Anomalies and Supersymmetry (I & II)

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East Asia Symposium on Fields and Strings 2021, Osaka City Univ.
& KIAS-YITP 2021: String Theory & Quantum Gravity

Minasian, Papadimitriou, P.Y. 2021
...
Kaiser 1988~1989
Itoyama, Nair, Ren 1985~1986
Bardeen, Zumino 1984

Anomalies vs. Anomalous Ward Identities

Diffeomorphism and
Covariant Gauge/Flavor Currents

Supersymmetrized Anomaly

Anomaly Inflow with SUSY

Extended BRST and New Anomaly Descent

Anomalies vs. Anomalous Ward Identities

effective action, gauge symmetry, and anomaly

$$e^{-W(\mathcal{A})} = \int [d\Psi]\, e^{-S(\Psi;\mathcal{A})}$$

$$S(\Psi + \delta_\Lambda \Psi; \mathcal{A} + \delta_\Lambda \mathcal{A}) = S(\Psi; \mathcal{A})$$

$$\delta_\Lambda \mathcal{A} = d_{\mathcal{A}} \Lambda$$

$$\delta_\Lambda W(\mathcal{A}) = \frac{\delta W}{\delta \Lambda} = \delta_\Lambda \mathcal{A} \cdot \frac{\delta W}{\delta \mathcal{A}} = -\Lambda \cdot \langle \nabla_\mu \mathcal{J}^\mu \rangle \;\neq\; 0 \;\; ?$$

$$\frac{\delta S}{\delta \mathcal{A}} = \mathcal{J}$$

Wess-Zumino consistency → anomaly descent

$$[\delta_{\Lambda'}, \delta_{\Lambda}] W(\mathcal{A}) = \delta_{[\Lambda', \Lambda]} W(\mathcal{A})$$

δ_{Λ} never acts directly on parameters
it acts only on field variables and
the commutator on the right
is induced via an integration by part

BRST (Becchi-Rouet-Stora-Tyutin)

$$\mathcal{B} = \mathcal{A} + v$$

gauge function, valued in
anticommuting numbers

$$\mathcal{G} = (d + \mathbf{s})\mathcal{B} + \mathcal{B} \wedge \mathcal{B}$$

$$= d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = \mathcal{F}$$

$$\mathbf{s}\mathcal{A} = -dv - \{\mathcal{A}, v\}$$

$$\mathbf{s}\mathcal{F} = \mathcal{F}v - v\mathcal{F}$$

$$\mathbf{s}\, v = -v^2 \;= -(v_a T^a)(v_b T^b) = -v_a v_b\, [T^a, T^b]/2$$

unlike δ_Λ

$$0 = (d + \mathbf{s})^2$$

$$0 = d^2 = \mathbf{s}^2 = d\mathbf{s} + \mathbf{s}d$$

anomaly polynomials

$$P_{d+2}(\mathcal{F}) = \sum_{\sum n_j = d/2+1} \# \, \mathrm{tr}\mathcal{F}^{n_1} \wedge \cdots \wedge \mathrm{tr}\mathcal{F}^{n_k}$$

$$= dw_{d+1}(\mathcal{A}, \mathcal{F})$$

anomaly polynomials and anomaly descent

$$\mathcal{B} = \mathcal{A} + v$$

$$\mathcal{G} \equiv (d + \mathbf{s})\mathcal{B} + \mathcal{B} \wedge \mathcal{B}$$

$$= d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = \mathcal{F}$$

$$(d + \mathbf{s})\mathcal{G} + \mathcal{B}\mathcal{G} - \mathcal{G}\mathcal{B} = 0$$

$$P_{d+2}(\mathcal{F}) = P_{d+2}(\mathcal{G}) = (d + \mathbf{s})w_{d+1}(\mathcal{B}, \mathcal{G})$$

$$(d + \mathbf{s})P_{d+2}(\mathcal{F}) = (d + \mathbf{s})^2 w_{d+1}(\mathcal{B}, \mathcal{G}) = 0$$

expanding in terms of the ghost number

$$w_{d+1}(\mathcal{B},\mathcal{G}) = w^{(0)}_{d+1}(\mathcal{A},\mathcal{F}) + w^{(1)}_{d}(v;\mathcal{A},\mathcal{F}) + w^{(2)}_{d-1}(v;\mathcal{A},\mathcal{F}) + \cdots$$

$$P_{d+2}(\mathcal{F}) = P_{d+2}(\mathcal{G}) = (d+\mathbf{s})w_{d+1}(\mathcal{B},\mathcal{G})$$

$$(d+\mathbf{s})P_{d+2}(\mathcal{F}) = (d+\mathbf{s})^2 w_{d+1}(\mathcal{B},\mathcal{G}) = 0$$

