3 \\ b_2 b_3 & b_1 b_2 & b_1 \\ b_3 & b_1 & b_3^2 \end{pmatrix}$$

Overall mass scale Λ と3つの複素パラメータ b_1, b_2, b_3 で neutrino mass matrixは記述される

Note

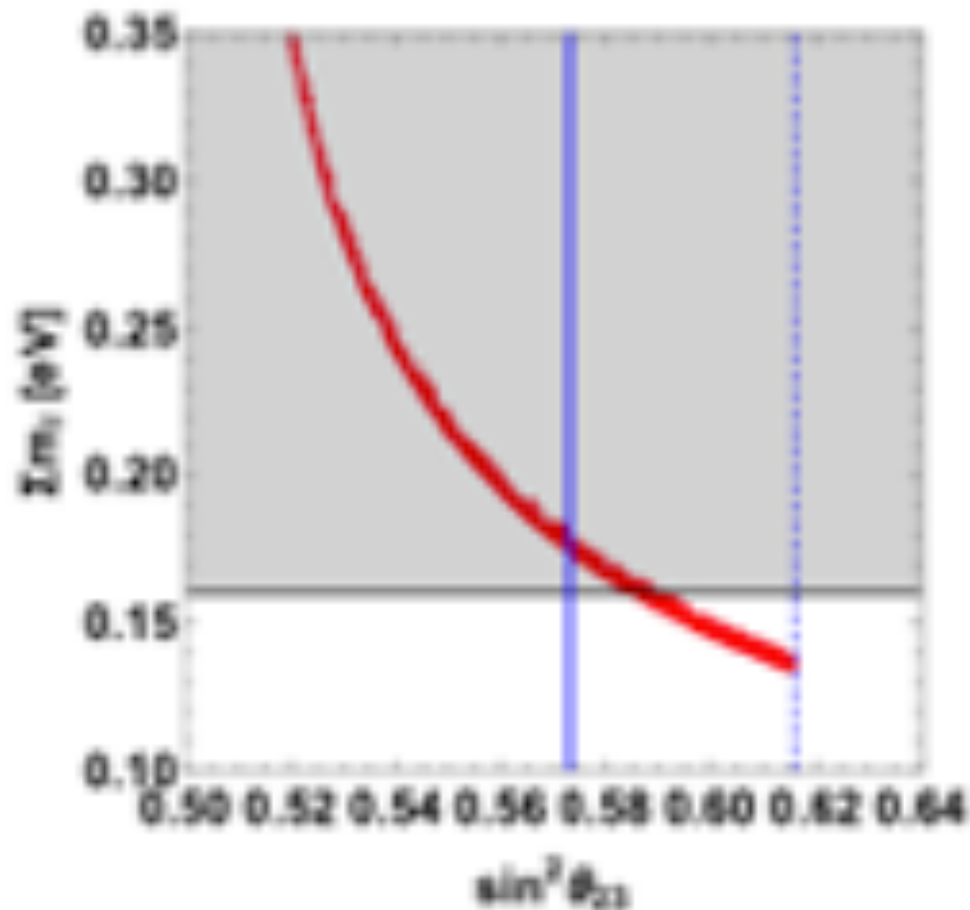
Kinetic normalizationの効果は Λ, b_1, b_2, b_3 の再定義で吸収できる

