Holographic β function in de Sitter space

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dS entropy and asymptotic freedom

Gibbons Hawking $\qquad$ dS_4 is rotated into S_4

$$\text{The volume of } S_4 \text{ is } V_{S^4} = 8\pi^2/3H^4$$

S= $V_{S^4} 6H^2/16\pi G_N = 1/g \qquad g = G_N H^2/\pi$

Distribution function $\qquad \rho(\xi(t),\omega) = \sqrt{\dfrac{4\xi(t)}{\pi g}}\exp\Big(-\dfrac{4\xi(t)}{g}\omega^2\Big).$

$$S = -\mathrm{tr}(\rho\log\rho) \sim 1/2\log(1/\xi)$$

$$<\omega^2> = \frac{3g}{4}N(t) = \frac{g}{8\xi}$$

Renormalization group and FP equation.

IR log

$$< \omega^2 >_{\mathrm{bulk}} = -\frac{3g}{4}\int_{Ha(t)}^{\Lambda}\frac{dk}{k}$$

$$= \frac{3g}{4}Ht = \frac{3g}{4}N(t).$$

Probability density at boundary

IR logs are subtracted

$$\rho_B = \exp\Big(-\frac{3g}{4}\frac{Ht}{2}\frac{\partial^2}{\partial\omega^2}\Big)\rho,$$

FP equation

Renormalization group

$$\dot{\rho} - \frac{3g}{4}\cdot\frac{H}{2}\frac{\partial^2}{\partial\omega^2}\rho = 0,$$

We assume generic Gaussian $$\frac{\partial}{\partial t}\frac{1}{2}\log(\frac{\xi}{g})+3H\xi=0$$

Master equation $$\frac{\partial}{\partial N}\log\frac{g}{\xi}=6\xi.$$

$$\frac{\partial P}{\partial t}=A\Sigma_1^N(\frac{\partial}{\partial x_i})^2P \qquad P(x_i,0)=\prod_1^N\delta(x_i-x(0))$$

FP equation g: constant Large N limit

$$\dot{\xi}=-6H\xi^2. \qquad \xi=\frac{1}{1+6Ht}.$$

We recall the following identity holds at the horizon exit $t = t_*$

$$\dot{\rho}(t_*)e^{\rho(t_*)} = k$$

It is nothing but choosing our renormalization scale as $\log k = Ht$.

$$< \omega^2(t) > = \frac{g}{8\xi} = \frac{3g}{4}N(t)$$

$$k^{1-n_s} \qquad < \zeta^2 > \propto < \omega^2 > \text{ as } \xi \propto \epsilon.$$

$$g = N^{\frac{m}{2}}, \quad \xi = \frac{m+2}{12N}.$$

Explicit solutions of power potentials $V = f^{(1/n)}$

$$\epsilon = -\frac{1}{2}\frac{\partial}{\partial N} log(H^2) = \frac{1}{4n\tilde{N}},$$

$$1 - n_s = \frac{\frac{1}{n} + 2}{2\tilde{N}}.$$

Concave solutions maybe obtained by $m \Rightarrow 1/n$: (m,n integers)

$$\epsilon = \frac{m}{4\tilde{N}} = \frac{3m}{m+2}\xi$$

UV fixed point

$$g = \frac{2}{\log N}\Big(1 - \frac{1}{\log N}\Big), \quad \xi = \frac{1}{6N}\Big(1 - \frac{1}{\log N}\Big).$$