gives a series of descent relations

$$w_{d+1}(\mathcal{B},\mathcal{G}) = w^{(0)}_{d+1}(\mathcal{A},\mathcal{F}) + w^{(1)}_{d}(v;\mathcal{A},\mathcal{F}) + w^{(2)}_{d-1}(v;\mathcal{A},\mathcal{F}) + \cdots$$

$$dw^{(0)}_{d+1}(\mathcal{A},\mathcal{F}) = P_{d+2}(\mathcal{F})$$

$$\mathbf{s}\, w^{(0)}_{d+1}(\mathcal{A},\mathcal{F}) + dw^{(1)}_{d}(v;\mathcal{A},\mathcal{F}) = 0$$

$$\boxed{\mathbf{s}\, w^{(1)}_{d}(v;\mathcal{A},\mathcal{F}) + dw^{(2)}_{d-1}(v;\mathcal{A},\mathcal{F}) = 0}$$

$$\mathbf{s}\, w^{(2)}_{d-1}(v;\mathcal{A},\mathcal{F}) + dw^{(3)}_{d-2}(v;\mathcal{A},\mathcal{F}) = 0$$

BRST algera offers solutions to the WZ, bypassing path integral

$$0 = \mathbf{s}\int w_d^{(1)}(v;\mathcal{A},\mathcal{F}) + \int dw_{d-1}^{(2)}(v;\mathcal{A},\mathcal{F})$$

$$v = \vartheta'\Lambda' + \vartheta\Lambda$$

$$0 = -\vartheta'\vartheta[\delta_{\Lambda'},\delta_{\Lambda}]\int_X w_{d+1}^{(0)}(\mathcal{A},\mathcal{F}) + \vartheta'\vartheta\,\delta_{[\Lambda',\Lambda]}\int_X w_{d+1}^{(0)}(\mathcal{A},\mathcal{F})$$

$$\delta_{\Lambda'}\int w_d^{(1)}(\Lambda;\mathcal{A},\mathcal{F}) - \delta_{\Lambda}\int w_d^{(1)}(\Lambda';\mathcal{A},\mathcal{F}) = \int w_d^{(1)}([\Lambda',\Lambda];\mathcal{A},\mathcal{F})$$

anomaly descent → consistent anomaly

$$-\delta_\Lambda W(\mathcal{A}) = \Lambda \cdot \langle \nabla_\mu \mathcal{J}^\mu \rangle = \int w_d^{(1)}(v;\mathcal{A},\mathcal{F})\Big|_{v\to\Lambda}$$

↑

$$0 = \mathbf{s} \int w_d^{(1)}(v;\mathcal{A},\mathcal{F})$$

for some $P_{d+2}(\mathcal{F})$
to be determined by
one-loop computation

Alvarez-Gaume & Witten 1984

how to deal with more generic variations of gauge fields

$$\Delta_a \mathcal{A} = a\ , \ \ \Delta_a \mathcal{F} = da + \mathcal{A}a + a\mathcal{A}\ , \ \ \Delta_a v = 0$$

$a = \mathcal{L}_\xi \mathcal{A}$ diffeomorphism transformation

$a = \delta_\epsilon \mathcal{A}$ supersymmetry transformation

$\vdots$

Bardeen, Zumino 1984

an anti-derivative proves very useful

$$\Delta_a \mathcal{A} = a \ , \quad \Delta_a \mathcal{F} = da + \mathcal{A}a + a\mathcal{A} \ , \quad \Delta_a v = 0$$

$$\Delta_a = dl_a + l_a d \qquad\qquad l_a \mathcal{F} = a \ , \quad l_a \mathcal{A} = 0 \ , \quad l_a v = 0$$

Bardeen, Zumino 1984

the Bardeen-Zumino current

to be found everywhere in today's talk

$$\Delta_a \mathcal{A} = a \ , \ \ \Delta_a \mathcal{F} = da + \mathcal{A}a + a\mathcal{A} \ , \ \ \Delta_a v = 0$$

$$\Delta_a = dl_a + l_a d \qquad\qquad l_a \mathcal{F} = a \ , \ \ l_a \mathcal{A} = 0 \ , \ \ l_a v = 0$$

$$\Delta_a w^{(0)}_{d+1}(\mathcal{A}, \mathcal{F}) = d \boxed{\left(l_a w^{(0)}_{d+1}(\mathcal{A}, \mathcal{F}) \right) \equiv a \cdot X_{\mathrm{BZ}}} + l_a P_{d+2}(\mathcal{F})$$