→ 予言は変わらない

Neutrino masses

- Normal hierarchyのときのみneutrino dataと合う領域を見つけた
- Neutrino massの和

$$\sum m_i \geq 0.13\text{eV}$$



Cosmological bound

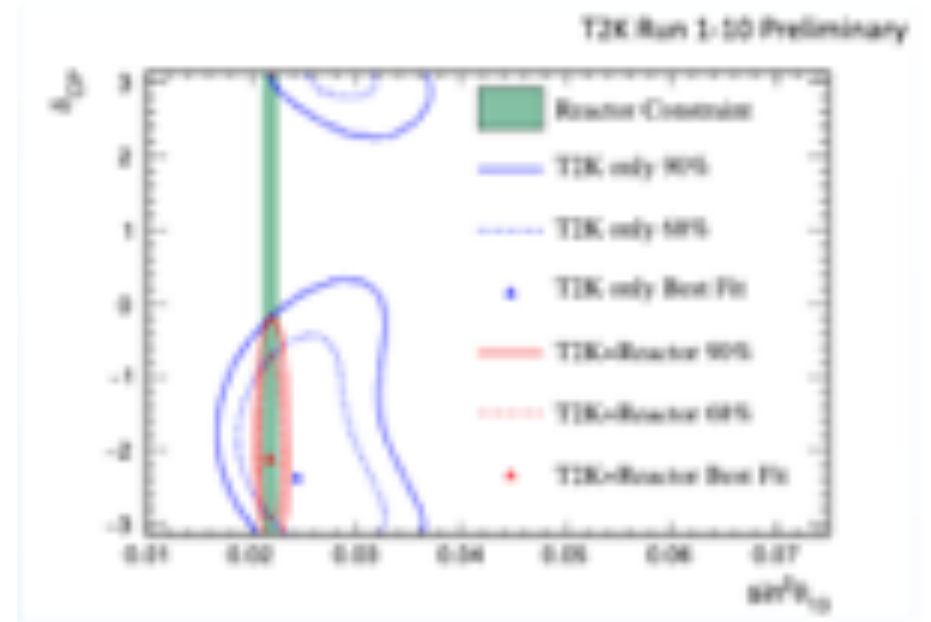
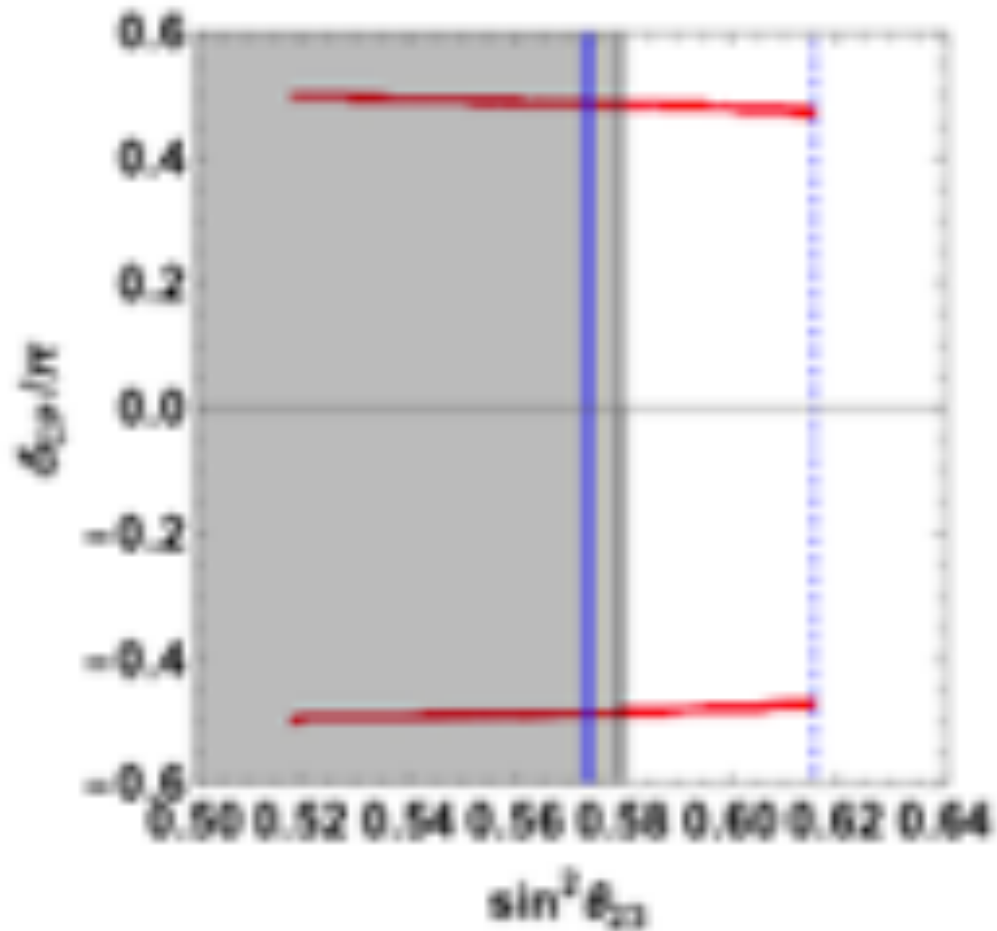
$$\sum m_i \leq 0.16\text{eV} \quad [\text{Planck 2018}]$$