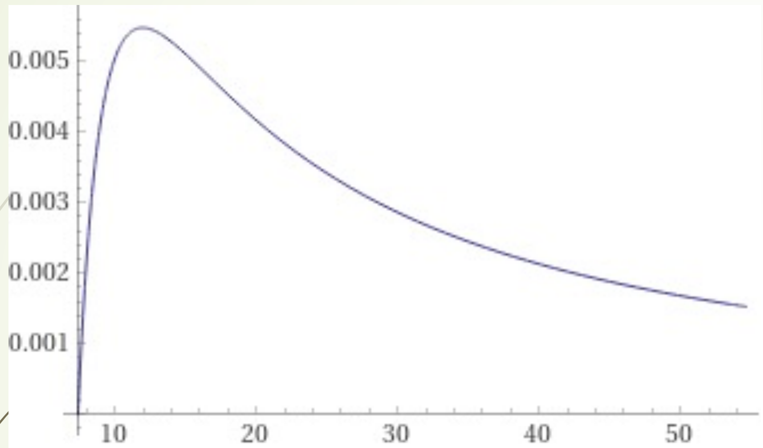
$$\beta = \frac{\partial}{\partial \log N} g = -\frac{2}{\log^2 N} + \frac{4}{\log(N)^3}$$

$\beta = 0$ *is the UV fxed point*

$$\epsilon = -\frac{1}{2}\frac{\partial}{\partial N}\log(g) = -\frac{1}{2gN}\beta_g$$

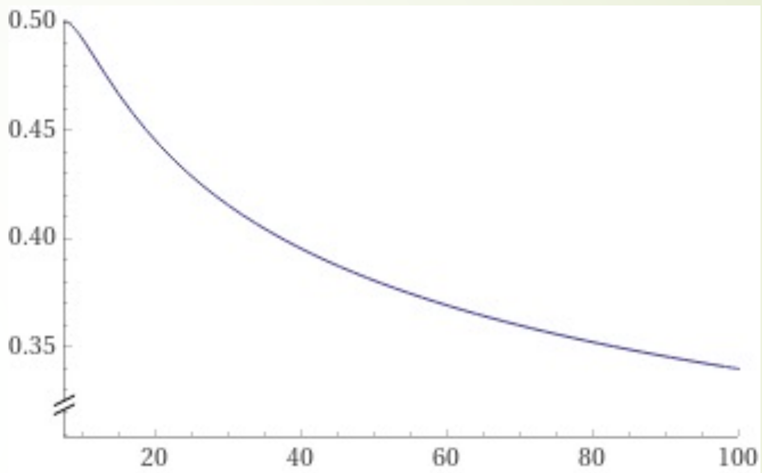
ϵ *vanishes near UV fixed point*

ϵ



N

g



N

There is a maximum in ϵ at N~10

It is small: 1/200

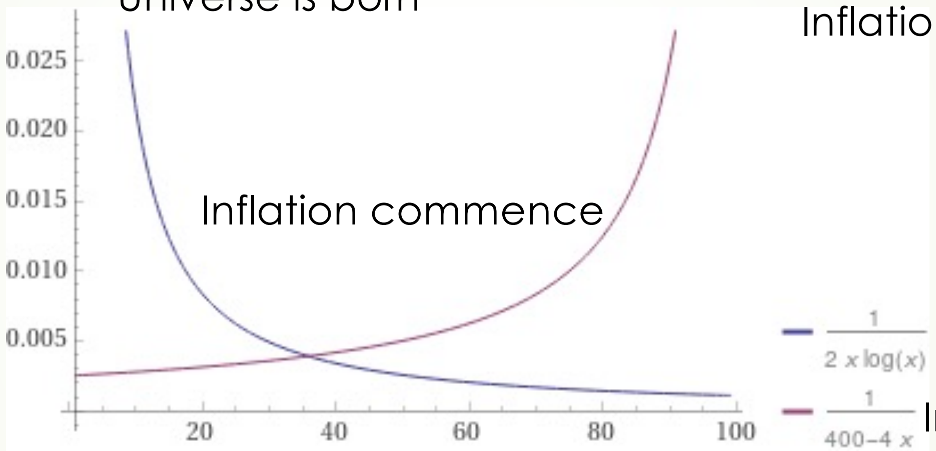
Gravitational coupling is still substantial