Bardeen, Zumino 1984

which adds to & covariantizes the consistent current

$$J_{\text{covariant}} = J + X_{\text{BZ}}(A, F; \cdots)$$

$$P_6(F) = \#\text{tr}F^3$$

$$w_5^{(0)}(A, F) = \#\text{tr}\left(AFF - \frac{1}{2}A^3F + \frac{1}{10}A^5\right)$$

l_a

$$a \cdot X_{\text{BZ}} = \int 6 \cdot \#\text{tr}\left(aA\left(\frac{1}{3}F - \frac{1}{12}A^2\right)\right)$$

$$= \int \#\text{tr}\left(a\left(AdA + dAA + \frac{3}{2}A^3\right)\right)$$

Bardeen, Zumino 1984

arbitrary gauge-field variation of the consistent anomaly
→ gauge variation of Bardeen-Zumino current

$$\Delta_a \mathcal{A} = a \ , \quad \Delta_a \mathcal{F} = da + \mathcal{A}a + a\mathcal{A} \ , \quad \Delta_a v = 0$$

$$\Delta_a = dl_a + l_a d$$

$$\boxed{0 = \mathbf{s} l_a + l_a \mathbf{s}}$$

$$\Delta_a w_d^{(1)}(v; \mathcal{A}, \mathcal{F}) = d\left(l_a w_d^{(1)}(v; \mathcal{A}, \mathcal{F})\right) - l_a \left(\mathbf{s} w_{d+1}^{(0)}(\mathcal{A}, \mathcal{F})\right)$$

$$= d\left(l_a w_d^{(1)}(v; \mathcal{A}, \mathcal{F})\right) + \mathbf{s}\left(l_a w_{d+1}^{(0)}(\mathcal{A}, \mathcal{F})\right)$$

$$\Delta_a \int w_d^{(1)}(v; \mathcal{A}, \mathcal{F}) = \mathbf{s} \int l_a w_{d+1}^{(0)}(\mathcal{A}, \mathcal{F}) = \mathbf{s}\,(a \cdot X_{\mathrm{BZ}})$$

the question: what happens if one symmetry affects gauge fields of different symmetries?

$$\delta_\Phi W = \delta_\Phi \mathcal{A} \cdot \frac{\delta W}{\delta \mathcal{A}} + \sum_{A \neq \mathcal{A}} \delta_\Phi A \cdot \frac{\delta W}{\delta A}$$

δ_Φ diffeomorphisms or supersymmetries

how do the anomalies of internal symmetries
affect Ward identities of spacetime symmetries

$$\delta_\Phi W = \delta_\Phi \mathcal{A} \cdot \frac{\delta W}{\delta \mathcal{A}} + \sum_{A \neq \mathcal{A}} \delta_\Phi A \cdot \frac{\delta W}{\delta A}$$

$$= -\Phi \cdot \langle \nabla_\mu \mathcal{J}^\mu \rangle + \sum_{A \neq \mathcal{A}} \delta_\Phi A \cdot \langle J \rangle$$

Wess-Zumino consistency, again

$$[\delta_{\Lambda'}, \delta_{\Lambda}] W(\mathcal{A}) = \delta_{[\Lambda', \Lambda]} W(\mathcal{A})$$

$$[\delta_{\Phi'}, \delta_{\Phi}] W(\mathcal{A}) = \delta_{[\Phi', \Phi]} W(\mathcal{A})$$

$$[\delta_{\Phi}, \delta_{\Lambda}] W(\mathcal{A}) = \delta_{?????} W(\mathcal{A})$$

δ_{Φ} diffeomorphisms or supersymmetries

Diffeomorphism and
Covariant Gauge/Flavor Currents

an aside: two equivalent forms of Riemannian geometry

with $SO(d)$ spin connection

$$\Omega^{ab} \equiv \Omega^{ab}_{\mu} dx^{\mu} \qquad \mathcal{R}^{a}{}_{b} = d\Omega^{a}{}_{b} + \Omega^{a}{}_{c} \wedge \Omega^{c}{}_{b}$$

with $GL(d)$ Christoffel connection

$$\Gamma_{\beta}{}^{\alpha} \equiv -\Gamma^{\alpha}_{\mu\beta} dx^{\mu} \qquad R_{\beta}{}^{\alpha} = d\Gamma_{\beta}{}^{\alpha} + \Gamma_{\beta}{}^{\gamma} \wedge \Gamma_{\gamma}{}^{\alpha}$$

$$\Downarrow$$

$$(R_{\beta}{}^{\alpha})_{\nu\mu} = e^{\alpha}_{a} e^{b}_{\beta} (\mathcal{R}^{a}{}_{b})_{\mu\nu}$$