$$\sin^2 \theta_{23} \geq 0.58$$

将来のneutrino振動実験
で検証可能

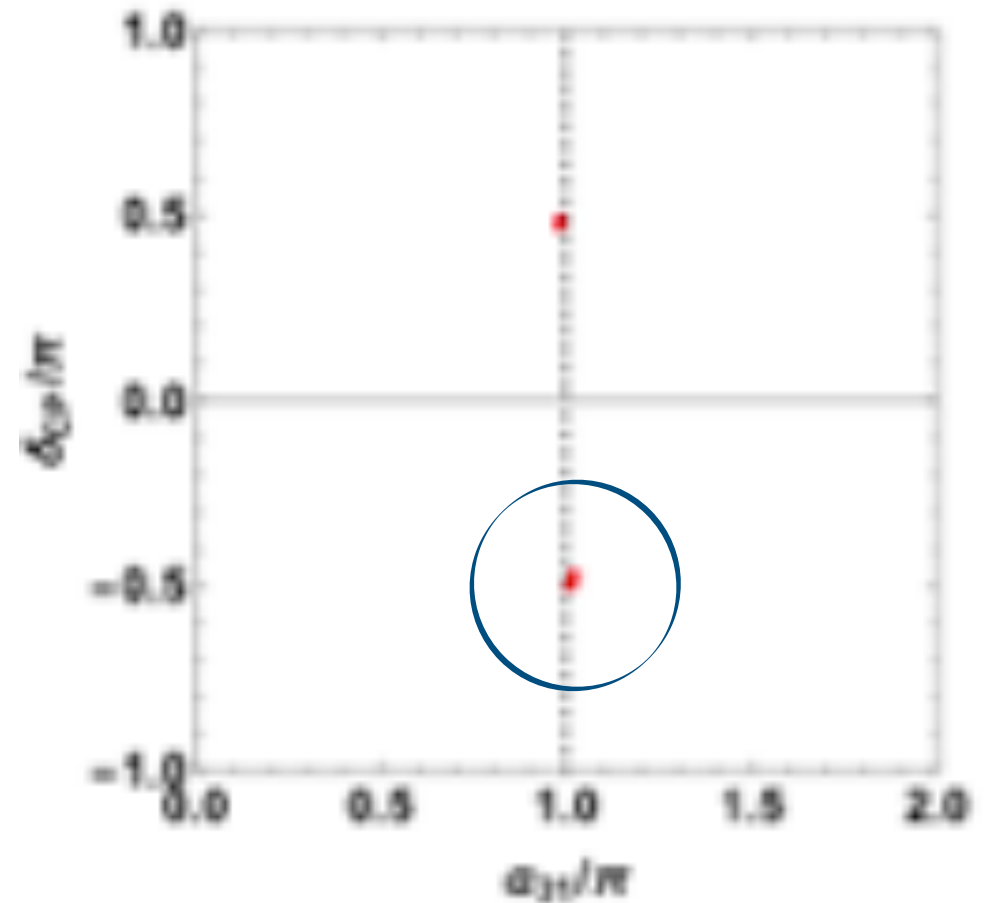
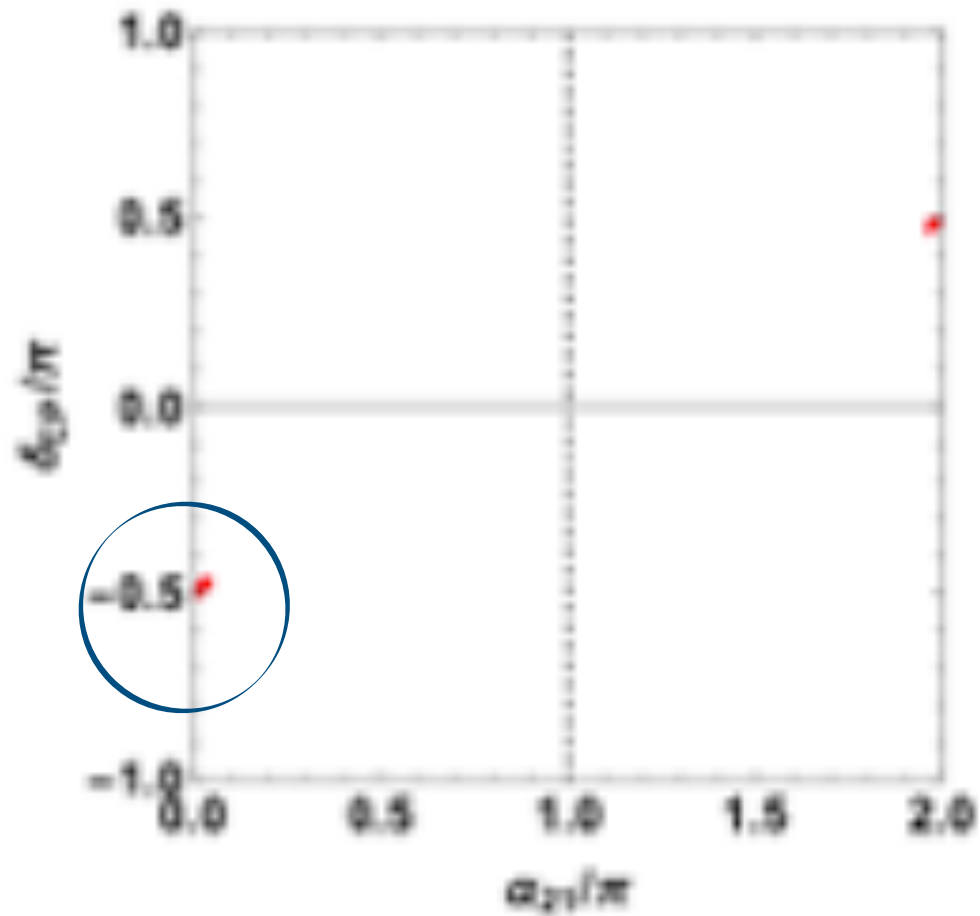
Dirac phase