ϵ Problem and solution

ϵ

Universe is born

Inflation terminates



Inflation commence

UV finite

Inflation

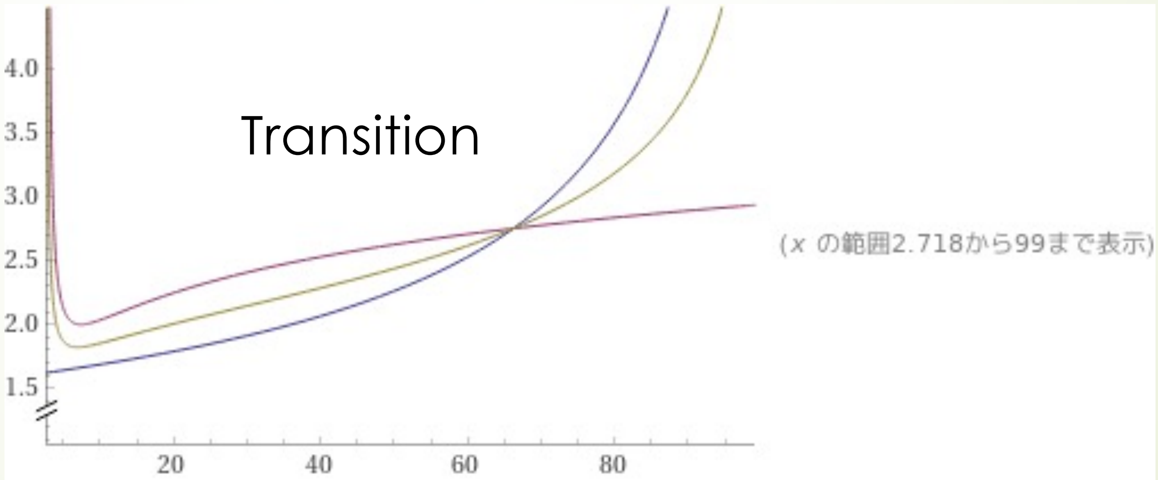
N

Pre-Inflation to Inflation

$16\,(100-x)^{-1/2}$	Blue	
$\dfrac{\frac{\log(x)}{2}}{1-\frac{1}{\log(x)}}$	Pink	$x = e$から99まで
$4\,(100-x)^{-1/4}\left(1-\dfrac{1}{\log(x)}\right)^{-1/2}\times\dfrac{\sqrt{\log(x)}}{\sqrt{2}}$	Green	

Entropy of composite Universe

Birth of Universe



Conclusions and discussions

There may be a pre-inflation era

The entropy grows logarithmically log N N:e-foldings

In the inflationary Universe, the entropy glows power like: $(N_e\text{-}N)^{(-1/n)}$

The inflationary universe inevitably dominates due to larger entropy

The inflation era lasts until g ~ 10^{-10} ~ P(density perturbation).   N~10^{20}

The CMB isotropy N ~ 50 is a tiny part of it.

Slow roll and Brownian motion

$$\int \sqrt{-g} d^4x \frac{1}{16\pi G_N}\left[R - H^2 V(\varphi) - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right]$$

$$3H(t)\dot{\varphi} = -\frac{6H^2}{\kappa}\sqrt{\gamma}. \quad \varphi\sqrt{\gamma}\kappa = -2\gamma H t, \quad 4\epsilon N^2 \sim N$$

$$m/4N > \epsilon \sim 1/4N > 1/4nN$$

Convex potential

Concave potential

Trans-Planckian problem?

Safe regime?

δN formalism

$$\zeta = \delta N = \frac{H}{\dot{\varphi}}\delta\varphi,$$

$$< \delta\varphi(t)\delta\varphi(t') > = (\frac{H^2}{4\pi^2})H\delta(t-t').$$

$$< \zeta^2(t) > =< (\frac{H}{\dot{\varphi}})^2\delta\varphi^2 >$$

$$= \frac{1}{2\epsilon M_P^2} < \delta\varphi^2 > = < \frac{H^2}{8\pi^2\epsilon M_P^2} >$$

$$= \frac{g}{\epsilon} = P$$

Inflation terminates

Inflation commence

Universe is born