SO(d) Lorentz anomaly

$$\Omega^{ab} \equiv \Omega^{ab}_{\mu} dx^{\mu} \qquad \delta_{\mathcal{O}} \Omega = d_{\Omega} \mathcal{O}$$

$$\delta_{\mathcal{O}_1} \int w^{(1)}_{2n}(\mathcal{O}_2; \Omega, \mathcal{R}; F) - \delta_{\mathcal{O}_2} \int w^{(1)}_{2n}(\mathcal{O}_1; \Omega, \mathcal{R}; F) = \int w^{(1)}_{2n}([\mathcal{O}_1, \mathcal{O}_2]; \Omega, \mathcal{R}; F)$$

cancelation of this Lorentz anomaly implies
cancelation of the diffeomorphism anomaly and vice versa;
however, the former is not the expression
that enters the right hand side of the latter Ward identity !

diffeomorphisms generate both rotations and translations

with $SO(d)$ spin connection

$$\Omega^{ab} \equiv \Omega^{ab}_{\mu} dx^{\mu}$$

$$\delta_{\xi}\Omega = \hat{\mathcal{L}}_{\xi}\Omega + d_{\Omega}\hat{\xi}_K$$

$$\hat{\xi}^{ab}_K \equiv \nabla^{[b}\xi^{a]} - \xi^{\mu}\Omega^{ab}_{\mu}$$

Kosman lift, circa 1970's

$$\delta_{\mathcal{O}}\Omega = d_{\Omega}\mathcal{O} \neq \delta_{\xi}\Omega$$

regardless of the choice for $\mathcal{O}$; because diffeo must involve a translation as well

Christoffel connection proves to be more amenable for this

with $GL(d)$ Christoffel connection

$$\Gamma_{\beta}{}^{\alpha} \equiv -\Gamma^{\alpha}_{\mu\beta} dx^{\mu}$$

$$\delta_{\xi}\Gamma = \mathcal{L}'_{\xi}\Gamma + d_{\Gamma}(-\partial\xi)$$

$$= \mathcal{L}'_{\xi}\Gamma + \delta^{GL(d)}_{-\partial\xi}\Gamma$$

$$(\partial\xi)_{\beta}{}^{\alpha} = \partial_{\beta}\xi^{\alpha}$$

Wess-Zumino for diffeomorphism anomaly

$$[\delta_\zeta, \delta_\xi] W(\mathcal{A}) = \delta_{-[\zeta,\xi]_{\rm Lie}} W(\mathcal{A})$$

with $GL(d)$ Christoffel connection

$$\Gamma_\beta{}^\alpha \equiv -\Gamma^\alpha_{\mu\beta} dx^\mu$$

$$\delta_\xi \Gamma = \mathcal{L}'_\xi \Gamma + d_\Gamma(-\partial\xi)$$

$$= \mathcal{L}'_\xi \Gamma + \delta^{GL(d)}_{-\partial\xi} \Gamma$$

$$(\partial\xi)_\beta{}^\alpha = \partial_\beta \xi^\alpha$$

Bardeen, Zumino 1984

GL(d) anomaly descent solves the diffeomorphism WZ

although it does not follow from the standard BRST algebra

$$-\delta_\xi W(\mathcal{A}) = \int w_{2n}^{(1)}(-\partial\xi; \Gamma, R; F)$$

$$\delta_\zeta \int w_{2n}^{(1)}(-\partial\xi; \Gamma, R; F) - \delta_\xi \int w_{2n}^{(1)}(-\partial\zeta; \Gamma, R; F) \;=\; \int w_{2n}^{(1)}(\partial[\zeta,\xi]_{\mathrm{Lie}}; \Gamma, R; F)$$

$$\delta_\xi \Gamma = \mathcal{L}'_\xi \Gamma + \boxed{\delta_{-\partial\xi}^{GL(d)}} \Gamma$$

Bardeen, Zumino 1984

WZ consistency for diffeomorphisms + gauge/flavor symmetries

$$[\delta_{\Lambda'}, \delta_{\Lambda}] W(\mathcal{A}, \Gamma) = \delta_{[\Lambda', \Lambda]} W(\mathcal{A}, \Gamma)$$

$$[\delta_{\xi'}, \delta_{\xi}] W(\mathcal{A}, \Gamma) = \delta_{-[\xi', \xi]_{\rm Lie}} W(\mathcal{A}, \Gamma)$$

$$[\delta_{\xi}, \delta_{\Lambda}] W(\mathcal{A}, \Gamma) = \delta_{-\mathcal{L}_{\xi}\Lambda} W(\mathcal{A}, \Gamma)$$

the last is solved universally as

$$\delta_\Lambda \left(\delta_\xi W(\mathcal{A}, \Gamma) \right) = \delta_\Lambda \int w_d^{(1)}(-\partial\xi; \Gamma, R; F) \ = \ 0$$

$$\delta_\xi \delta_\Lambda W(\mathcal{A}, \Gamma) + \delta_{\mathcal{L}_\xi \Lambda} W(\mathcal{A}, \Gamma) \ = \ 0$$