T2K @Neutrino 2020
Slide by P. Dunne

$$\delta_{CP} \approx -0.5\pi, \quad +0.5\pi$$

Majorana phases

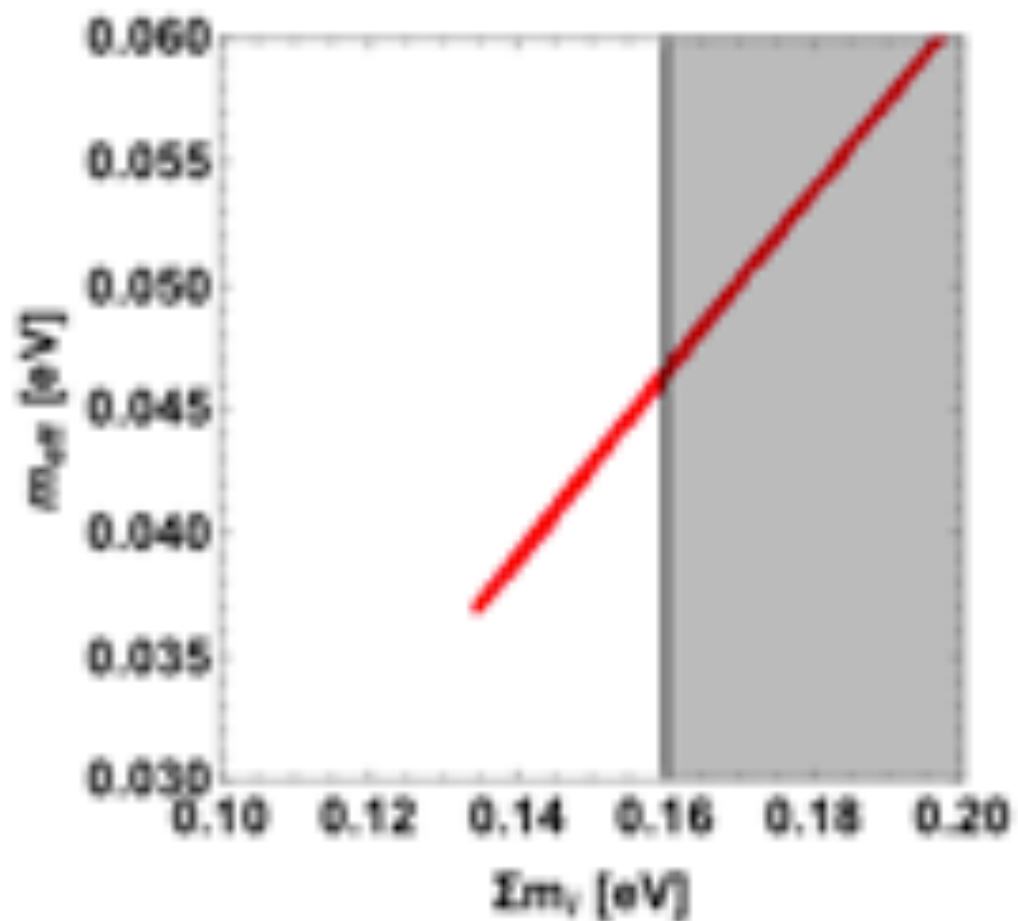


Neutrino 振動実験からfavor

$$\alpha_{21} \approx 0 + \epsilon, \quad \alpha_{31} \approx \pi + \epsilon$$

$$0 < \epsilon \ll 1$$

Neutrinoless double beta decay



$\sum m_i \leq 0.16$ eVを課すと

$$0.037 \text{ eV} \leq m_{\text{eff}} \leq 0.047 \text{ eV}$$

Summary

Summary

- Modular A_4 対称性を持つ lepton flavor modelを提案した
 - Large volume limit ($\text{Im } \tau \gg 1$)で振動実験の結果を説明可能
 - Singlet のVEVがHiggs μ -termと右巻きニュートリノmassの起源となる
 - Kähler potentialからの余計な混合を避けることができる

- 模型の予言

- NHの場合のみが振動実験の結果と合う

$$\sin^2 \theta_{23} \geq 0.58$$

$$\sum m_i \geq 0.13 \text{eV}$$

$$0.037 \text{ eV} \leq m_{\text{eff}} \leq 0.047 \text{ eV}$$

favor

$$\delta_{CP} \approx -0.5\pi, \quad +0.5\pi$$

$$\alpha_{21} \approx 0 + \epsilon, \quad 2\pi - \epsilon$$

$$\alpha_{21} \approx \pi + \epsilon, \quad \pi - \epsilon$$