the diffeomorphism anomaly, and thus the other side of
the Ward identity, must be explicitly and separately gauge-invariant

$$-\delta_\xi W = \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu} \rangle - \sum \mathcal{L}_\xi A \cdot \langle J \rangle \qquad = \int w_d^{(1)}(-\partial\xi; \Gamma, R; F)$$

the last is solved universally as

$$\delta_\Lambda\left(\delta_\xi W(\mathcal{A},\Gamma)\right) = \delta_\Lambda \int w_d^{(1)}(-\partial\xi;\Gamma,R;F) \;=\; 0$$

$$\delta_\xi \delta_\Lambda W(\mathcal{A},\Gamma) + \delta_{\mathcal{L}_\xi \Lambda} W(\mathcal{A},\Gamma) \;=\; 0$$

the diffeomorphism anomaly, and thus the other side of
the Ward identity, must be explicitly and separately gauge-invariant

$$-\delta_\xi W = \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu}\rangle \boxed{- \sum \mathcal{L}_\xi A \cdot \langle J \rangle} = \int w_d^{(1)}(-\partial\xi;\Gamma,R;F)$$

$$= \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu}\rangle \boxed{- \sum \xi^\mu F_{\mu\nu} \cdot \langle J^\nu_{\text{covariant}}\rangle}$$

How?

Minasian, Papadimitriou, P.Y. 2021

covariant current = consistent current + Bardeen-Zumino

$$J_{\text{covariant}} = J + X_{\text{BZ}}(A, F; \cdots)$$

$$a \cdot X_{\text{BZ}} \equiv \int l_a \left[w_{d+1}^{(0)}(A, F; \cdots) \right]$$

$$l_a F = a \ , \ \ l_a A = 0 \ , \ \ l_a v = 0$$

$$-\delta_\xi W = \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu} \rangle \boxed{- \sum \xi^\mu F_{\mu\nu} \cdot \langle J^\nu_{\text{covariant}} \rangle} = \int w_d^{(1)}(-\partial\xi; \Gamma, R; F)$$

How?

for the example of U(1)

$$\mathcal{L}_\xi A = \xi \lrcorner dA + d(\xi \lrcorner A) = \xi \lrcorner F + d(\xi \lrcorner A)$$

$$\mathcal{L}_\xi A \cdot \langle J \rangle = \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle - (\xi^\nu A_\nu) \cdot \langle \partial_\mu J^\mu \rangle$$

$$= \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle - \int w_d^{(1)}(\Lambda \to (\xi^\nu A_\nu); A, F; \cdots)$$

$$= \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle + \xi^\mu F_{\mu\nu} \cdot X^\nu_{BZ}$$

Jensen, Loganayagam, Yarom, 2012

Papadimitriou 2017

$$-\delta_\xi W = \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu} \rangle \boxed{- \sum \xi^\mu F_{\mu\nu} \cdot \langle J^\nu_{\text{covariant}} \rangle} = \int w_d^{(1)}(-\partial\xi; \Gamma, R; F)$$

the same happens for non-Abelian cases

$$\mathcal{L}_\xi A = \xi \lrcorner dA + d(\xi \lrcorner A) = \xi \lrcorner F - (\xi \lrcorner A)A + A(\xi \lrcorner A) + d(\xi \lrcorner A)$$

$$\begin{aligned}
\mathcal{L}_\xi A \cdot \langle J \rangle &= \xi^\mu (dA)_{\mu\nu} \cdot \langle J^\nu \rangle - (\xi^\nu A_\nu) \cdot \langle \partial_\mu J^\mu \rangle \\
&= \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle - (\xi^\nu A_\nu) \cdot \langle \nabla_\mu J^\mu \rangle \\
&= \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle - \int w_d^{(1)}(\Lambda \to (\xi^\nu A_\nu); A, F; \cdots) \\
&= \xi^\mu F_{\mu\nu} \cdot \langle J^\nu \rangle + \xi^\mu F_{\mu\nu} \cdot X^\nu_{BZ}
\end{aligned}$$

$$-\delta_\xi W = \xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu} \rangle \boxed{- \sum \xi^\mu F_{\mu\nu} \cdot \langle J^\nu_{\text{covariant}} \rangle} = \int w_d^{(1)}(-\partial\xi; \Gamma, R; F)$$

covariant current vs. consistent current

the covariant current & the covariant anomaly
played many intermediate roles historically;
most crucially in Alvarez-Gaume & Witten's
general derivation of gauge & gravitational anomaly

they are often deemed unphysical by themselves
since the consistent current (anomaly) is the one
produced by variation of the (effective) action

covariant (not the consistent) gauge/flavor current
are naturally induced in the diffeomorphism Ward identity

$$0 \;=\; \delta_\Lambda \int w_d^{(1)}(-\partial\xi;\Gamma,R;F)$$

$$\delta_\xi W = -\xi_\nu \cdot \langle \nabla_\mu T^{\mu\nu}\rangle + \sum \xi^\mu F_{\mu\nu}\cdot\langle J^\nu_{\text{covariant}}\rangle = \int w_d^{(1)}(-\partial\xi;\Gamma,R;F)$$

such occurrences of covariant gauge/flavor currents on the left
are expected in the diffeomorphism Ward identity since
the anomaly on the right is itself gauge/flavor invariant

Minasian, Papadimitriou, P.Y. 2021

Supersymmetrized Anomaly

WZ consistency for supersymmetry + gauge/flavor symmetries

$$[\delta_{\Lambda'}, \delta_{\Lambda}]W(\mathcal{A}, \psi) = \delta_{[\Lambda', \Lambda]}W(\mathcal{A}, \psi)$$

$$[\delta_{\epsilon'}, \delta_{\epsilon}]W(\mathcal{A}, \psi) = \delta_{\bar{\epsilon}'\gamma\epsilon}W(\mathcal{A}, \psi)$$

δ_ϵ supersymmetry transformations

$$[\delta_{\epsilon}, \delta_{\Lambda}]W(\mathcal{A}, \psi) = 0$$ in the WZ gauge choice

$$\mathcal{Q} = \frac{\delta S}{\delta \psi}$$

$$\delta_\epsilon W = -\epsilon \cdot \langle \nabla_\mu \mathcal{Q}^\mu \rangle + \sum \delta_\epsilon A \cdot \langle J \rangle \ = ?$$

the WZ gauge choice fixes part of the supersymmetric gauge transformation of the vector supermultiplet

$$V \;\to\; V + \Psi + \Psi^*$$

and leaves behind gauge transformations of the gauge field

$$\mathcal{A} \;\to\; \mathcal{A} + d_{\mathcal{A}}\Lambda$$

so "undo" supersymmetry completion of the latter transformation

WZ consistency for supersymmetry + gauge/flavor symmetries

$$[\delta_{\Lambda'}, \delta_\Lambda] W(\mathcal{A}, \psi) = \delta_{[\Lambda', \Lambda]} W(\mathcal{A}, \psi)$$

$$[\delta_{\epsilon'}, \delta_\epsilon] W(\mathcal{A}, \psi) = \delta_{\bar{\epsilon}' \gamma \epsilon} W(\mathcal{A}, \psi)$$

$$[\delta_\epsilon, \delta_\Lambda] W(\mathcal{A}, \psi) = 0$$ *in the WZ gauge choice*

$$\mathcal{Q} = \frac{\delta S}{\delta \psi}$$

$$\delta_\epsilon W = -\epsilon \cdot \langle \nabla_\mu \mathcal{Q}^\mu \rangle + \sum \delta_\epsilon A \cdot \langle J \rangle \neq 0$$

Itoyama, Nair, Ren 1985

WZ consistency for supersymmetry + gauge/flavor symmetries

$$\delta_\Lambda\left(\delta_\epsilon W(\mathcal{A},\psi)\right)=\delta_\epsilon\left(\delta_\Lambda W(\mathcal{A},\psi)\right)\quad=-\delta_\epsilon\int w_d^{(1)}(\Lambda;A,F;\cdots)$$

$$=-\int\Delta_{\delta_\epsilon A}w_d^{(1)}(\Lambda;A,F;\cdots)$$

$$\Delta_a=dl_a+l_ad\qquad\qquad=-\int l_{\delta_\epsilon A}dw_d^{(1)}(\Lambda;A,F;\cdots)$$

$$\Delta_aA=d(l_aA)+l_a(dA)=0+l_a(F-A^2)=l_a(F)=a$$

$$\Delta_aF=d(l_aF)+l_a(dF)=da+l_a(-AF+FA)=d_Aa$$

Itoyama, Nair, Ren 1985

WZ consistency for supersymmetry + gauge/flavor symmetries

$$\delta_\Lambda\left(\delta_\epsilon W(\mathcal{A},\psi)\right) = \delta_\epsilon\left(\delta_\Lambda W(\mathcal{A},\psi)\right) \quad = -\delta_\epsilon \int w_d^{(1)}(\Lambda; A, F; \cdots)$$

$$= -\int \Delta_{\delta_\epsilon A} w_d^{(1)}(\Lambda; A, F; \cdots)$$

$$\Delta_a = dl_a + l_a d \qquad = -\int l_{\delta_\epsilon A} d w_d^{(1)}(\Lambda; A, F; \cdots)$$

$$dw_d^{(1)}(\Lambda; A, \cdots) = \delta_\Lambda w_{d+1}^{(0)}(\Lambda; A, \cdots) \qquad = -\int l_{\delta_\epsilon A} \delta_\Lambda w_{d+1}^{(0)}(A, F; \cdots)$$

$$= -\delta_\Lambda \int l_{\delta_\epsilon A} w_{d+1}^{(0)}(A, F; \cdots)$$

Itoyama, Nair, Ren 1985

supersymmetry variation of the effective action in the presence of gauge/flavor anomaly

$$-\delta_\epsilon W(\mathcal{A},\psi) \;=\; \sum_A \int l_{\delta_\epsilon A} w^{(0)}_{d+1}(A,F;\cdots) \;+\; \text{gauge invariant terms}$$

determines these

$$[\delta_{\epsilon'},\delta_\epsilon]W(\mathcal{A},\psi) = \delta_{\bar{\epsilon}'\gamma\epsilon}W(\mathcal{A},\psi)$$

Itoyama, Nair, Ren 1985

supersymmetry variation of the effective action in the presence of gauge/flavor anomaly

$$-\delta_\epsilon W(\mathcal{A},\psi) \;=\; \sum_A \int l_{\delta_\epsilon A} w^{(0)}_{d+1}(A,F;\cdots) \;+\; \text{gauge invariant terms}$$

determines these

$$[\delta_{\epsilon'},\delta_\epsilon]W(\mathcal{A},\psi) = \delta_{\bar{\epsilon}'\gamma\epsilon}W(\mathcal{A},\psi)$$

$$= \; \sum_A \delta_\epsilon A\cdot X_{\rm BZ} + \; \text{gauge invariant terms}$$

Itoyama, Nair, Ren 1985

again, the gauge/favor current become covariantized as in the diffeomorphism Ward identity

$$-\delta_\epsilon W(\mathcal{A},\psi) \;=\; \sum_A \delta_\epsilon A \cdot X_{\mathrm{BZ}} + \text{ gauge invariant terms}$$

determines these

$$[\delta_{\epsilon'},\delta_\epsilon]W(\mathcal{A},\psi) = \delta_{\bar{\epsilon}'\gamma\epsilon}W(\mathcal{A},\psi)$$

$$\epsilon\cdot\langle\nabla_\mu \mathcal{Q}^\mu\rangle - \sum \delta_\epsilon\mathcal{A}\cdot\langle J_{\text{covariant}} = J + X_{\mathrm{BZ}}\rangle \;=\; \text{gauge invariant terms}$$

Minasian, Papadimitriou, P.Y. 2021

what happens if we do not take
the WZ gauge for the vector supermultiplet ?

$$-\delta_\epsilon W(\mathcal{A},\psi,\cdots) \;=\; 0$$

no real difference in the Ward identity, other than
that the variation is attributed to more auxiliary fields
and that all terms naturally belong to the left hand side

$$\epsilon\cdot\langle\nabla_\mu \mathcal{Q}^\mu\rangle - \underline{\sum \delta_\epsilon\mathcal{A}\cdot\langle J\rangle \;-\; \text{terms involving the rest of vector multiplet}} \;=\; 0$$

gauge-invariant altogether, similar to the diffeomorphism Ward identity

these different viewpoints happen because
the WZ gauge choice fixes part of the supersymmetric
gauge transformation of the vector supermultiplet

$$V \;\to\; V + \Psi + \Psi^*$$

and leaves behind gauge transformations of the gauge field

$$\mathcal{A} \;\to\; \mathcal{A} + d_{\mathcal{A}}\Lambda$$

so "undo" supersymmetry completion of the latter transformation

Itoyama, Nair, Ren 1985

if an anomaly is canceled by
anomaly inflow from variation of $\mathcal{S}_{\mathrm{bulk}}$,
what happens to Itoyama-Nair-Ren?

Minasian, Papadimitriou, P.Y. 2021

upon susy variation,

the leading Bardeen-Zumino current follows

via precisely the same math as in Itoyama-Nair-Ren

while the remainder must organize into boundary terms

since the bulk action is susy-invariant

$$\delta_\epsilon \left(\mathcal{S}_{\rm bulk} + \text{susy completion}\right) \;=\; \sum \delta_\epsilon \mathcal{A} \cdot X_{\rm BZ} + \text{gauge invariant terms}$$

Minasian, Papadimitriou, P.Y. 2021

if an anomaly is canceled by

anomaly inflow from variation of $\mathcal{S}_{\rm bulk}$,

what happens to Itoyama-Nair-Ren?

complete cancelation!

$$\delta_\epsilon \left(W(\mathcal{A}) + \mathcal{S}_{\rm bulk} + \text{susy completion} \right) \;=\; 0$$

Minasian, Papadimitriou, P.Y. 2021

supersymmetric completion of anomalies,
rather than anomalies of supersymmetry

nevertheless, they have palpable consequences
such as in exact supersymmetric partition functions
anomalously coupled to flavor vector supermultiplet

Closset, di Pietro, Kim 2019

so, how would you compute
these supersymmetrized anomalies?

solve Wess-Zumino consistency, sequentially

Itoyama, Nair, Ren 1985
Papadimitriou et. al. 2019

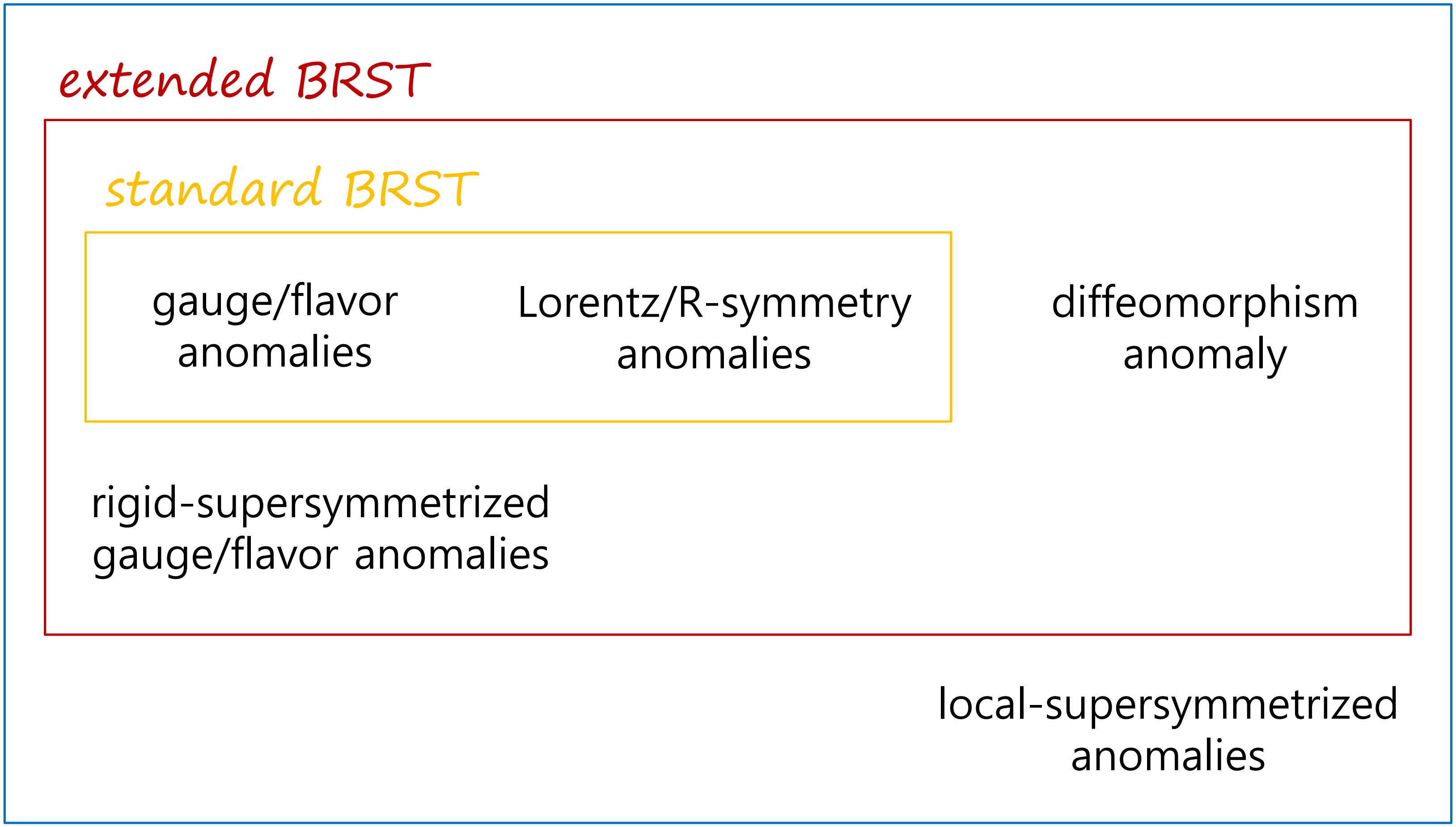
more systematic approaches,
on par with the usual anomaly descent?

anomaly inflow and
extended anomaly descent

Minasian, Papadimitriou, P.Y. 2021

further extentions of BRST ?

anomaly inflow ? (C.S. anomaly inflow for a limited subset with few susy)



Minasian, Papadimitriou, P.Y. 2021